Boson description of the spin-orbit interaction in nuclei

P. Van Isacker, GANIL, France

Motivation

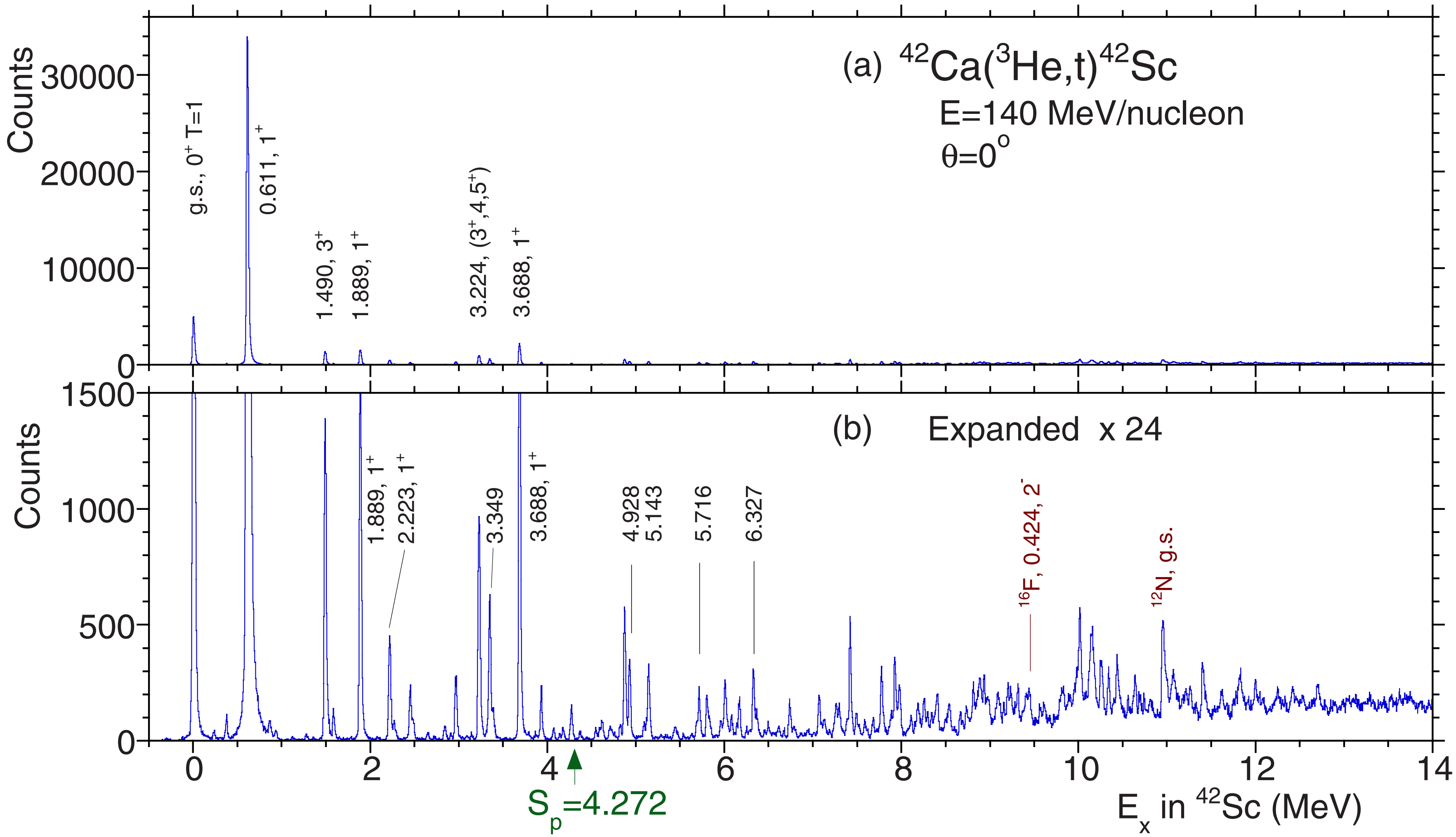
Two-shell fermionic model

Models with spatially unfavoured bosons

Gamow-Teller strength in $N=Z$ nuclei

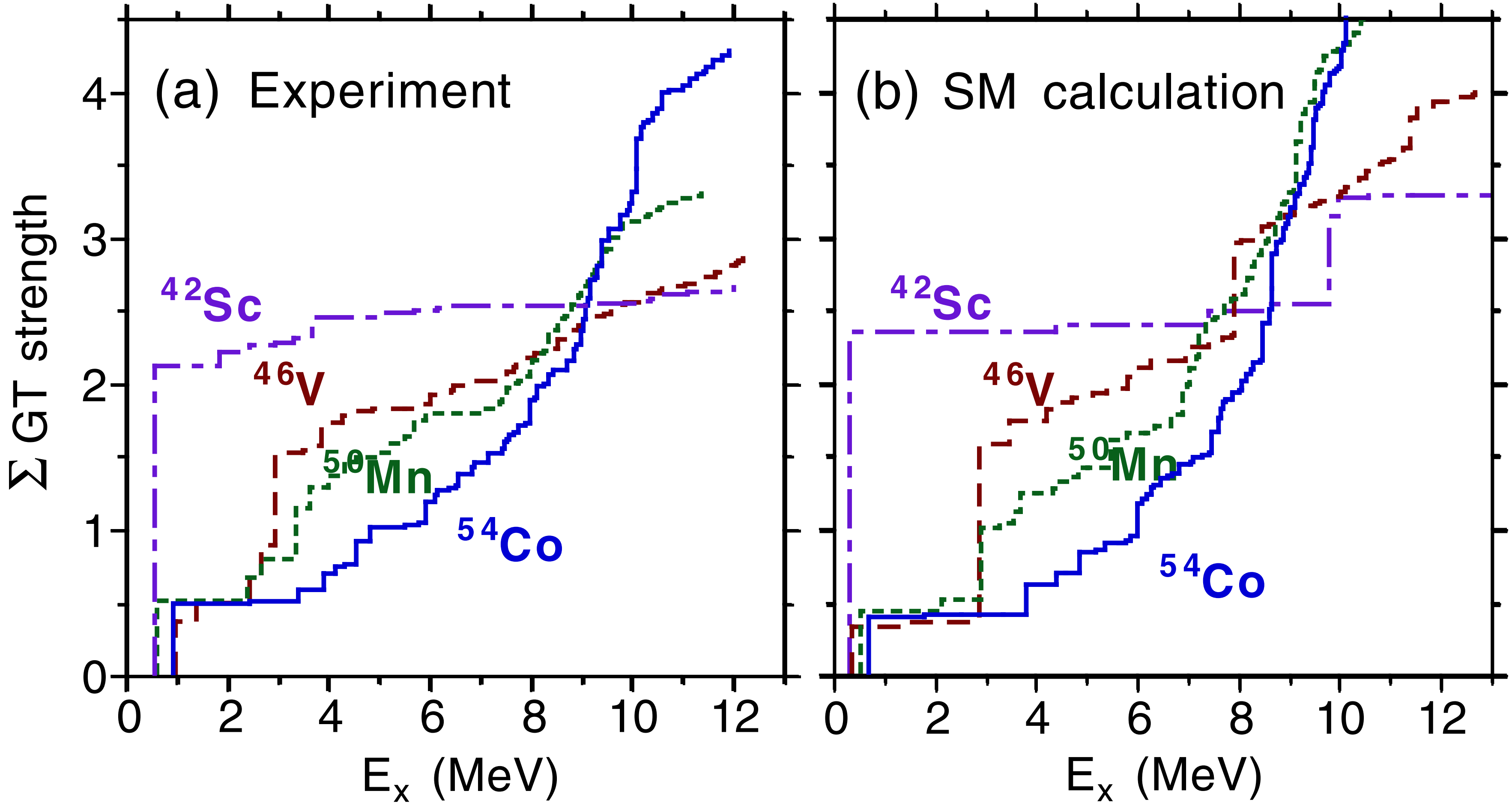
Arima88, Shanghai, September 2018

Charge-exchange reactions



Y. Fujita *et al.*, Phys. Rev. Lett. **112** (2014) 112502

Charge-exchange reactions



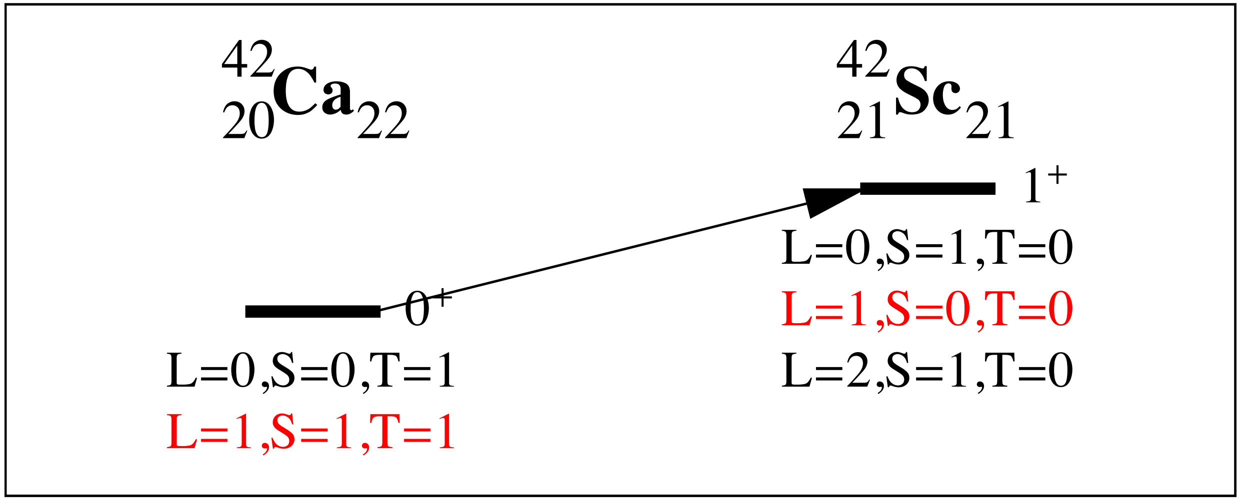
Y. Fujita *et al.*, Phys. Rev. Lett. **112** (2014) 112502

Two-shell model for A=42

Consider two j shells with $j=l\pm\frac{1}{2}$.

In LS coupling two-nucleon states $|l^2LSJT\rangle$ have $L+S+T$ = odd.

The initial and final states in ^{42}Ca and ^{42}Sc are admixtures:



Two-shell model for A=42

The spin-orbit interaction mixes favoured and unfavoured $|l^2LSJT\rangle$ states:

$$\hat{H}_{\mathrm{so}} = \varepsilon_- \hat{n}_- + \varepsilon_+ \hat{n}_+ = \Delta\varepsilon \tfrac{1}{2}\left(\hat{n}_- - \hat{n}_+\right) + \bar{\varepsilon}\hat{n}$$

Energy matrices:

$$^{42}\mathrm{Ca}\left(0^+\right): \quad 2\bar{\varepsilon} + \frac{\Delta\varepsilon}{2l+1}\begin{bmatrix} -1 & \sqrt{4l(l+1)} \\ \sqrt{4l(l+1)} & +1 \end{bmatrix}$$

$$^{42}\mathrm{Sc}\left(1^+\right): \quad 2\bar{\varepsilon} + \frac{\Delta\varepsilon}{3(2l+1)}\begin{bmatrix} -3 & \sqrt{12l(l+1)} & 0 \\ \sqrt{12l(l+1)} & -3 & \sqrt{6(2l-1)(2l+3)} \\ 0 & \sqrt{6(2l-1)(2l+3)} & +6 \end{bmatrix}$$

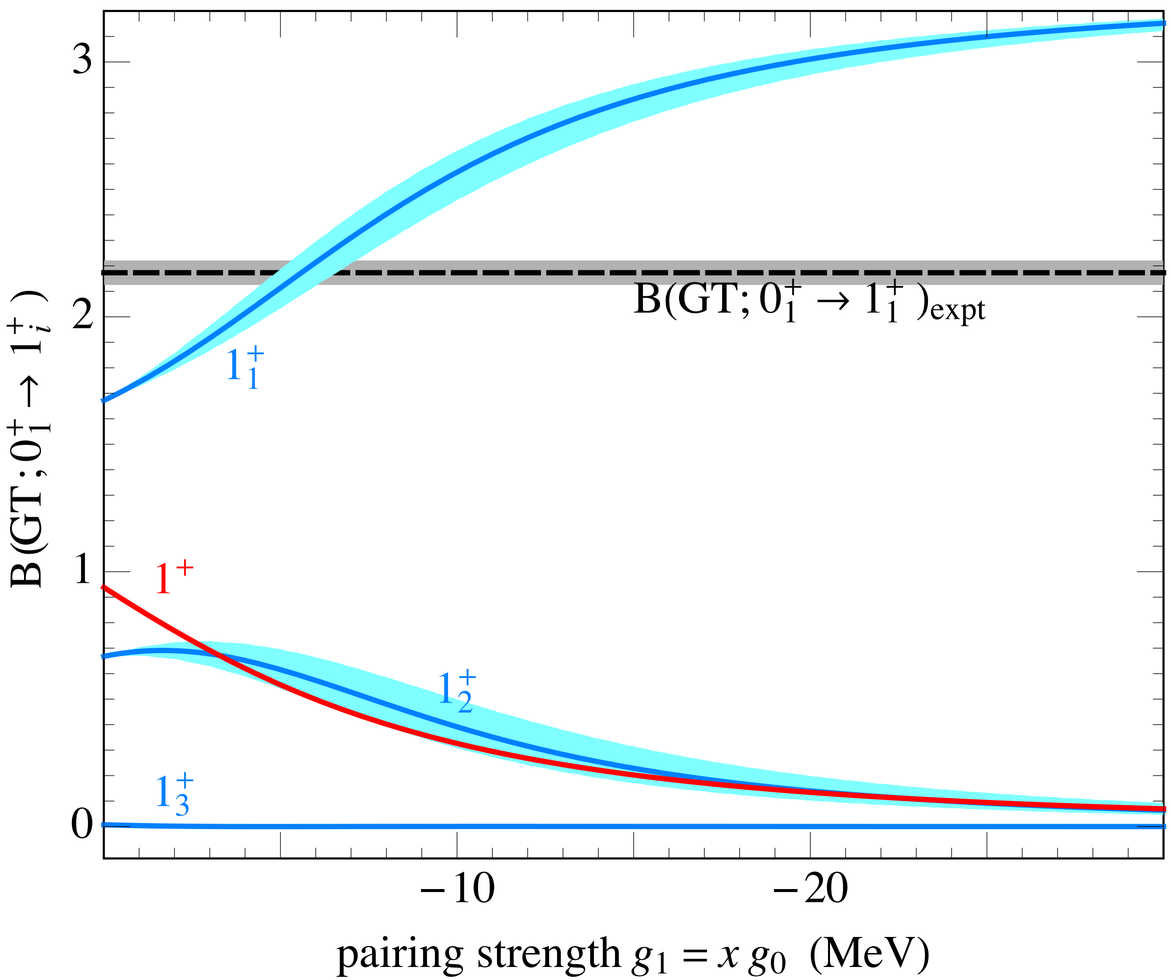
Two-shell model for A=42

A schematic SM hamiltonian with a spin-orbit interaction, isoscalar and isovector pairing, and quadrupole pairing:

$$\hat{H} = \hat{H}_{\mathrm{so}} + g_0 P^{+}_{T=0} P_{T=0} + g_1 P^{+}_{T=1} P_{T=1} + g_2 P^{+}_{2} \cdot P_{2} + \cdots$$

Description of energies, M1 transitions, GT strength and transfer cross sections.

GT strength for ^{42}Ca -> ^{42}Sc



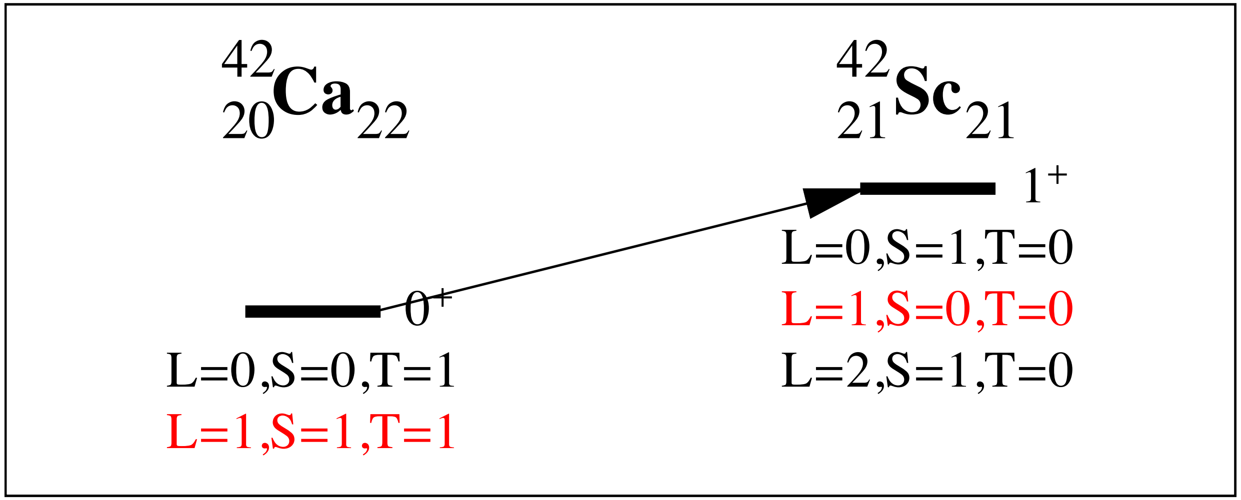
P. Van Isacker & A.O. Macchiavelli, Phys Lett. B 780 (2018) 414

Two-shell model for A=42

Consider two j shells with $j=l\pm½$.

In LS coupling two-nucleon states $|l^2LSJT\rangle$ have $L+S+T$ = odd.

The initial and final states in ^{42}Ca and ^{42}Sc are admixtures:

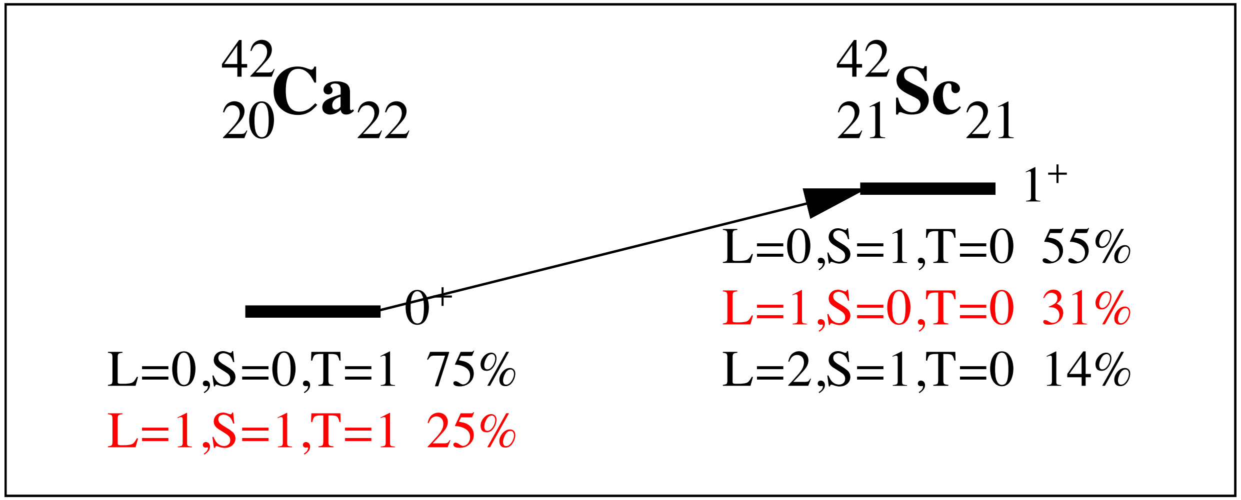


Two-shell model for A=42

Consider two j shells with $j=l\pm½$.

In LS coupling two-nucleon states $|l^2LSJT\rangle$ have $L+S+T$ = odd.

The initial and final states in ^{42}Ca and ^{42}Sc are admixtures:



Boson mapping

Associate pairs of fermions with bosons:

$$\left(a^{+}_{l\frac{1}{2}\frac{1}{2}} \times a^{+}_{l\frac{1}{2}\frac{1}{2}}\right)^{(LST)}_{M_L M_S M_T} \mapsto b^{+}_{LM_L SM_S TM_T} \equiv b^{+}_{\ell m_\ell s m_s t m_t}$$

Remember: *L+S+T* must be odd!

T. Otsuka et al. (OAI), Nucl. Phys. A 309 (1978) 1

Favoured-boson classification

Many-boson states are classified by

$$\mathrm{U}\left[6(2\ell+1)\right]\supset\left(\mathrm{SU}_{ST}(6)\supset\mathrm{SU}_{ST}(4)\supset\mathrm{SU}_{S}(2)\otimes\mathrm{SU}_{T}(2)\right)$$
$$\otimes\left(\mathrm{U}_{L}(2\ell+1)\supset\cdots\supset\mathrm{SO}_{L}(3)\right)$$

This classification is at the basis of Elliott & Evans' IBM-4 with spatially favoured bosons with l=0,2.

J.P. Elliott and J.A. Evans, Phys. Lett. B **101** (1981) 216

Unfavoured-boson classification

Many-boson states are classified by

$$\mathrm{U}\left[10(2\ell+1)\right] \supset \left(\mathrm{SU}_{ST}(10) \supset \mathrm{SU}_{ST}(4) \supset \mathrm{SU}_{S}(2) \otimes \mathrm{SU}_{T}(2)\right)$$
$$\otimes\left(\mathrm{U}_{L}(2\ell+1) \supset \cdots \supset \mathrm{SO}_{L}(3)\right)$$

This classification is new.

P. Van Isacker, unpublished

Generic algebraic structure

The most general IBM classification reads

$$\begin{aligned}\mathrm{U}(6\Lambda_\mathrm{e}+10\Lambda_\mathrm{o}) &\supset \mathrm{U}(6\Lambda_\mathrm{e})\otimes\mathrm{U}(10\Lambda_\mathrm{o})\supset\\ &\supset\left(\mathrm{SU}^\mathrm{e}_{ST}(6)\otimes\mathrm{SU}^\mathrm{o}_{ST}(10)\supset\mathrm{SU}_{ST}(4)\right)\\ &\otimes\left(\mathrm{U}_L(\Lambda_\mathrm{e})\otimes\mathrm{U}_L(\Lambda_\mathrm{o})\supset\cdots\supset\mathrm{SO}_L(3)\right)\end{aligned}$$

Bosons needed for states involved in the charge-exchange reaction:

L=0,2 and L=1 ➔ Λ_e=1+5=6 and Λ_o=3 ➔ U(66).

P. Van Isacker, unpublished

Summary of the remaining slides

Map realistic shell-model hamiltonian (KB3G) and GT operator for A=42 and 44.

Mapping of

bare SM operators -> IBMb

effective SM operators (OLS) -> IBMe

Use mapped operators to calculate GT strength for A=46, 50 and 54.

Study the importance of the pairs included in the mapping.

Nucleon-pair shell model (NPSM)

Pairs of fermions

$$P^{+}_{j_1 j_2 JM} = \left(a^{+}_{j_1} \times a^{+}_{j_2}\right)^{(J)}_{M} \equiv P^{+}_{\alpha JM}$$

Basis states for 2*n* nucleons in NPSM

$$\left|\alpha_1 J_1 \ldots \alpha_n J_n; L_2 \ldots L_n\right\rangle \equiv \left(\cdots\left(\left(P^{+}_{\alpha_1 J_1} \times P^{+}_{\alpha_2 J_2}\right)^{(L_2)} \times P^{+}_{\alpha_3 J_3}\right)^{(L_3)} \times \cdots \times P^{+}_{\alpha_n J_n}\right)^{(L_n)} \left|\mathrm{O}\right\rangle$$

LST formulation: *J* -> *LST*.

Matrix elements in NPSM can be calculated with a recursive technique.

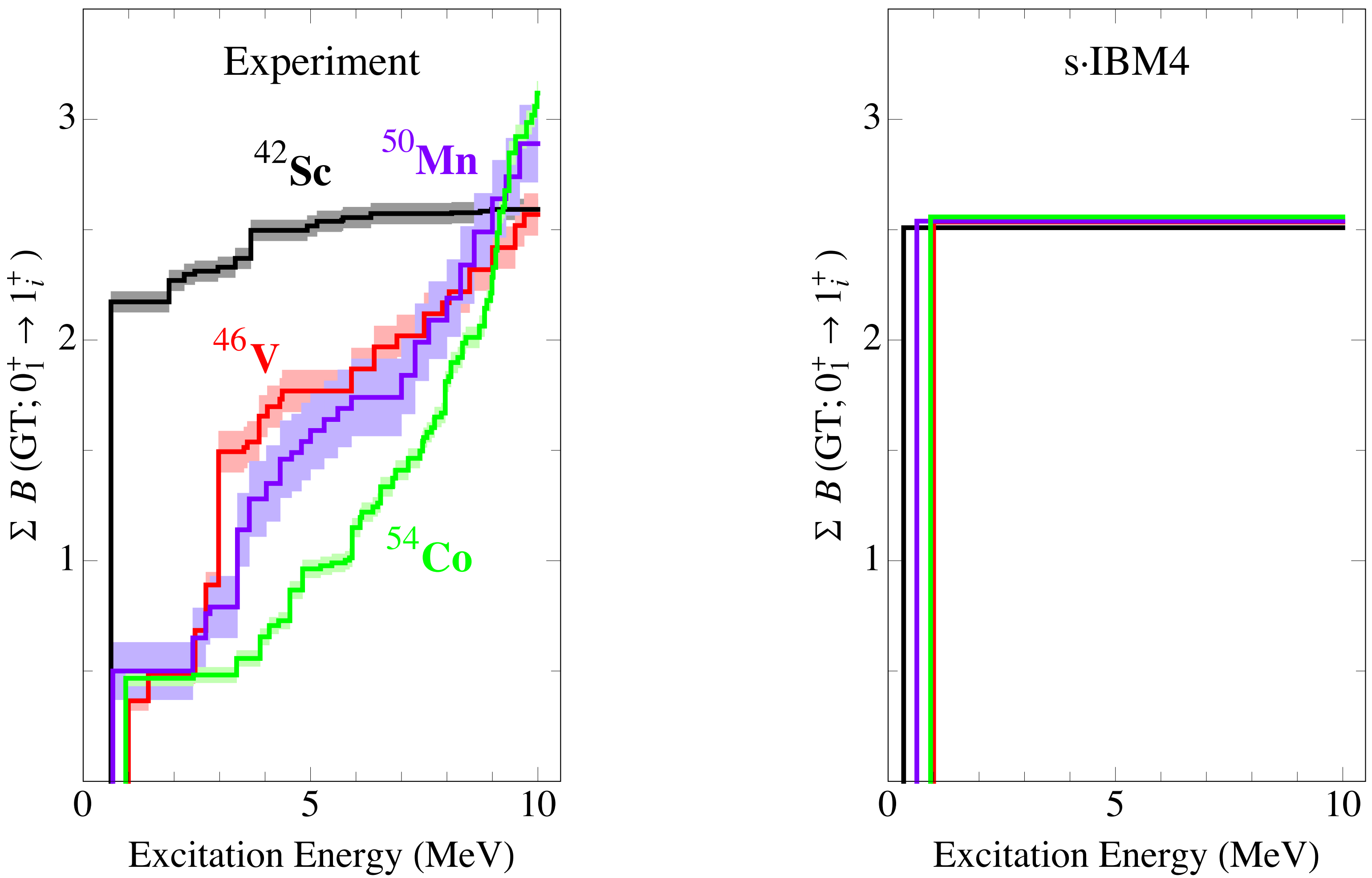
Use of this overcomplete & non-orthogonal basis requires diagonalization of overlap matrix.

J.-Q. Chen, Nucl. Phys. A **562** (1993) 218; **626** (1997) 686
Y. Zhao and A. Arima, Phys. Reports **545** (2014) 1
G.J. Fu *et al.*, Phys. Rev. C **87** (2013) 044310

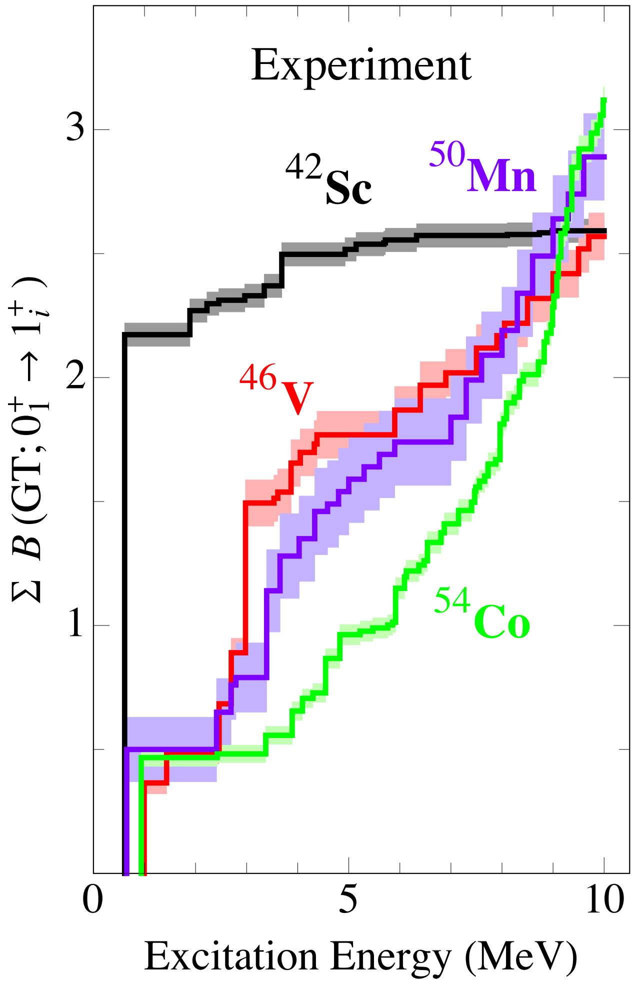
Mapping with different bosons

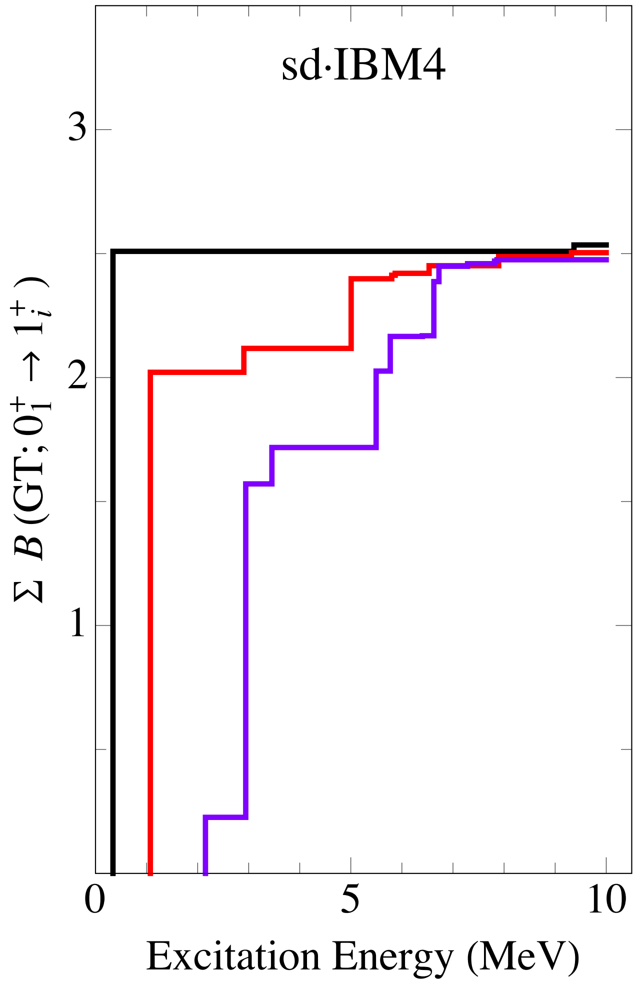
1. Pairs with L=0 (s) ➔ S=0,T=1 & S=1,T=0.
2. Pairs with L=2 (d) ➔ S=0,T=1 & S=1,T=0.
3. Pairs with L=1 (p) ➔ S=0,T=0 & S=1,T=1. These are spatially unfavoured pairs.

IBM-4 with *s* pairs

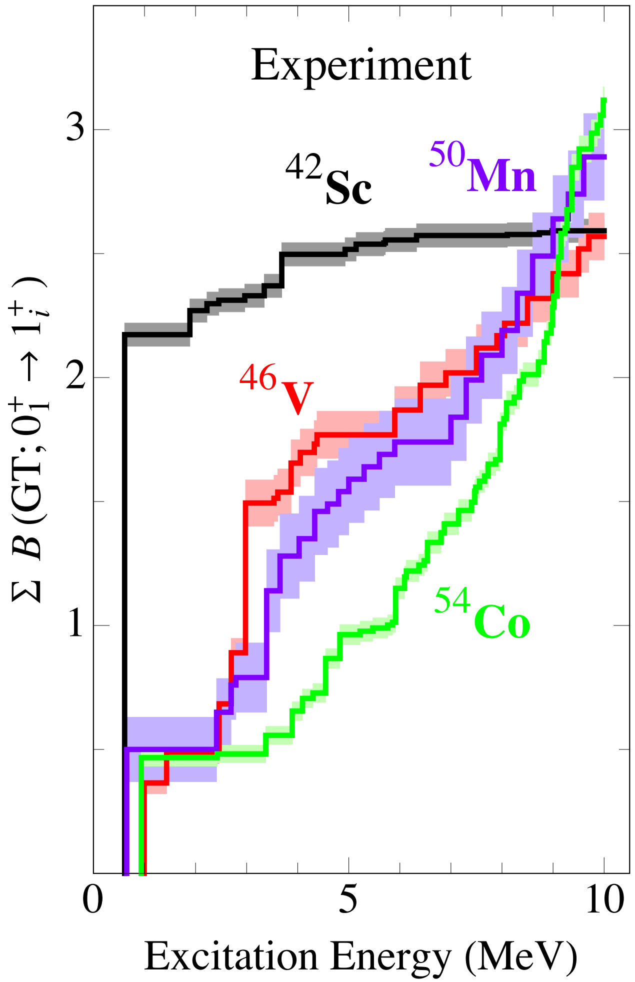


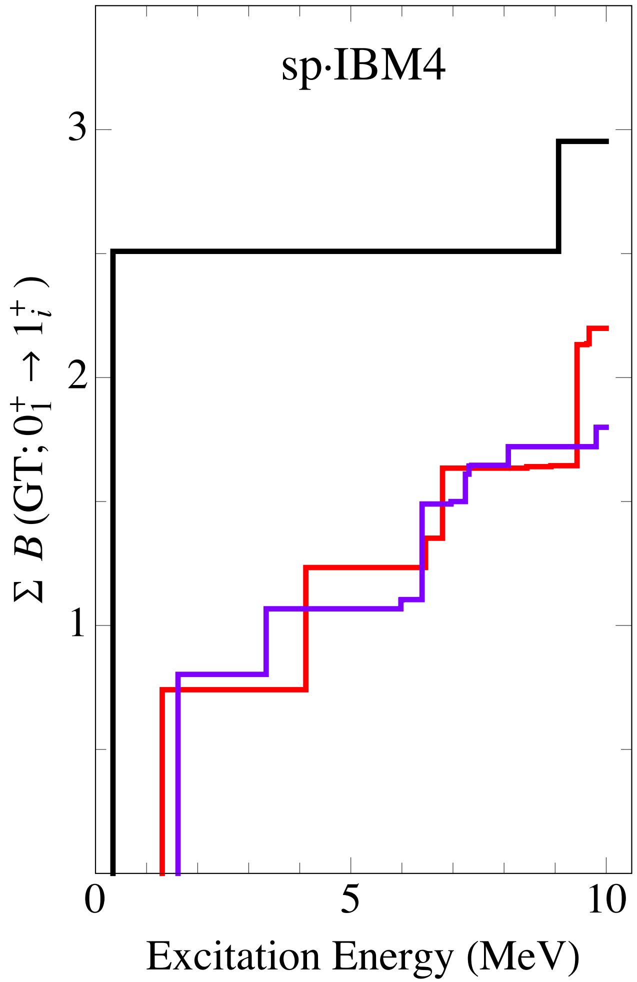
IBM-4 with *sd* pairs



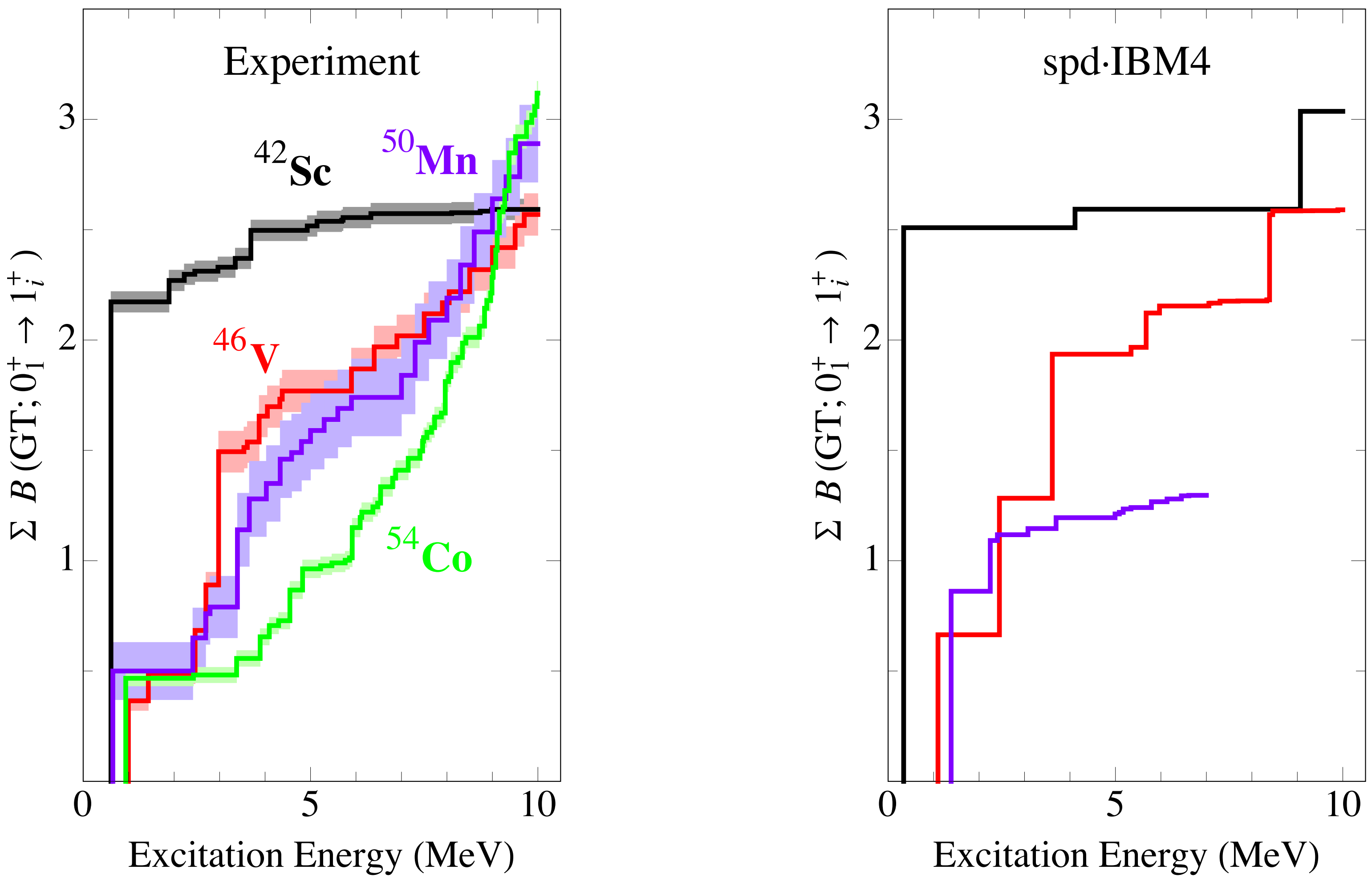


IBM-4 with *sp* pairs





IBM-4 with *spd* pairs



Conclusions

The spin-orbit interaction mixes spatially favoured (L even) and unfavoured (L odd) pairs.

Up to now only spatially favoured pairs have been mapped onto bosons (IBM-4).

A general formulation of the IBM, which includes unfavoured bosons, is possible.

Unfavoured bosons are at the basis of the suppression of GT strength in N=Z nuclei.