# Dirac Brueckner Hartree-Fock Approach

# From Nuclear matter to Finite Nuclei

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# Happyness and Long Live

## 恭贺 Prof. Arima 八十八米寿诞！

# 福如东海

# 寿比南山

# Contents

- Introduction
- DBHF in Nuclear Matter
- ROMP in DBHF
- Structure of finite nuclei
- Summary

# Introduction

- One of the main goals of the Nuclear Physics

  Study the nuclear structure and reactions from the fundamental interactions.

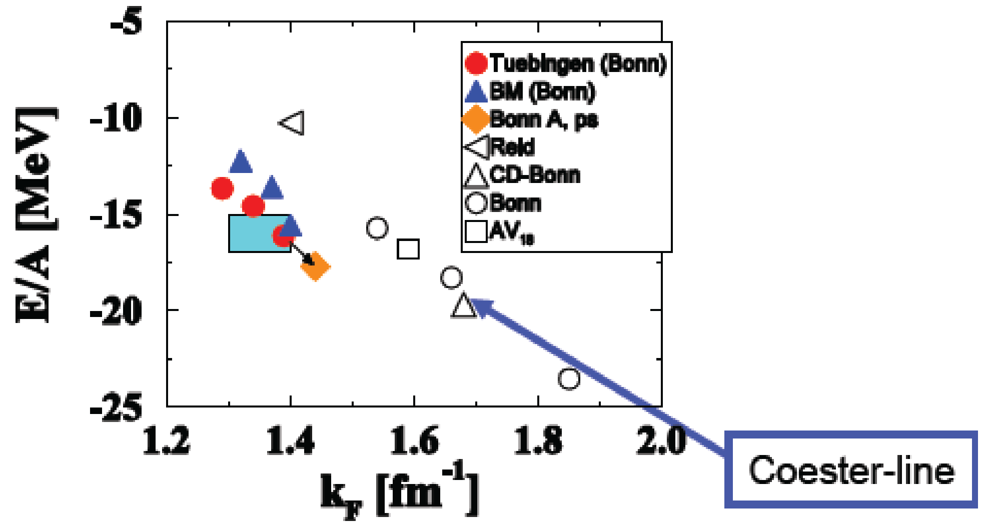
  Great success : study NN Ints. as well as the system with a few nucleons in EFT , CPT , Lattice QCD etc. .

- An important first step in describing the dynamics of a multi-nucleon system is to start the interaction between just two-nucleons, i.e. the two-nucleon scattering amplitude.

# DBHF approach

- BHF

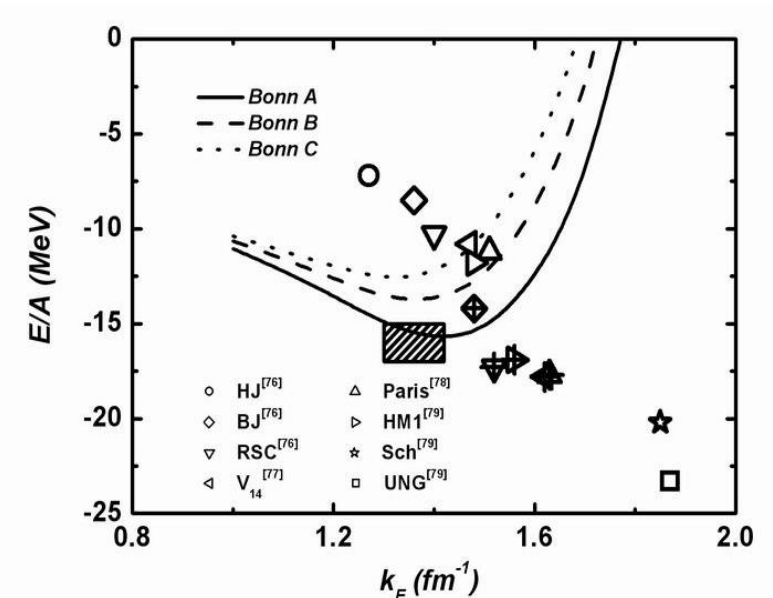
To reproduce the saturation properties one has to add 3-body fore




- Relativistic approaches

NN + DBHF

Success in NM saturation properties



# Dirac structure of nucleon self-energies

- DBHF have to be solved consistently

  $T \xrightarrow{?} U_s, U_o \rightarrow$ Dirac eq. $\rightarrow$ s.p. wf $\rightarrow$ BS eq.

  T matrix --- do not keep the track of rel. structure

- Two methods:

  the fit method

  the projection technique

- Fit method:

  Extract the nucleon self-energy with single particle potentials

  Assume constant scalar and vector potentials due to the weak momentum dep.

R. Brockmann and R. Machleidt, Phys. Rev. C 42 (1990) 1965

# G → $U_s$ $U_o$

- Single particle energy

$$\varepsilon_i(k) = \sqrt{k^2 + [M + U_s^i(k)]^2} - U_0^i(k)$$

R. Brockmann, R. Machleidt PRC 42(90)1965

Momentum dep. of $U_s$ & $U_0$ are neglected

$$\varepsilon_i(k) = \sqrt{k^2 + [M + U_s^i]^2} - U_0^i$$

- Works somewhat reliable in SNM, fails to determine the correct behavior of the isospin dependence of the nucleon self-energies

inconsistent results in ASNM

wrong sign of the isospin dependence

# Projection method

- Projection method accurate, but more involved

$$T \to \left\{1, \gamma^{\mu}, \sigma^{\mu\nu}, \gamma^{5}\gamma^{\mu}, \gamma^{5}\right\} \longrightarrow U_s, U_0$$

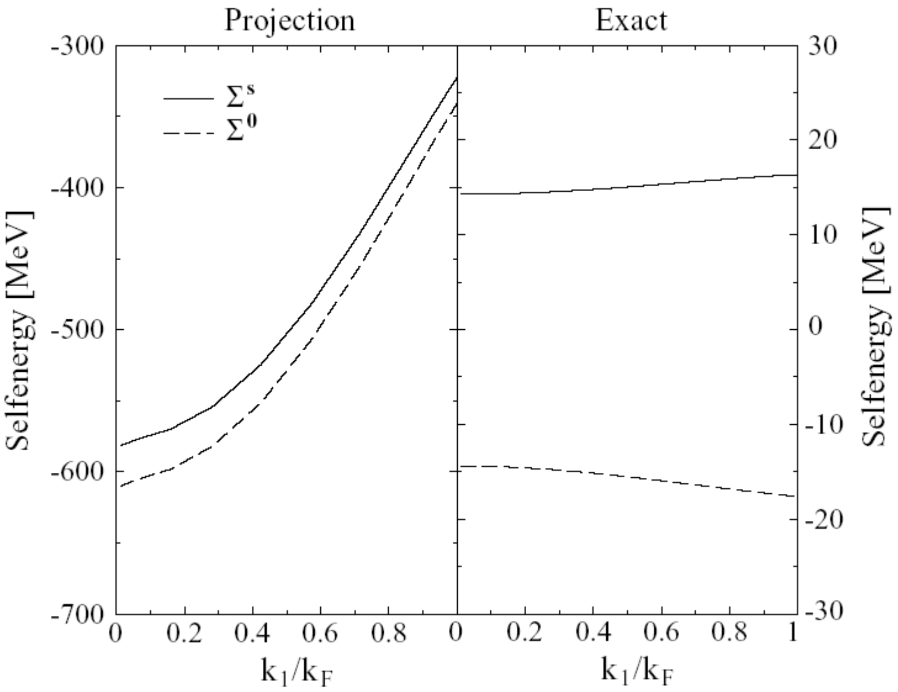
F. Boersma, R. Malfliet,

PRC 49(94)233

Ambiguity results are

obtained

for $\pi$ with PS and PV




Strong momentum dependence
is mainly due to one pion exchange

Shiller,Muether, EPJ. A11(2001)15

# Subtracted T matrix scheme

- Subtracted T-matrix representation (STM)

$$T = T_{sub} + V_{\pi,\eta}$$

The ps representation for the $T_{sub}$ to get the most favorable representation

$$T_{sub} = F_S S + F_V V + F_T T + F_A A + F_P P,$$

→ Real and Imaginary $\Sigma_s^t(k, k_F, \beta)$ $\Sigma_0^t(k, k_F, \beta)$ $\Sigma_v^t(k, k_F, \beta)$

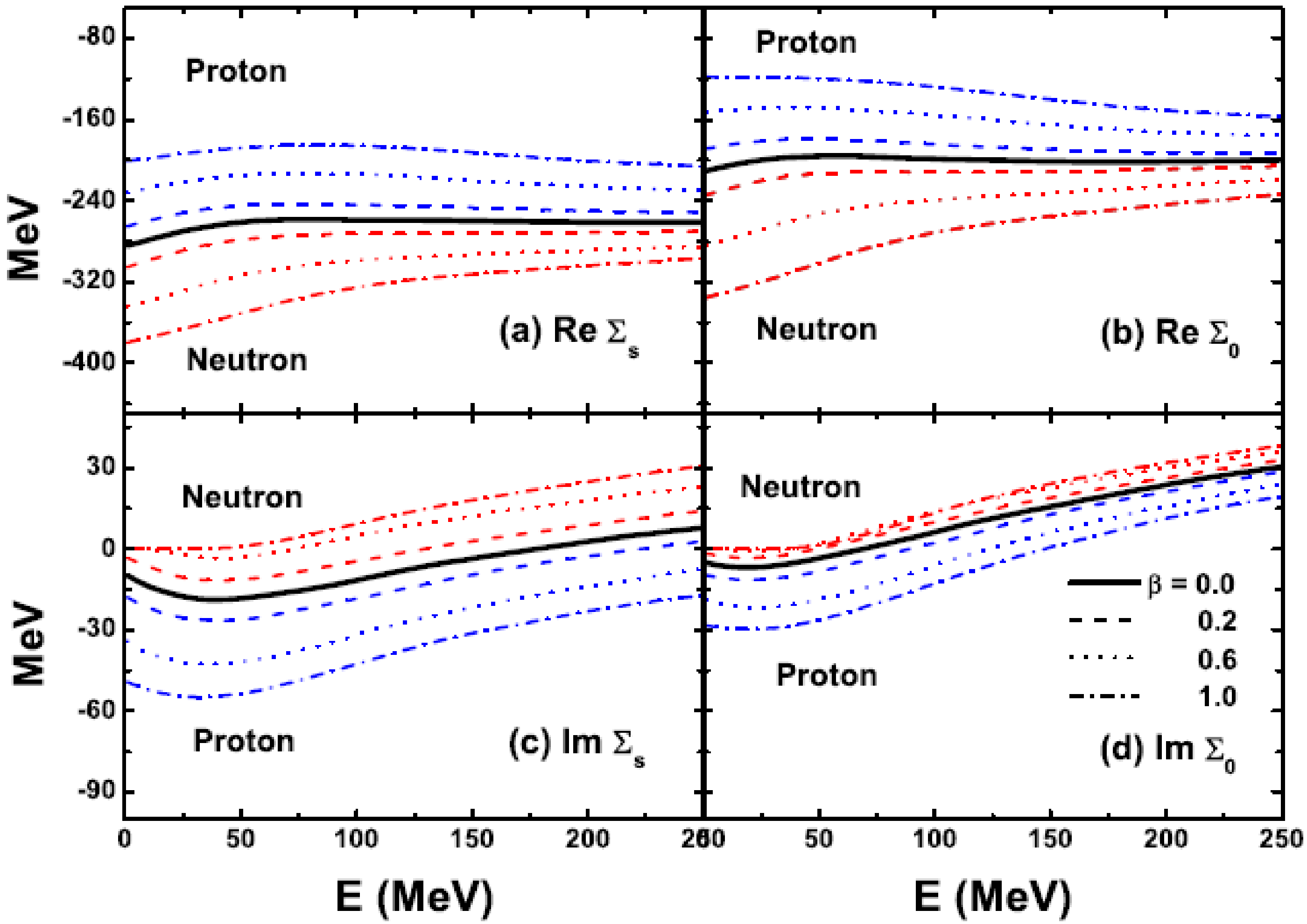
→ Real and Imaginary $U_s(k, k_F, \beta), U_0(k, k_F, \beta)$


E. N. E. van Dalen, C. Fuchs, and A. Faessler, Nucl. Phys. A 744, 227 (2004)

E. N. E. van Dalen, C. Fuchs, and A. Faessler, Eur. Phys. J. A 31, 29 (2007).

# Nucleon self-energies

$k_F$=1.34fm$^{-1}$



# ROMP in DBHF

- The optical model is one of the essential tools of exploring the physics of nuclear reactions.
- The optical model potential (OMP) of a nucleon in the nuclear medium corresponds to the nucleon self-energy
- The nucleon self-energy in the nuclear medium in the DBHF approach is complex, dependent on density, energy and isospin

# ROMP in DBHF

Local density approximation for finite nuclei

the r dep. at energy $E$ directly related to ρ, $\beta$ in nuclear matter by

$$U_{LDA}(r,E) = U_{NM}(k,\rho(r),\beta)$$

Improved LDA:

the finite range correction with Gaussian form to make modifications in the potentials via LDA approach

$$U_{ILDA}(r,E) = (t\sqrt{\pi})^{-3} \times \int U_{LDA}(r',E)exp(-|\vec{r}-\vec{r'}|^2/t^2)d^3r'$$

*t* is the parameter that represents the effective range of int.

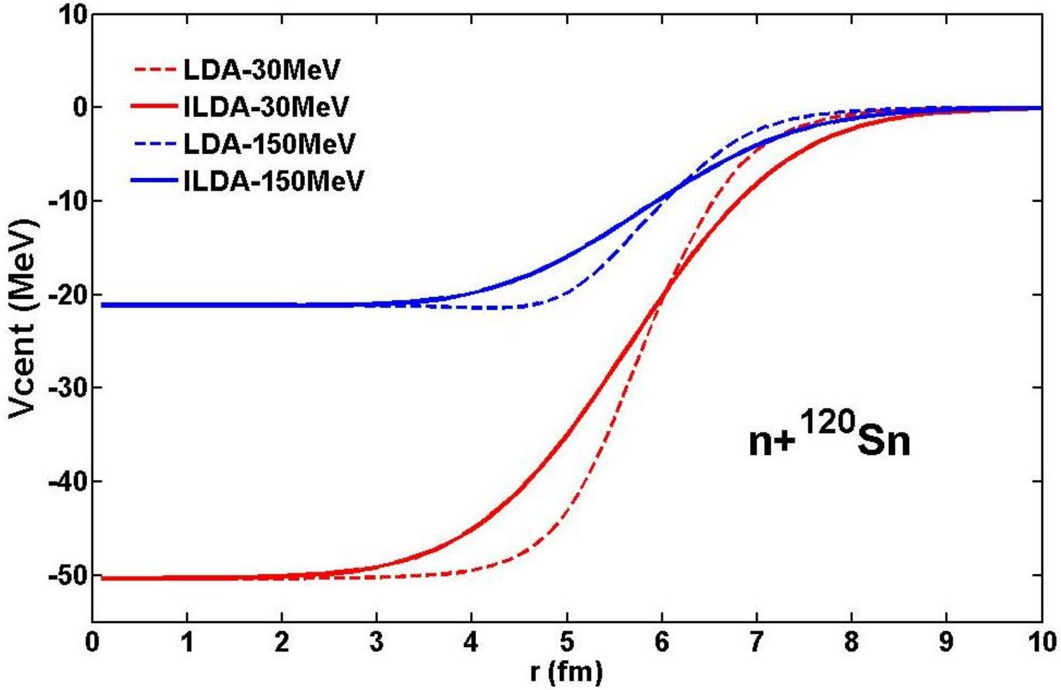
*t* =1.4 fm for all cases: nuclei, energies, densities, asymmetry

*RR Xu, ZY MA, E.N.E van Dalen, H. Muether. PRC85 (2012) 034613*

# Improved local density approximation

$$U_{LDA}(r,E) = U_{NM}(k,\rho(r),\beta),$$

$$U_{ILDA}(r,E) = (t\sqrt{\pi})^{-3} \times \int U_{LDA}(r',E)exp(-|\vec{r}-\vec{r'}|^2/t^2)d^3r' \ ,$$

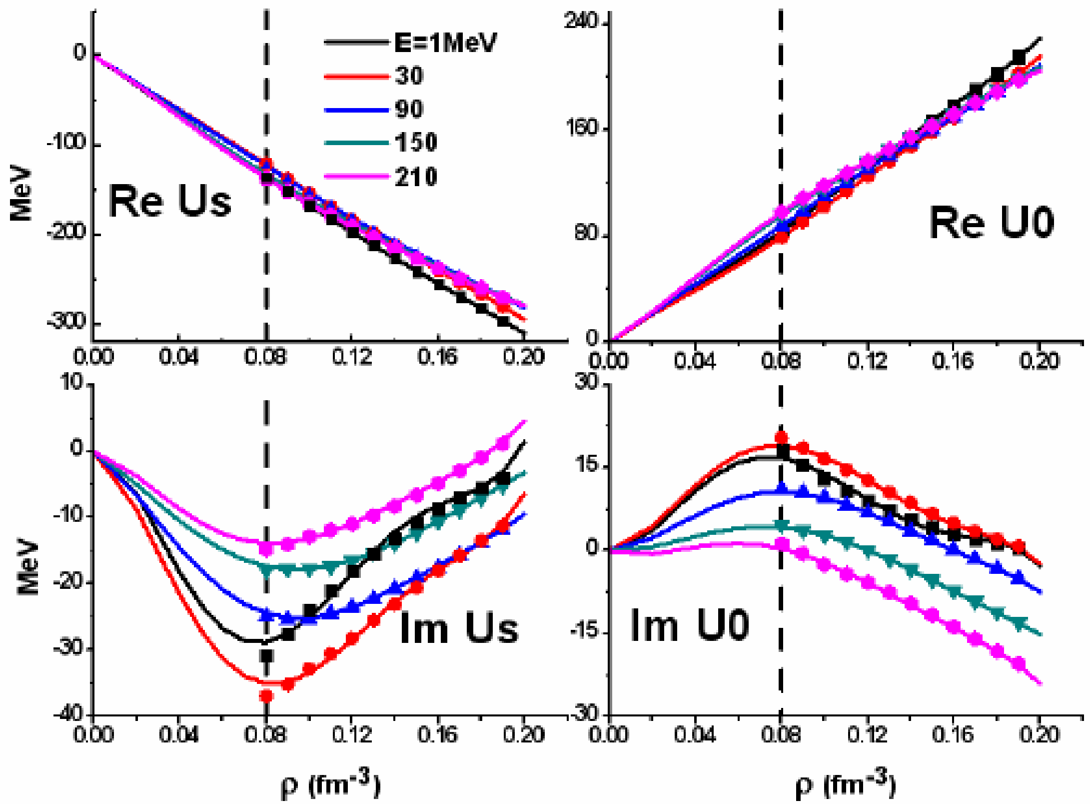



- Volume integral is conserved
- smooth distribution at the surface

# Extrapolation to low densities

Constrains:

- The scalar and vector potentials vanish at $\rho$=0
- Smooth connected by polynomial fits




- Guided by the p,n scattering data for a few magic nuclei, C, O, Pb

low density extrapolation and *t* values CTOM

# Systematic studies

- Systematic study of N+A scatterings with CTOM

n, p + $^{12}C$ - $^{208}Pb$ at $0.1\text{MeV} \le E \le 200\text{MeV}$

compared with all available data

$$\frac{d\sigma_{el}}{d\Omega} \qquad \sigma_t \qquad A_y \qquad Q$$

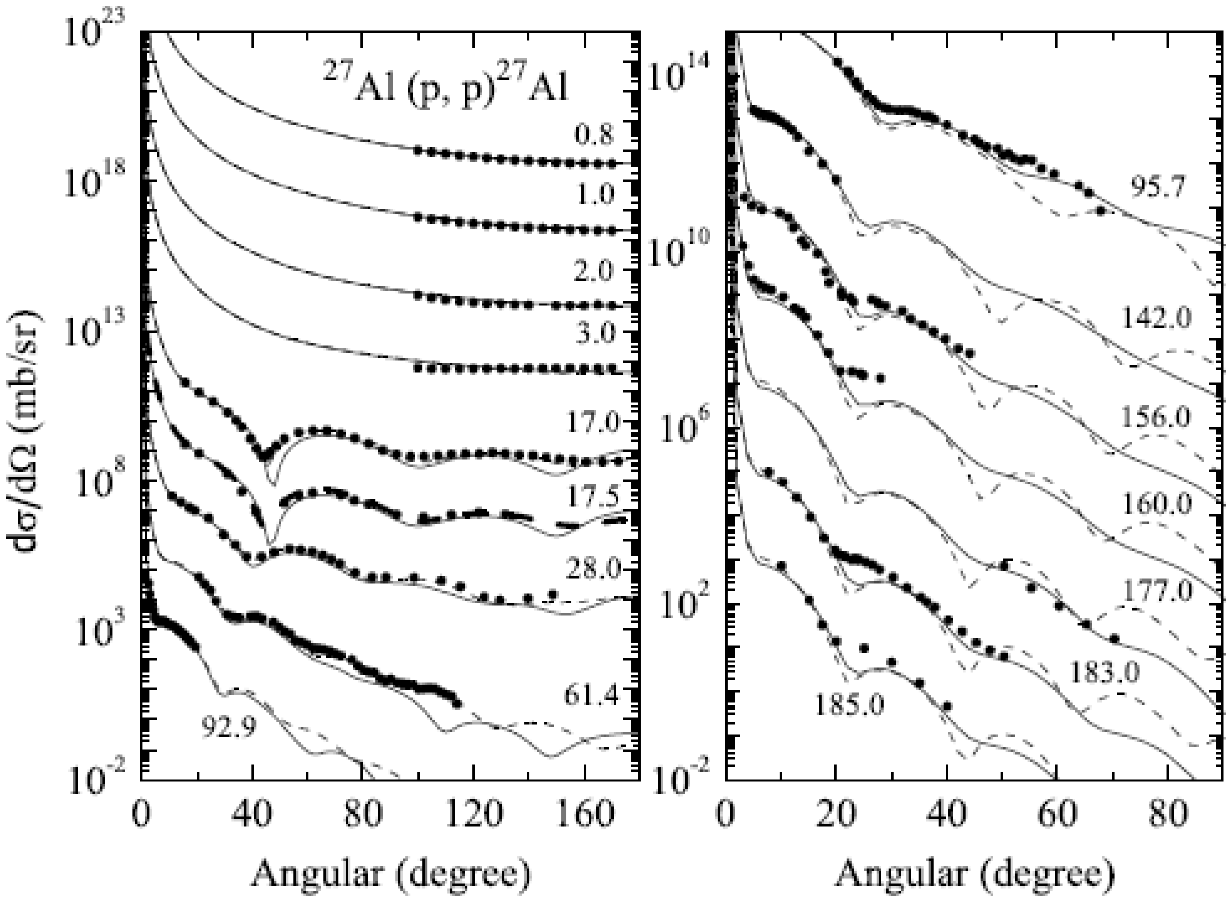
At low energies the elastic scatterings are contributed from both shape and compound elastic scattering processes.

To compare with the experimental data the Hauser-Feshbach model is used to determine the contribution from the compound nucleus

- Results are compared with the phenomenological OMP (Koning)

# Proton elastic scatterings

Compound elastic scatterings are added at low energies




Solid in CTOM
Dashed in KD

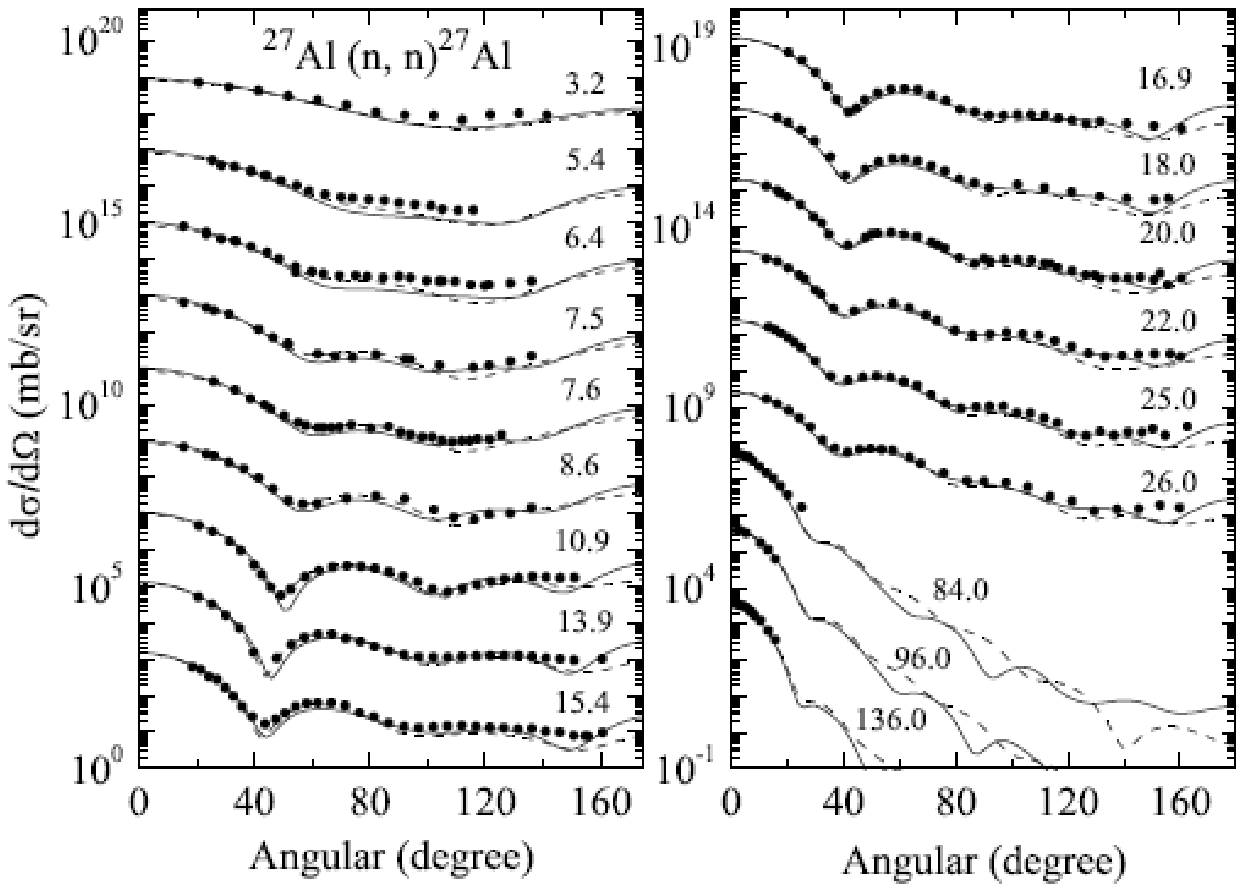
Offset=100

*RR Xu, ZY MA, E.N.E van Dalen, H. Muether. PRC85 (2012) 034613*

A. J. Koning and J. P. Delaroche, Nucl. Phys. A **713**, 231 (2003).

# Neutron elastic scatterings

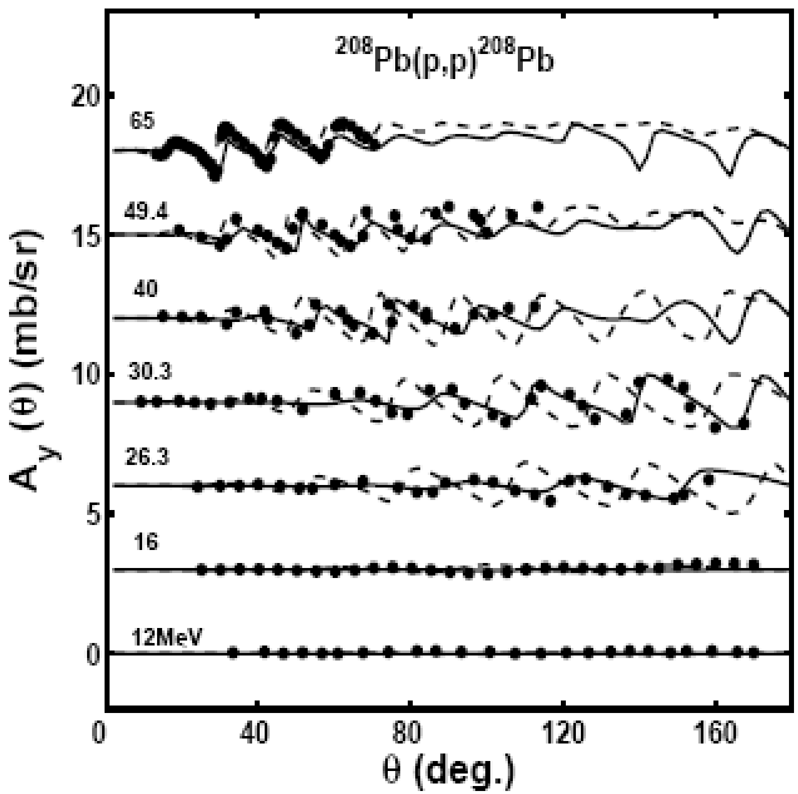
Compound elastic scatterings are added at low energies

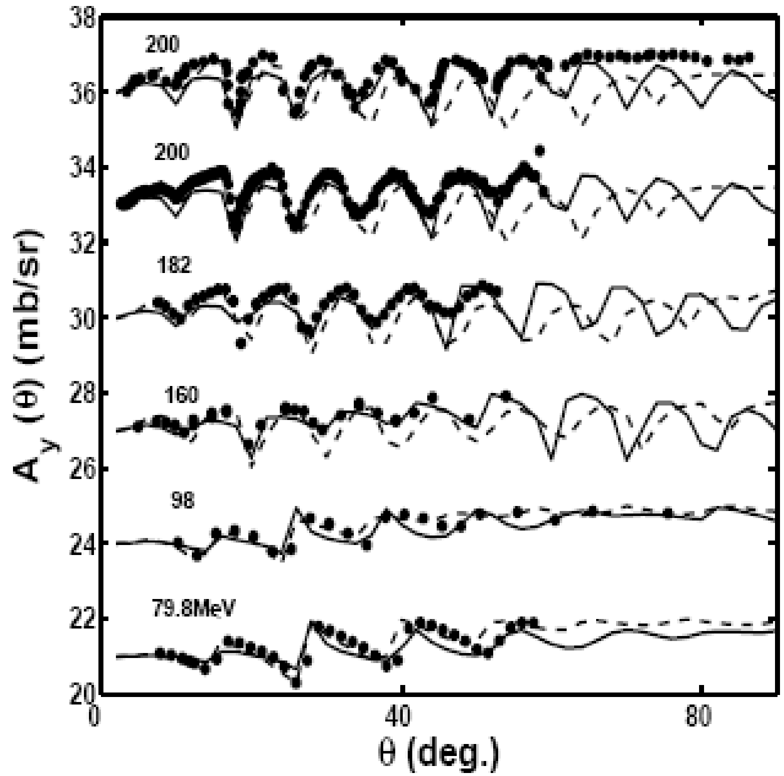



Offset=100

R.R. **Xu**, Z.Y. Ma, E. N. E. van Dalen, H. Müther, Phys. Rev. C 85, 034613, (2012)

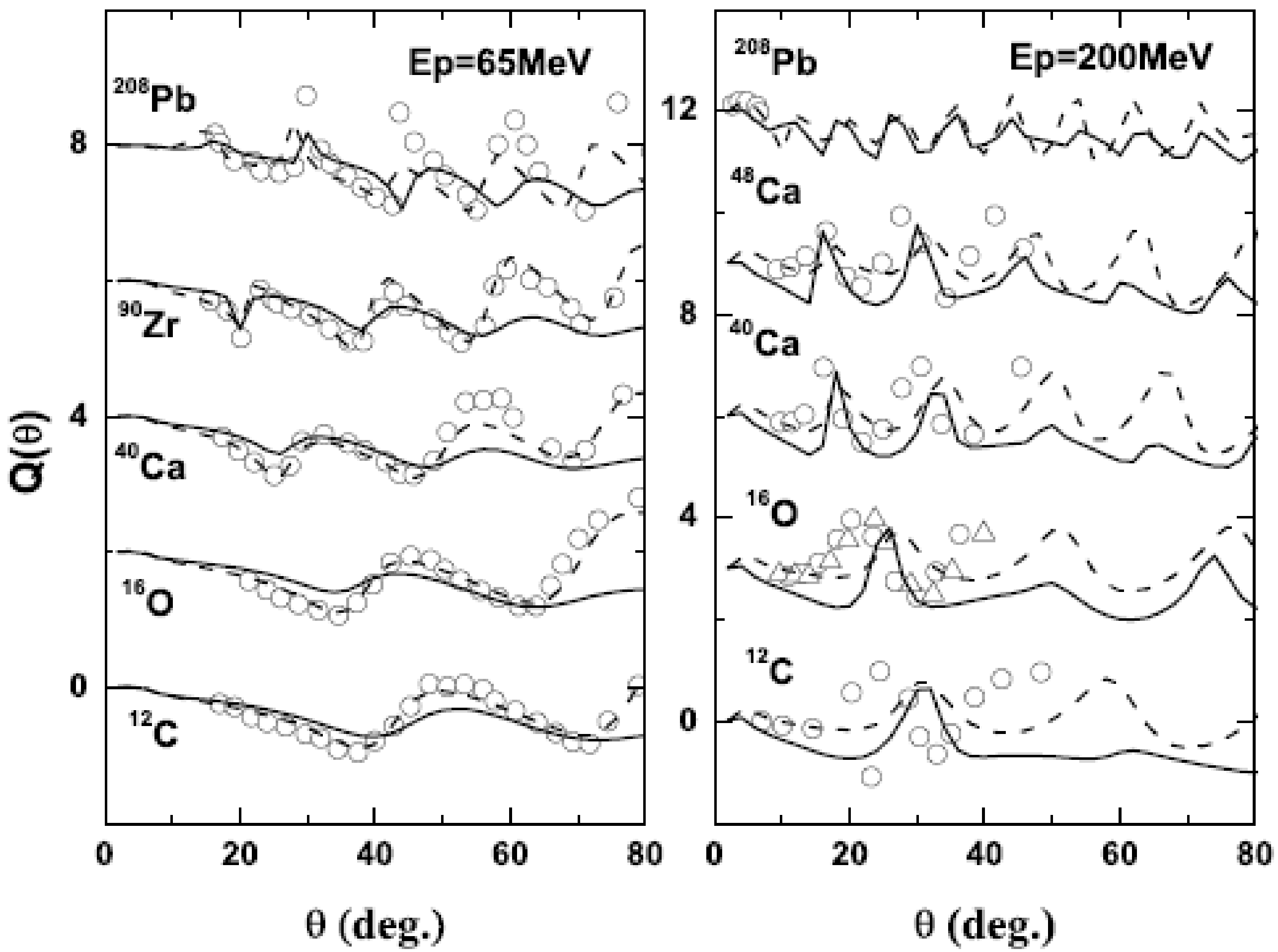
# Analyzing power



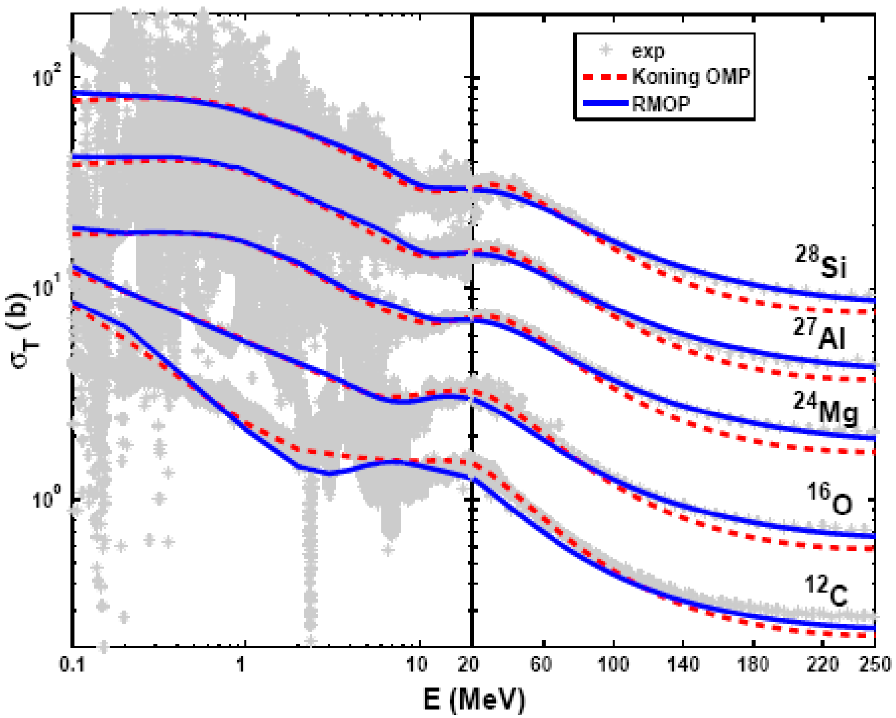




Offset=3

# Spin rotation functions



# Neutron total cross sections

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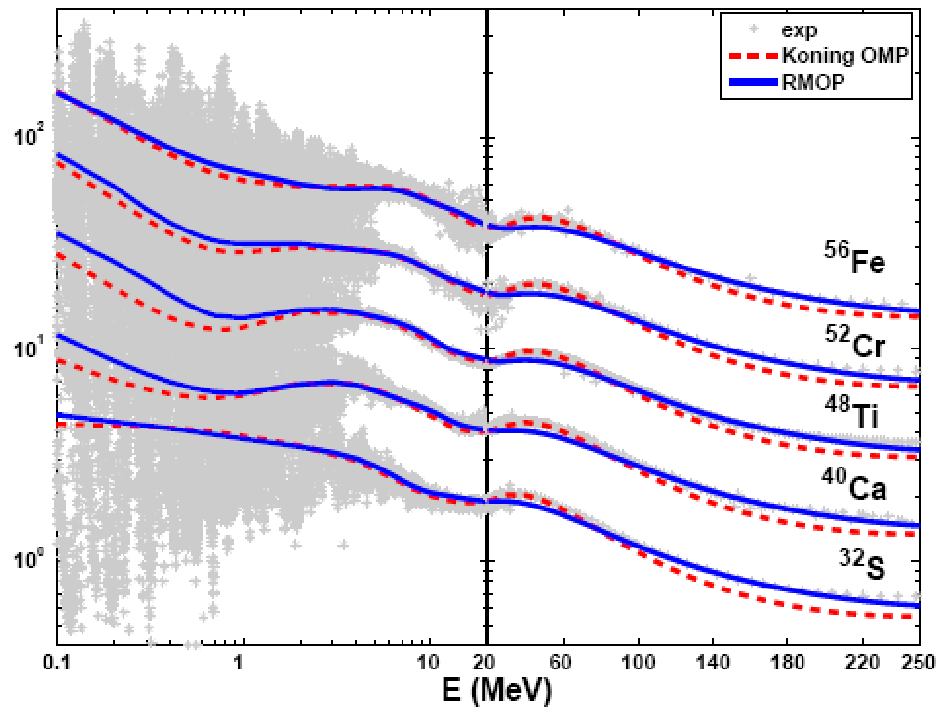
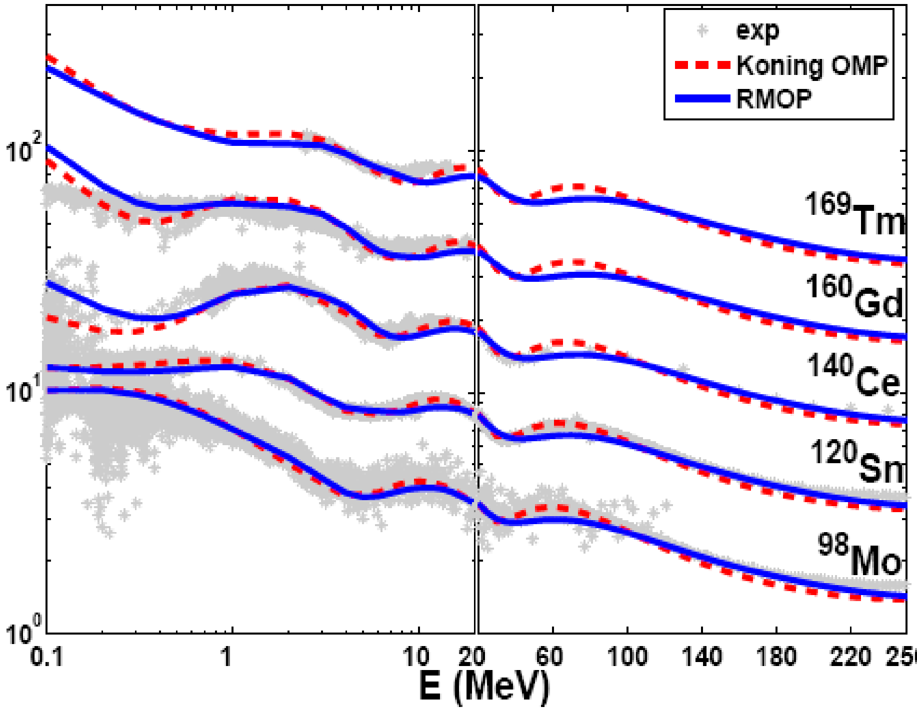


# Neutron total cross sections

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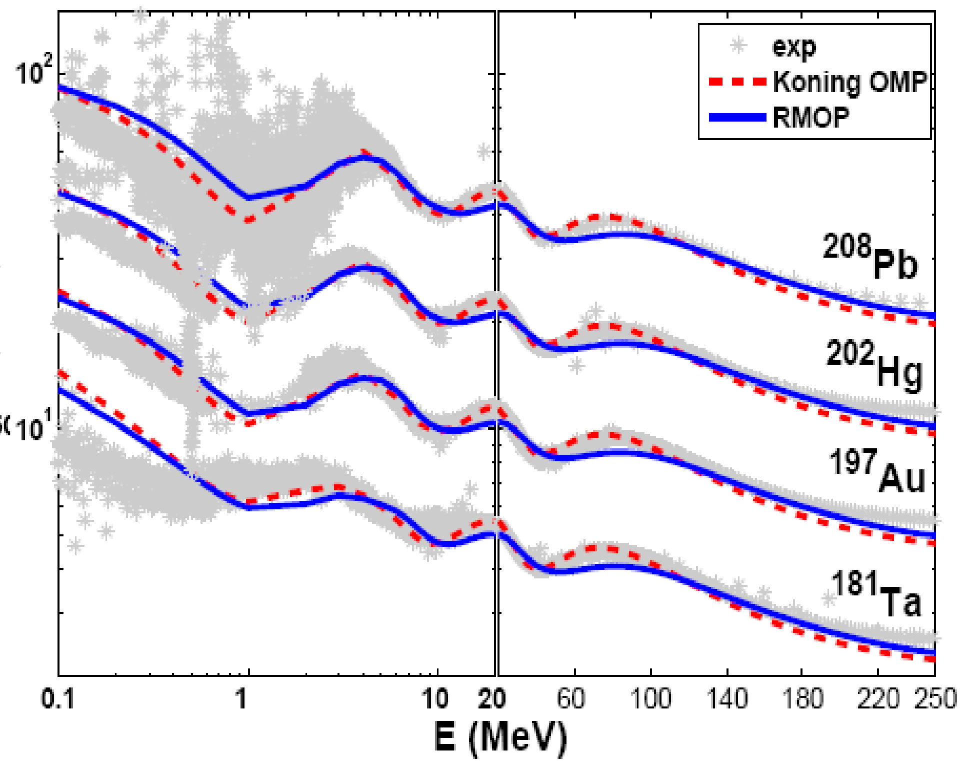
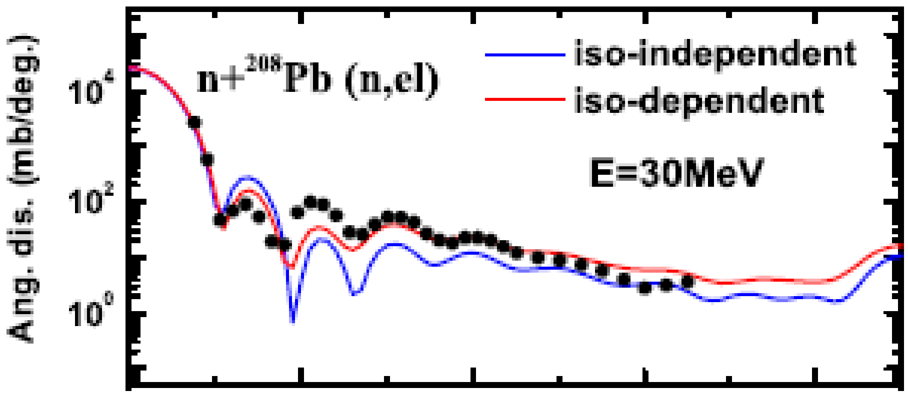
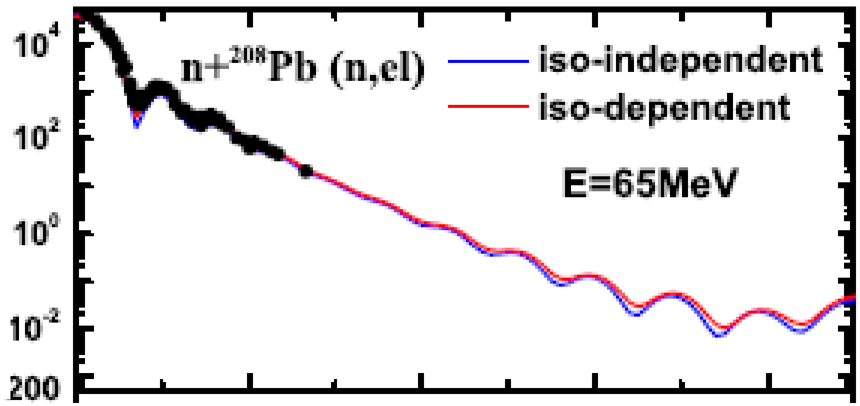


# Isospin dependence of OMP




ROMP with CTOM could well describe the nucleon-A scatterings, comparable to the KD pots.

http://www.nuclear.csdb.cn/ctom for testifying

# Structure of Finite nuclei

- *ab initio* investigations:

A full BHF and RBHF for finite nuclei starting with a bare NN interactions have been performed recently

BHF

PHYSICAL REVIEW C **95**, 034321 (2017)

**Brueckner-Hartree-Fock calculations for finite nuclei with renormalized realistic forces**

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(Received 13 January 2017; revised manuscript received 4 March 2017; published 27 March 2017)

One can adopt two-step $G$-matrix approximations for the Brueckner-Hartree-Fock (BHF) calculations. The first $G$ matrix is to soften the bare force, and the second one is to include the high-order correlations of the interaction in medium. The first $G$-matrix calculation for two-nucleon interaction should be done in the center-of-mass coordinate. As another alternative BHF approach, we have adopted the $V_{\text{low-}k}$ technique to soften the interaction and used the $G$ matrix to include high-order correlations. The $V_{\text{low-}k}$ renormalization leads to high-momentum and low-momentum components of the interaction decoupled. With the $V_{\text{low-}k}$ potential, we have performed the BHF calculations for finite nuclei. The $G$-matrix elements with exact Pauli exclusions are calculated in the self-consistent BHF basis. To see effects from further possible correlations beyond BHF, we have simultaneously performed renormalized BHF (RBHF) calculations with the same potential. In RBHF, the mean field derived from realistic forces is modified by introducing the particle-occupation depletion resulting from many-body correlations. The ground-state energies and radii of the closed-shell nuclei, $^{4}$He, $^{16}$O, and $^{40}$Ca, have been investigated. The convergences of the BHF and RBHF calculations have been discussed and compared with other *ab initio* calculations with the same potential.

# Structure of Finite nuclei

## DBHF

CHIN. PHYS. LETT. Vol. 33, No. 10 (2016) 102103 Express Letter

**Relativistic Brueckner–Hartree–Fock Theory for Finite Nuclei** *

Shi-Hang Shen(申时行)[1,2], Jin-Niu Hu(胡金牛)[3], Hao-Zhao Liang(梁豪兆)[2,4], Jie Meng(孟杰)[1,5,6**], Peter Ring[1,7], Shuang-Quan Zhang(张双全)[1]

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(Received 17 September 2016)

*Starting with a bare nucleon-nucleon interaction, for the first time the full relativistic Brueckner–Hartree–Fock equations are solved for finite nuclei in a Dirac–Woods–Saxon basis. No free parameters are introduced to calculate the ground-state properties of finite nuclei. The nucleus $^{16}$O is investigated as an example. The resulting ground-state properties, such as binding energy and charge radius, are considerably improved as compared with the non-relativistic Brueckner–Hartree–Fock results and much closer to the experimental data. This opens the door for ab initio covariant investigations of heavy nuclei.*

*S. H. Shen, H. Z. Liang, J. Meng, P. Ring, S. Q. Zhang, PRC96, 014316 (2017)*

# Structure of Finite nuclei

- The extension of the DBHF self-energy in LDA to finite nuclei

Inspired by the success in the RMOP, the similar method is applied to nuclear structure

- Most studies

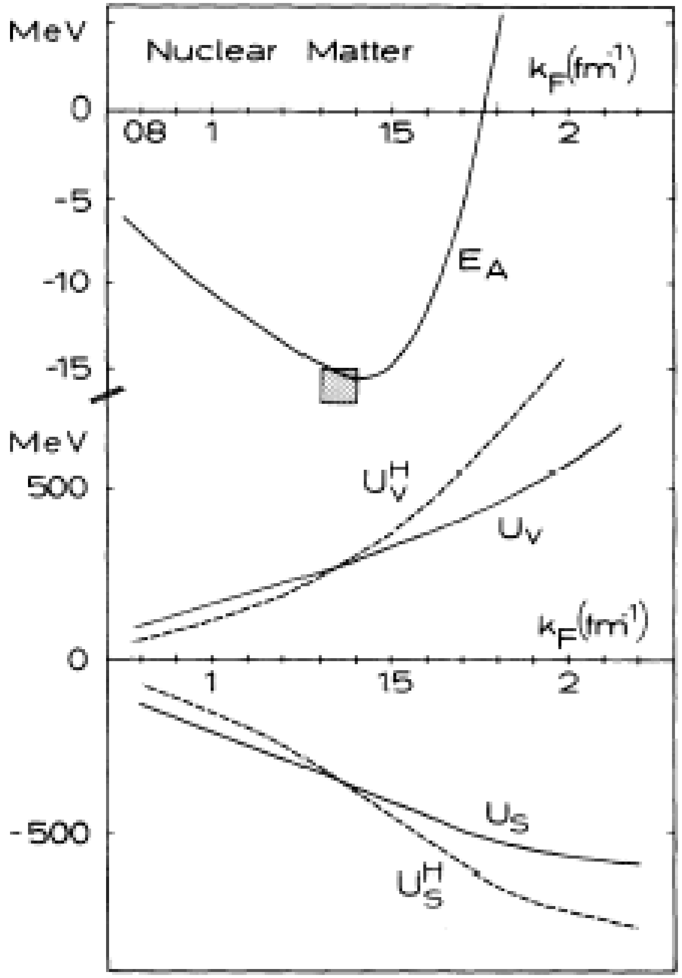
Introduce a density dependence of meson coupling constants $g_\sigma(\rho)$, $g_\omega(\rho)$ to reproduce the scalar and vector potentials in nuclear matter in RMF and RHF approaches

$$U_s,\ \ U_o \Longrightarrow g_\sigma(\rho),\ \ g_\omega(\rho)$$

no proper isospin and momentum dependence

# RDMF

Only density dependence included




| | RDDH | N-BHF | Experiment |
|---|---|---|---|
| $^{16}$O | | | |
| $BE/A$ (MeV) | −7.5 | −6.6 | −7.98 |
| $r_c$ (fm) | 2.66 | 2.29 | 2.70 ± 0.05 |
| $\epsilon(1s_{1/2})$ (MeV) | −43.5 | −50.5 | −40 ± 8 |
| $\epsilon(1p_{3/2})$ (MeV) | −21.8 | −22.9 | −18.4 |
| $\epsilon(1p_{1/2})$ (MeV) | −16.5 | −15.5 | −12.1 |
| $^{40}$Ca | | | |
| $BE/A$ (MeV) | −8.0 | ··· | −8.5 |
| $r_c$ (fm) | 3.36 | ··· | 3.5 |
| $\epsilon(1s_{1/2})$ (MeV) | −53.3 | ··· | − 50 ± 8 |
| $\epsilon(1p_{3/2})$ (MeV) | −36.0 | ··· | − 34 ± 5 |
| $\epsilon(1p_{1/2})$ (MeV) | −32.5 | ··· | − 34 ± 5 |
| $\epsilon(1d_{5/2})$ (MeV) | −19.3 | ··· | − 14 ± 2 |
| $\epsilon(2s_{1/2})$ (MeV) | −14.3 | ··· | − 10 ± 1 |
| $\epsilon(1d_{3/2})$ (MeV) | −13.6 | ··· | −7 ± 1 |

Brockmann & Toki,PRL68(92)3408

# RDHF

- Include isoscalar $\sigma$ $\omega$

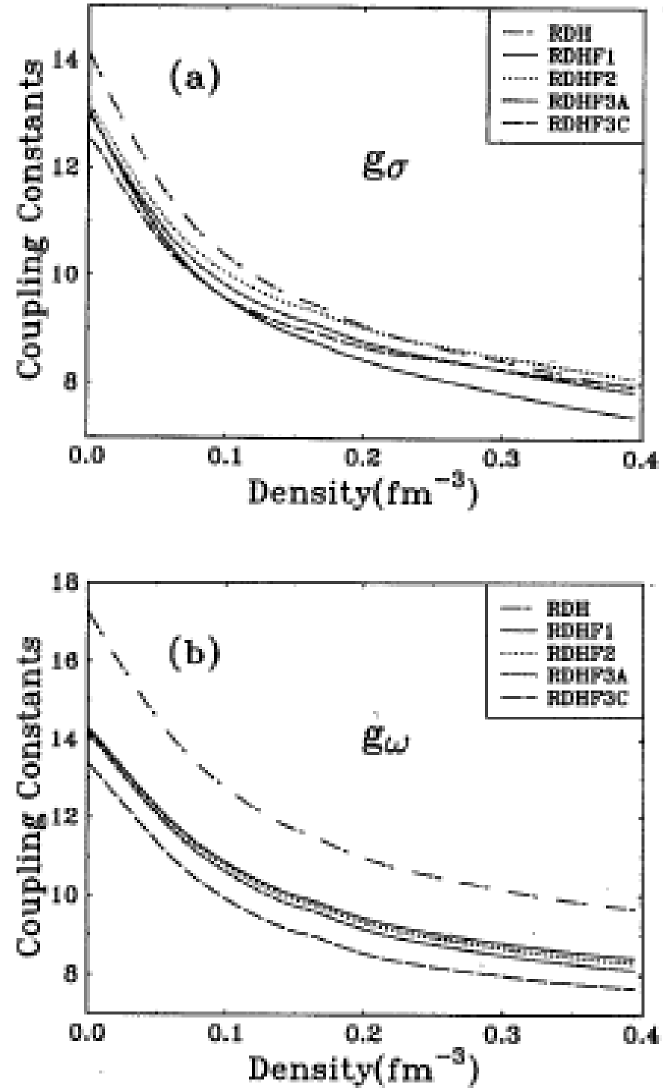
  isovector $\pi$ $\rho$

  $f_\pi$ , $g_\rho$ , $g_\rho/f_\rho$ values at

  $m_\pi$ , $m_\rho$ free space

- Isovector mesons play an important role in the spin-orbit splitting, isospin dep. Quantities, neutron skin etc.

Shi, Chen, Ma PRC52(95)144




RDH

RDHF1

RDHF2

RDHF3

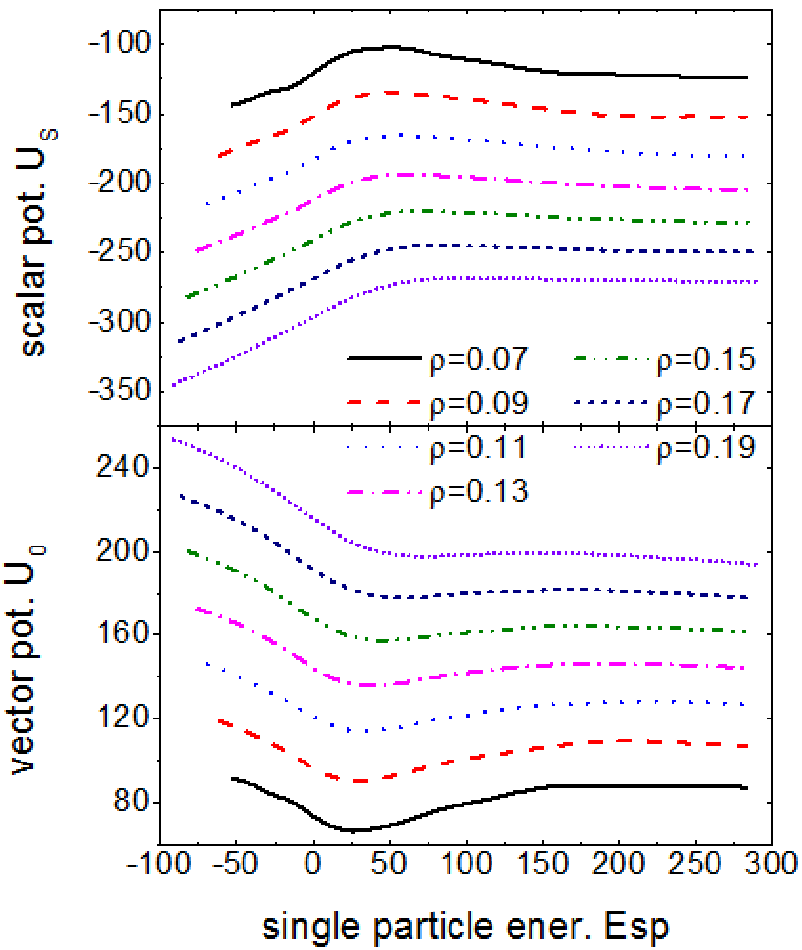
# DBHF approach

- The extension to bound states

  Inspired by the success in the ROMP

- The energy and density dependences of Us and Uo in DBHF with STM

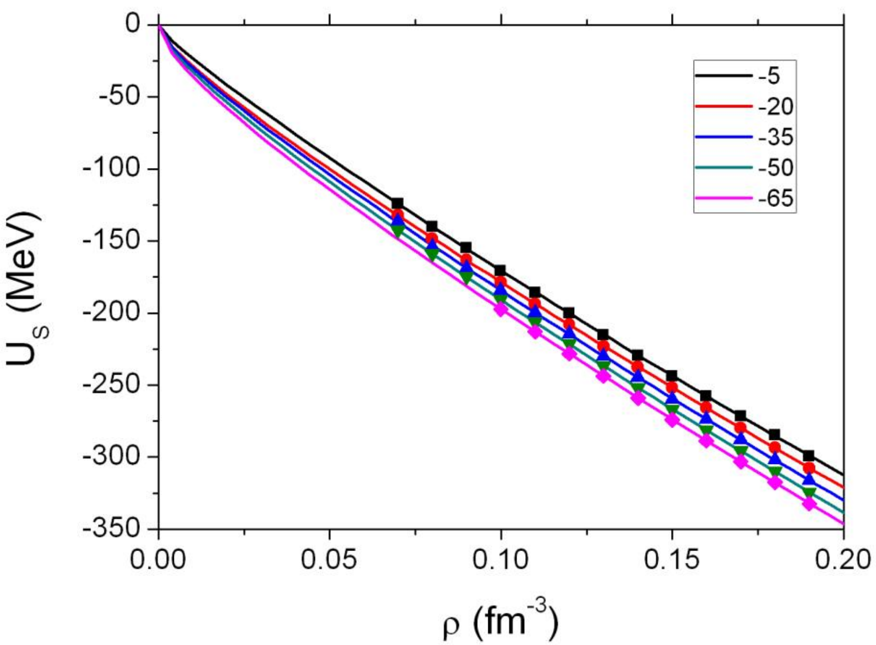
Stronger energy dependences for bound states

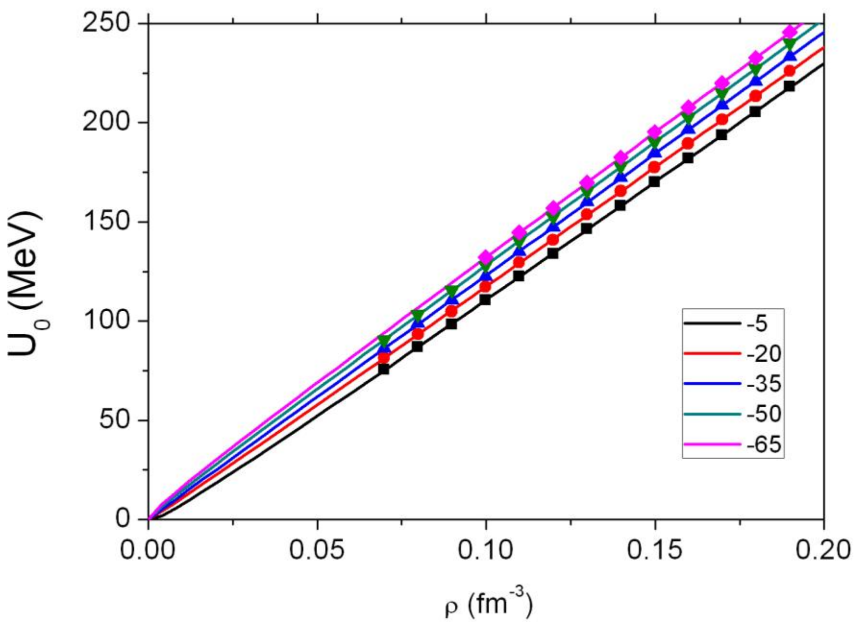



X.D.Sun, R.R.Xu, Y.Tian, Z.Y.Ma in preparation

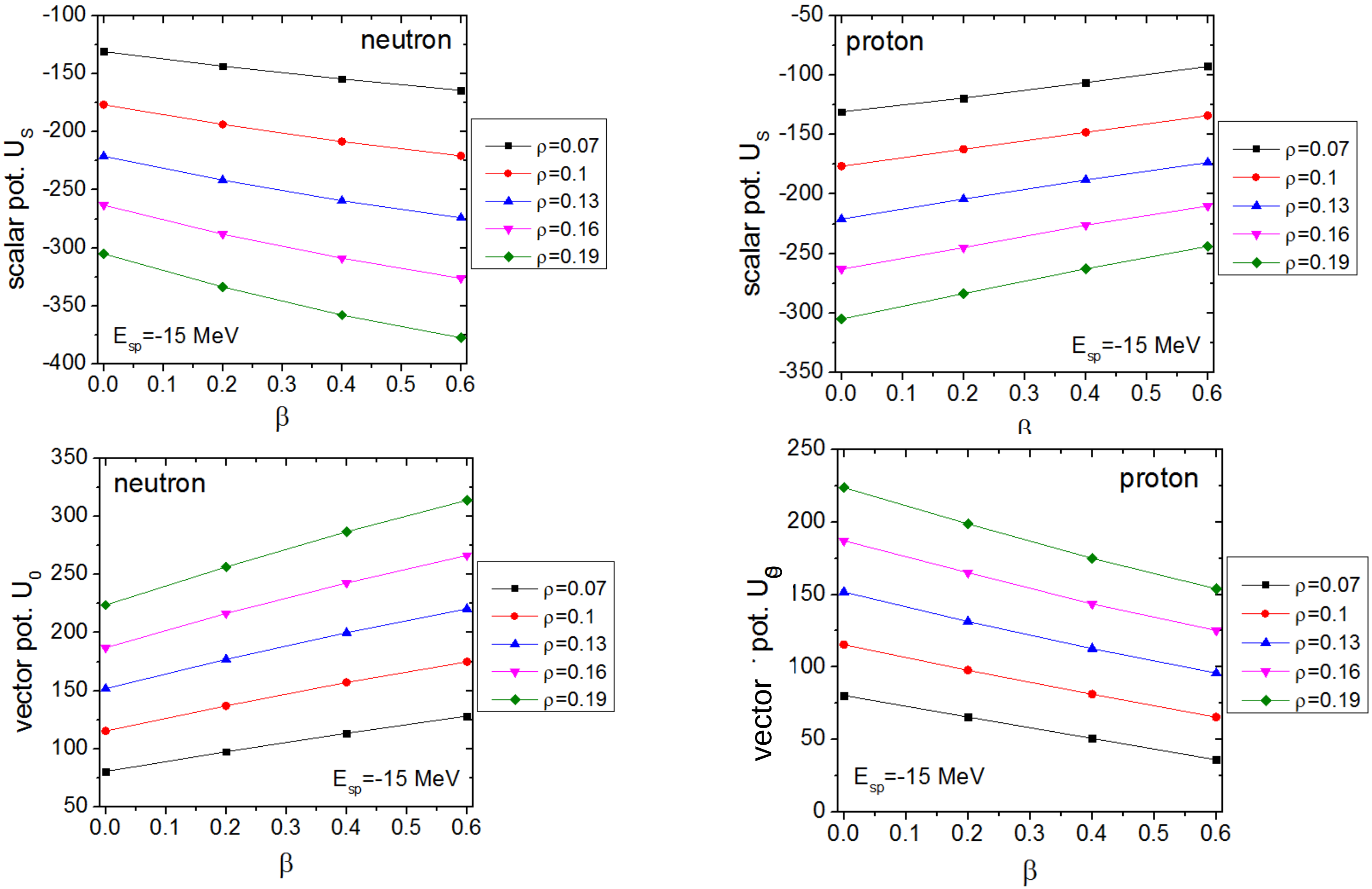
# Scalar and vector potentials

- The behavior of the potentials cannot be well constrained by the DBHF
- A polynomial fit ($\sqrt{\rho},\ \rho,\ \rho^2$) for the density dependences of $U_S$ , $U_0$



# Isospin dependences of pots



# Consistent calculation

- The local density approximation (LDA) is adopted to connect the density and isospin dependence of the self-energy to the radius of a finite nucleus.

$$Us(r,\varepsilon), Uo(r,\varepsilon) \longrightarrow Us(\rho,\beta,\varepsilon), Uo(\rho,\beta,\varepsilon)$$

- The energy dependence of the nucleon self-energy is considered for each single particle level.
- *A* consistent calculation is required for each single particle wave function and each nucleus.

# Ground state properties of nuclei

- Preliminary results are obtained

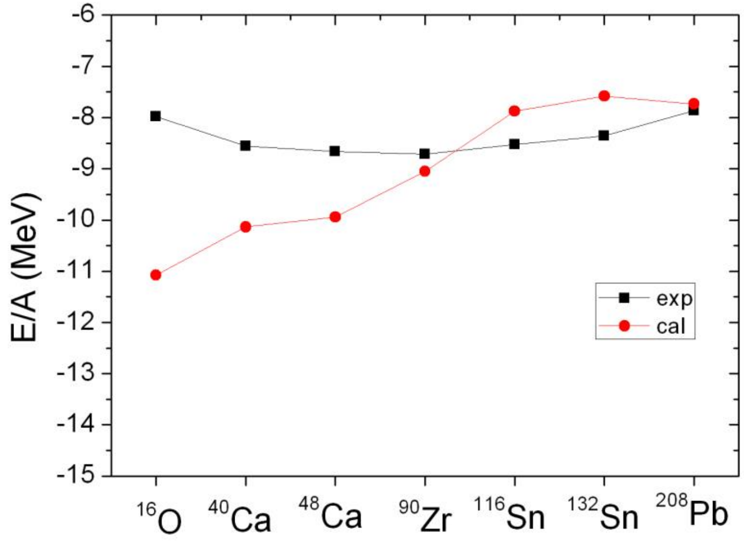
  without any free parameters

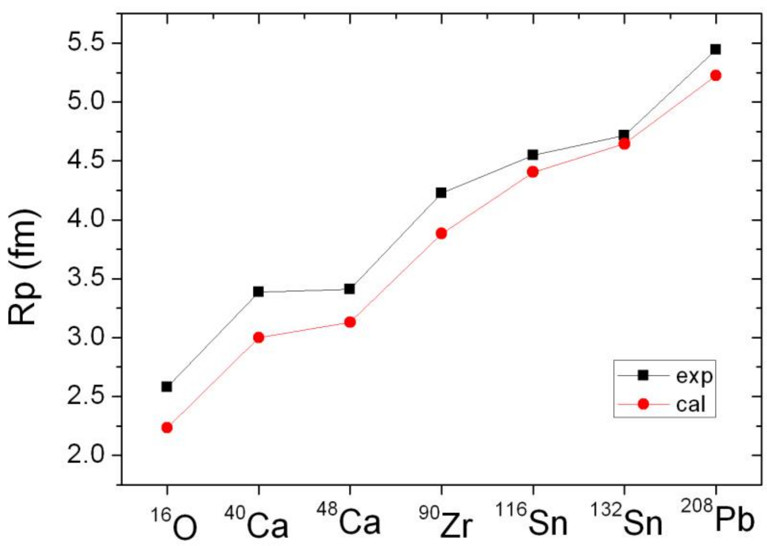
DBHF
+LDA

| | B/A(MeV) | Exp | Rp (fm) | Exp |
|---|---|---|---|---|
| 18O | 11.08 | 7.98 | 2.23 | 2.58 |
| 40Ca | 10.14 | 8.55 | 3.39 | 3.00 |
| 48Ca | 9.94 | 8.67 | 3.41 | 3.13 |
| 90Zr | 9.05 | 8.71 | 3.88 | 4.22 |
| 116Sn | 7.87 | 8.52 | 4.40 | 4.55 |
| 120Sn | 7.58 | 8.35 | 4.64 | 4.72 |
| 208Pb | 7.73 | 7.87 | 5.22 | 5.44 |

X.D.Sun, R.R.Xu, Y.Tian, Z.Y.Ma in preparation

# Ground state properties of nuclei




- Preliminary results are promising
- Further investigation is required

Surface effects are important in finite nuclei, but absent in nuclear matter.

# Summary

- DBHF with the subtracted T matrix approach could produce proper nucleon scalar and vector self-energies in the nuclear medium.
- The scalar and vector potentials are density, energy and isospin dependences
- The microscopic ROMP in ILDA CTOM could give a satisfactory description of nucleon scattering off nuclei
- The extension to the nuclear bound states with LDA in DBHF is performed. Results for the nuclear bound states are promising. Further investigation is required.

# Thanks