Dirac Brueckner Hartree-Fock Approach
From Nuclear matter to Finite Nuclei

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Happyness and Long Live

恭贺 Prof. Arima 八十八米寿诞！

福如东海

寿比南山
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Introduction

• One of the main goals of the Nuclear Physics
  Study the nuclear structure and reactions from
the fundamental interactions.

  **Great success**: study NN Ints. as well as the
system with a few nucleons in EFT, CPT, Lattice
QCD etc.

• An important first step in describing the dynamics
  of a multi-nucleon system is to start the
interaction between just two-nucleons, i.e. the two-
nucleon scattering amplitude.
DBHF approach

- BHF
  To reproduce the saturation properties one has to add 3-body fore.

- Relativistic approaches
  NN + DBHF
  Success in NM saturation properties.
Dirac structure of nucleon self-energies

- DBHF have to be solved consistently
  
  \[ T \rightarrow Us, Uo \rightarrow \text{Dirac eq.} \rightarrow \text{s.p. wf} \rightarrow \text{BS eq.} \]

  \[ T \text{ matrix} \quad \text{do not keep the track of rel. structure} \]

- Two methods:
  
  \[ \begin{align*}
  & \text{the fit method} \\
  & \text{the projection technique}
  \end{align*} \]

- Fit method:

  Extract the nucleon self-energy with single particle potentials

  Assume constant scalar and vector potentials due to the weak momentum dep.

G \rightarrow U_s U_o

- Single particle energy

\[ \varepsilon_i(k) = \sqrt{k^2 + [M + U_s^i(k)]^2} - U_0^i(k) \]

R. Brockmann, R. Machleidt PRC 42(90)1965

Momentum dep. of \( U_s \) & \( U_0 \) are neglected

\[ \varepsilon_i(k) = \sqrt{k^2 + [M + U_s^i]^2} - U_0^i \]

- Works somewhat reliable in SNM, fails to determine the correct behavior of the isospin dependence of the nucleon self-energies

inconsistent results in ASNM

wrong sign of the isospin dependence
Projection method

- Projection method accurate, but more involved

\[ T \rightarrow \{1, \gamma^\mu, \sigma^{\mu\nu}, \gamma^5 \gamma^\mu, \gamma^5\} \rightarrow U_s, U_0 \]

F. Boersma, R. Malfliet,
PRC 49(94)233

Ambiguity results are obtained
for \( \pi \) with PS and PV

Strong momentum dependence is mainly due to one pion exchange

Shiller, Muether, EPJ. A11(2001)15
Subtracted T matrix scheme

- Subtracted T-matrix representation (STM)

\[ T = T_{sub} + V_{\pi,\eta} \]

The ps representation for the \( T_{sub} \) to get the most favorable representation

\[ T_{sub} = F_S S + F_V V + F_T T + F_A A + F_P P \]

Real and Imaginary \( \Sigma^t_s(k, k_F, \beta) \)
Real and Imaginary \( \Sigma^t_0(k, k_F, \beta) \)
Real and Imaginary \( \Sigma^t_v(k, k_F, \beta) \)

Nucleon self-energies

$k_F = 1.34 \text{fm}^{-1}$
The optical model is one of the essential tools of exploring the physics of nuclear reactions. The optical model potential (OMP) of a nucleon in the nuclear medium corresponds to the nucleon self-energy. The nucleon self-energy in the nuclear medium in the DBHF approach is complex, dependent on density, energy and isospin.
Local density approximation for finite nuclei

the r dep. at energy $E$ directly related to $\rho$, $\beta$ in nuclear matter by

$$U_{LDA}(r, E) = U_{NM}(k, \rho(r), \beta)$$

Improved LDA:

the finite range correction with Gaussian form to make modifications in the potentials via LDA approach

$$U_{ILDA}(r, E) = (t\sqrt{\pi})^{-3} \times \int U_{LDA}(r', E)exp(-|\vec{r} - \vec{r}'|^2/t^2)d^3r'$$

$t$ is the parameter that represents the effective range of int.

$t = 1.4$ fm for all cases: nuclei, energies, densities, asymmetry

RR Xu, ZY MA, E.N.E van Dalen, H. Muether. PRC85 (2012) 034613
Improved local density approximation

\[ U_{LDA}(r, E) = U_{NM}(k, \rho(r), \beta), \]

\[ U_{ILDA}(r, E) = (t\sqrt{\pi})^{-3} \times \int U_{LDA}(r', E) \exp\left(-|\vec{r} - \vec{r}'|^2/t^2\right) d^3r', \]

- Volume integral is conserved
- Smooth distribution at the surface
Extrapolation to low densities

Constrains:

- The scalar and vector potentials vanish at $\rho=0$
- Smooth connected by polynomial fits
- Guided by the p,n scattering data for a few magic nuclei, C, O, Pb

low density extrapolation and $t$ values
Systematic studies

- Systematic study of N+A scatterings with CTOM

\[ n, \ p + ^{12}C - ^{208}Pb \ \text{at} \ \ 0.1\text{MeV} \leq E \leq 200\text{MeV} \]

compared with all available data

\[ \frac{d\sigma_{el}}{d\Omega} \quad \sigma_t \quad A_y \quad Q \]

At low energies the elastic scatterings are contributed from both shape and compound elastic scattering processes.

To compare with the experimental data the Hauser-Feshbach model is used to determine the contribution from the compound nucleus

- Results are compared with the phenomenological OMP (Koning)
Compound elastic scatterings are added at low energies

RR Xu, ZY MA, E.N.E van Dalen, H. Muether. PRC85 (2012) 034613
Neutron elastic scatterings

Compound elastic scatterings are added at low energies

\[ \sigma(\text{mb/sr}) \]

\[ \text{Angular (degree)} \]

Analyzing power

\[ ^{208}\text{Pb}(p,p)^{208}\text{Pb} \]

\[
\begin{array}{c}
\text{Energy (MeV)} \\
12 \quad 16 \quad 26.3 \quad 30.3 \quad 40 \quad 49.4 \quad 65
\end{array}
\]

\[ A_y(\theta) \text{ (mb/sr)} \]

\[
\begin{array}{c}
\theta (\text{deg.}) \\
0 \quad 40 \quad 80 \quad 120 \quad 160
\end{array}
\]
Spin rotation functions

![Graphs showing spin rotation functions for different elements at Ep=65MeV and Ep=200MeV.](image)
Neutron total cross sections

offset = 1.5
Isospin dependence of OMP with CTOM could well describe the nucleon-A scatterings, comparable to the KD pots.

http://www.nuclear.csdb.cn/ctom for testifying
Structure of Finite nuclei

• ab initio investigations:

A full BHF and RBHF for finite nuclei starting with a bare NN interactions have been performed recently

PHYSICAL REVIEW C 95, 034321 (2017)

Brueckner-Hartree-Fock calculations for finite nuclei with renormalized realistic forces

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One can adopt two-step $G$-matrix approximations for the Brueckner-Hartree-Fock (BHF) calculations. The first $G$ matrix is to soften the bare force, and the second one is to include the high-order correlations of the interaction in medium. The first $G$-matrix calculation for two-nucleon interaction should be done in the center-of-mass coordinate. As another alternative BHF approach, we have adopted the $V_{\text{low-}k}$ technique to soften the interaction and used the $G$ matrix to include high-order correlations. The $V_{\text{low-}k}$ renormalization leads to high-momentum and low-momentum components of the interaction decoupled. With the $V_{\text{low-}k}$ potential, we have performed the BHF calculations for finite nuclei. The $G$-matrix elements with exact Pauli exclusions are calculated in the self-consistent BHF basis. To see effects from further possible correlations beyond BHF, we have simultaneously performed renormalized BHF (RBHF) calculations with the same potential. In RBHF, the mean field derived from realistic forces is modified by introducing the particle-occupation depletion resulting from many-body correlations. The ground-state energies and radii of the closed-shell nuclei, $^4$He, $^{16}$O, and $^{40}$Ca, have been investigated. The convergences of the BHF and RBHF calculations have been discussed and compared with other ab initio calculations with the same potential.
Relativistic Brueckner–Hartree–Fock Theory for Finite Nuclei *

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(Received 17 September 2016)

Starting with a bare nucleon-nucleon interaction, for the first time the full relativistic Brueckner–Hartree–Fock equations are solved for finite nuclei in a Dirac–Woods–Saxon basis. No free parameters are introduced to calculate the ground-state properties of finite nuclei. The nucleus $^{16}\text{O}$ is investigated as an example. The resulting ground-state properties, such as binding energy and charge radius, are considerably improved as compared with the non-relativistic Brueckner–Hartree–Fock results and much closer to the experimental data. This opens the door for ab initio covariant investigations of heavy nuclei.

Structure of Finite nuclei

- The extension of the DBHF self-energy in LDA to finite nuclei
  Inspired by the success in the RMOP, the similar method is applied to nuclear structure
- Most studies
  Introduce a density dependence of meson coupling constants $g_\sigma(\rho), g_\omega(\rho)$ to reproduce the scalar and vector potentials in nuclear matter in RMF and RHF approaches
  
  $U_s, U_o \rightarrow g_\sigma(\rho), g_\omega(\rho)$

  no proper isospin and momentum dependence
Only density dependence included

RDMF

Brockmann & Toki, PRL68(92)3408
- Include isoscalar $\sigma$ $\omega$
  isovector $\pi$ $\rho$
  $f_\pi$, $g_\rho$, $g_\rho/f_\rho$ values at $m_\pi$, $m_\rho$ free space

- Isovector mesons play an important role in the spin-orbit splitting, isospin dep. Quantities, neutron skin etc.

Shi, Chen, Ma PRC52(95)144
The DBHF approach

- The extension to bound states
  Inspired by the success in the ROMP

- The energy and density dependences of $U_s$ and $U_0$ in DBHF with STM
  Stronger energy dependences for bound states

X.D. Sun, R.R. Xu, Y. Tian, Z.Y. Ma in preparation
Scalar and vector potentials

- The behavior of the potentials cannot be well constrained by the DBHF
- A polynomial fit ($\sqrt{\rho}, \rho, \rho^2$) for the density dependences of $U_s, U_0$
Isospin dependences of pots
The local density approximation (LDA) is adopted to connect the density and isospin dependence of the self-energy to the radius of a finite nucleus.

\[ U_s(r, \varepsilon), U_0(r, \varepsilon) \rightarrow U_s(\rho, \beta, \varepsilon), U_0(\rho, \beta, \varepsilon) \]

The energy dependence of the nucleon self-energy is considered for each single particle level.

A consistent calculation is required for each single particle wave function and each nucleus.
Ground state properties of nuclei

- Preliminary results are obtained without any free parameters

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<th>Exp</th>
<th>Rp (fm)</th>
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</table>

DBHF + LDA

X.D. Sun, R.R. Xu, Y. Tian, Z.Y. Ma in preparation
Ground state properties of nuclei

- Preliminary results are promising
- Further investigation is required

Surface effects are important in finite nuclei, but absent in nuclear matter.
Summary

- DBHF with the subtracted T matrix approach could produce proper nucleon scalar and vector self-energies in the nuclear medium.

- The scalar and vector potentials are density, energy and isospin dependences

- The microscopic ROMP in ILDA CTOM could give a satisfactory description of nucleon scattering off nuclei

- The extension to the nuclear bound states with LDA in DBHF is performed. Results for the nuclear bound states are promising. Further investigation is required.
Thanks