Happy Birthday to Prof. Arima

生 日 快 乐

Healthy and Happy

健康快乐

Shanghai, Sept. 28, 2018

北京大学物理学院
School of Physics, Peking University

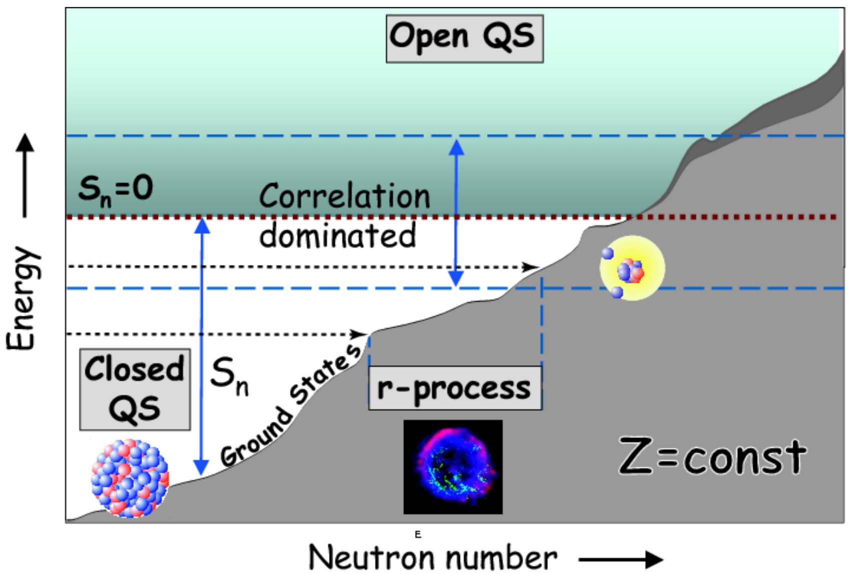
Resonance and continuum in nuclear drip lines

Furong Xu （许甫荣）

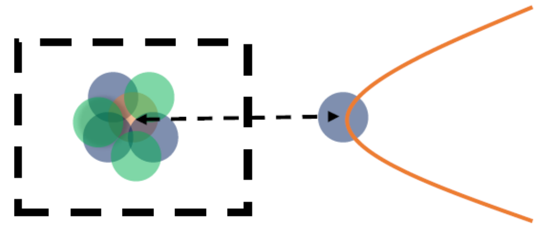
***Ab initio* calculations:**

- **Resonance and continuum Gamow Shell Model with CD Bonn**
- **Resonance and continuum Gamow IM-SRG with chiral N^2LO**

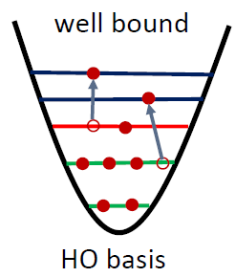
International Symposium on
Simplicity, Symmetry and Beauty of Atomic Nuclei,
in honor of Professor Akito Arima's 88 year-old birthday (米寿)
Shanghai, September 26th-28th, 2018

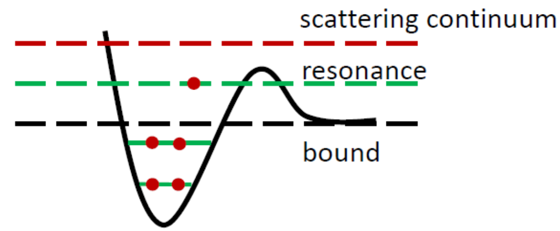


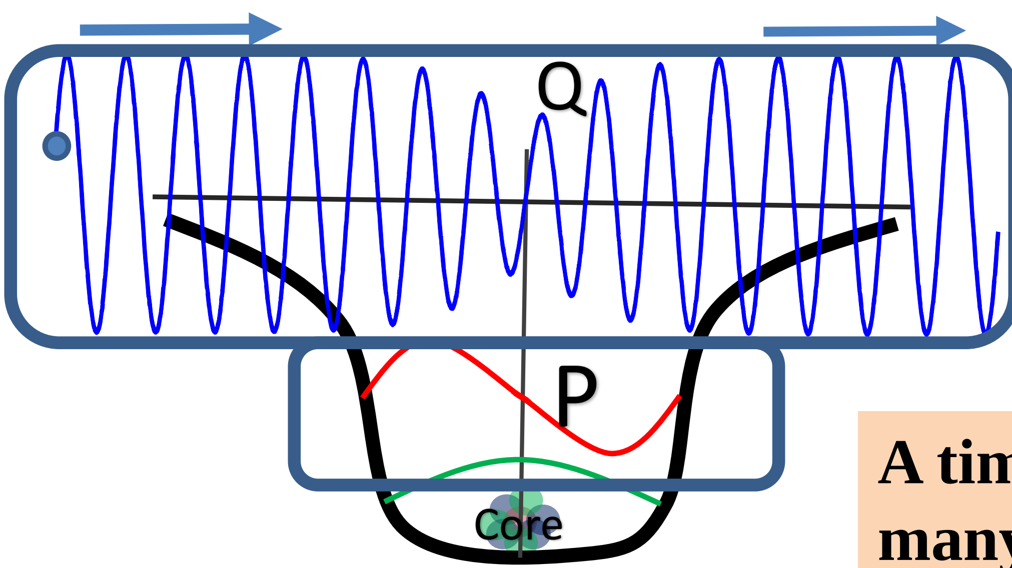
Closed quantum system



Open quantum system

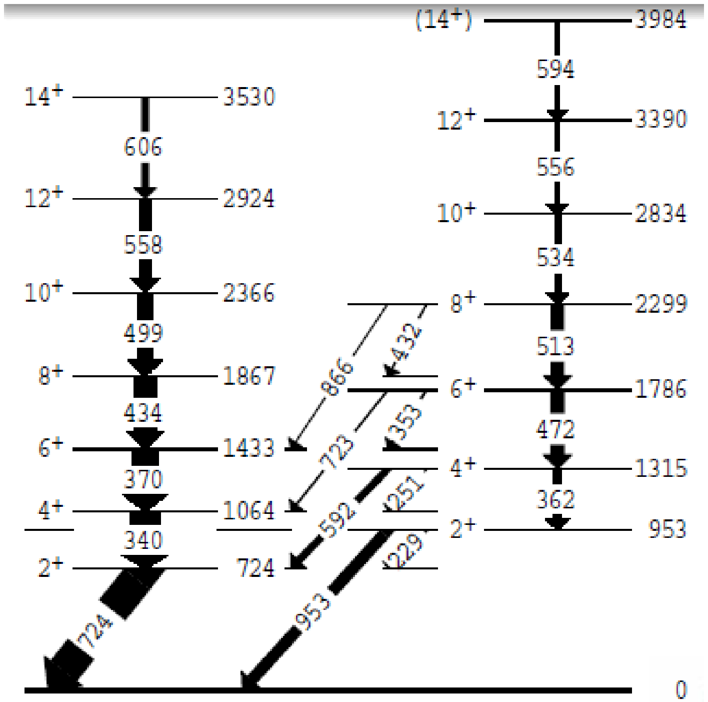






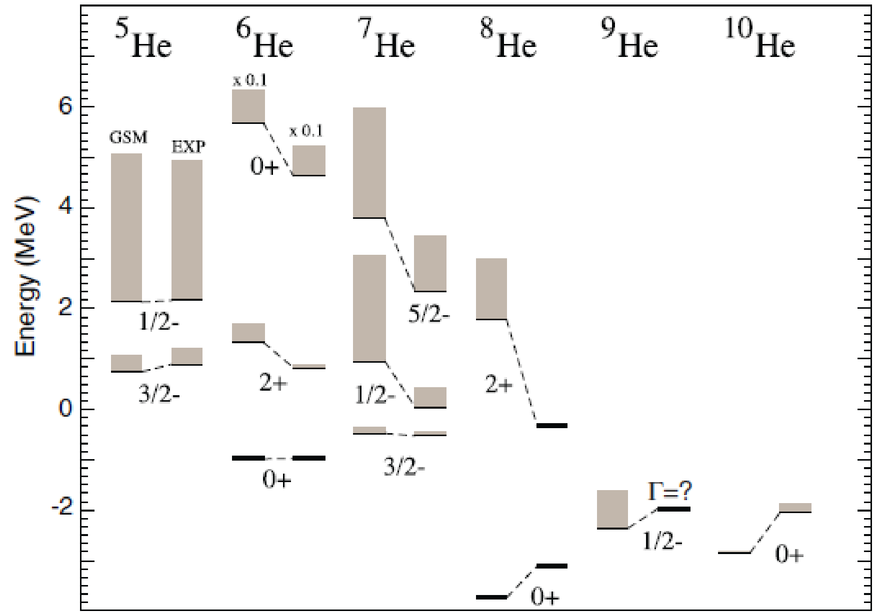
A time-dependent many-body problem

^{188}Pb



J. Pakarinen *et al.*, PRC 72, 011304(R) (2005)

Spectra of resonant states

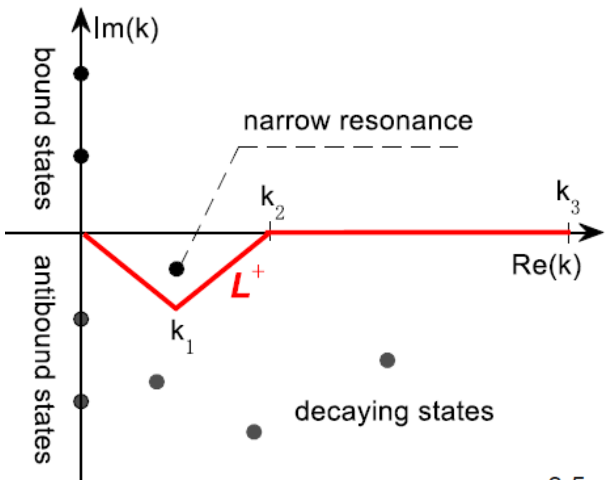


N. Michel *et al.*, Nucl. Phys. A 752, 335c (2005)

How to treat time-dependent quantum problems?

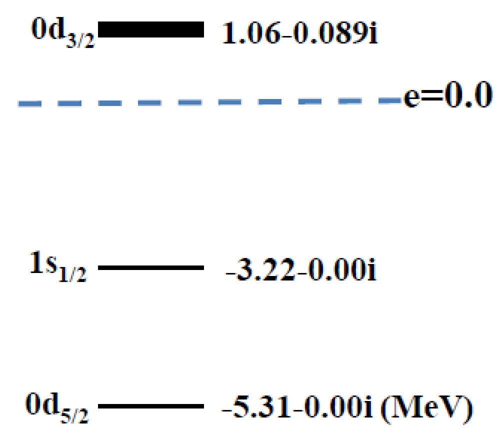
T. Berggren, Nucl. Phys. A109 (1968) 265

Complex-momentum space: **bound, resonance and continuum**

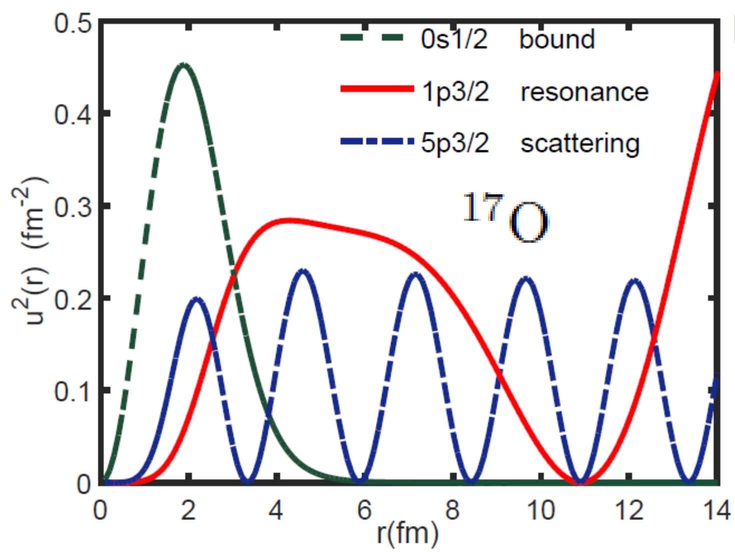


$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$

Woods-Saxon in complex-*k* space



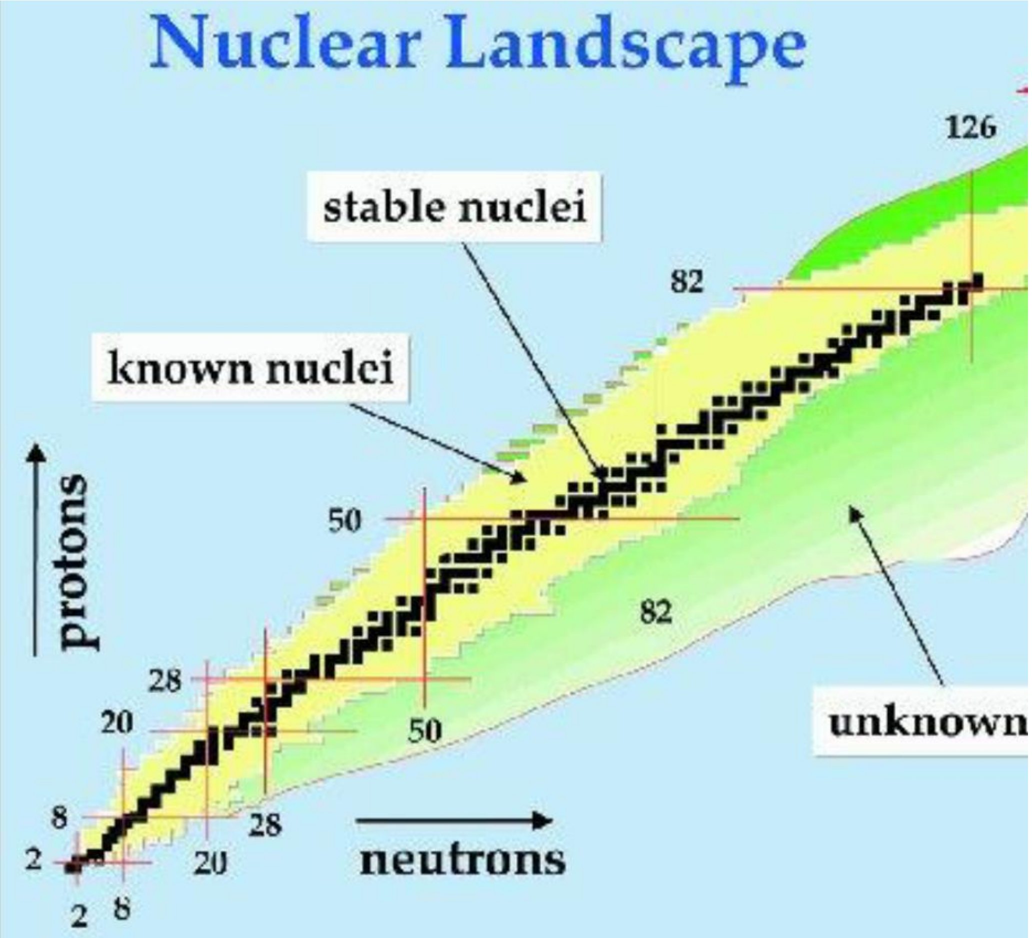
WS SPE's



***Two ab initio* calculations:**

- **Resonance and continuum Gamow Shell Model**

- **Resonance and continuum Gamow IM-SRG**



For nuclear structure calculations: there are two fundamental problems

1) nuclear force; 2) many-body problem

Ab initio

1. Starting from a realistic nuclear force

2. Renormalization (softening) to speed up convergence

3. "Rigorous" many-body methods

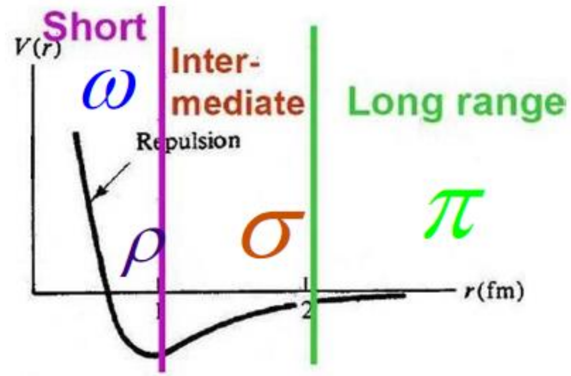
$$\hat{H}_{int} = \sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j}^{A} V_{NN,ij} + \sum_{i<j<k}^{A} V_{NNN,ijk}$$

$$H_{\text{int}} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|) - \frac{P^2}{2Am} \qquad \vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

How to solve the many-body problem? particularly for weakly-bound nuclei

Gamow shell model with an inert core

1. Start from CD Bonn



2. Choose a doubly-closed core (e.g., ^{16}O for *sd* shell)

Q-box + folded diagrams （Perturbation）

3. Calculate spectra with resonance and continuum

GSM based on realistic nuclear forces

Realistic nuclear forces ⇨ Gamow shell model calculations

Taking a doubly closed core

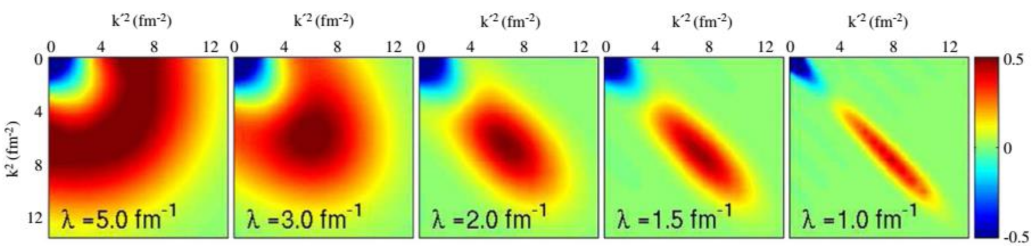
Bare forces: Strong repulsion, slow convergence

$V_{low\ k}$ or SRG

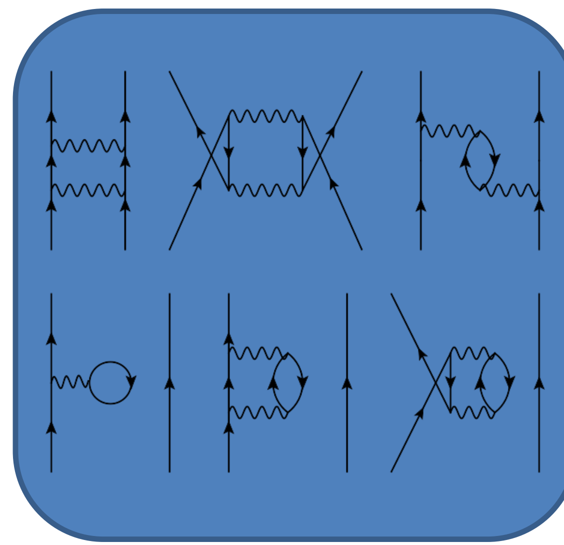
To remove hard core, but still keep good descriptions of NN scattering phase shifts

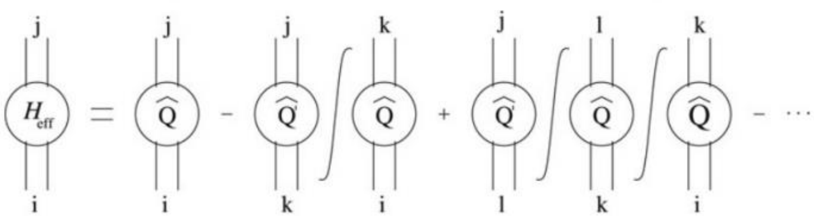
$$\langle \alpha_P | \bar{H}_{\rm eff} | \alpha_{P'} \rangle = \sum_{\alpha_{p''}} \sum_{\alpha_{p'''}} \sum_{kk'k'' \in \mathcal{K}} \langle \alpha_P | \tilde{k}'' \rangle \langle \tilde{k}'' | \alpha_{p''} \rangle \langle \alpha_{p''} | \tilde{k} \rangle E_k \langle \tilde{k} | \alpha_{p'''} \rangle \langle \alpha_{p'''} | \tilde{k}' \rangle \langle \tilde{k}' | \alpha_{P'} \rangle$$

$$\frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T_{\rm rel}, H_\lambda], H_\lambda]$$



Q

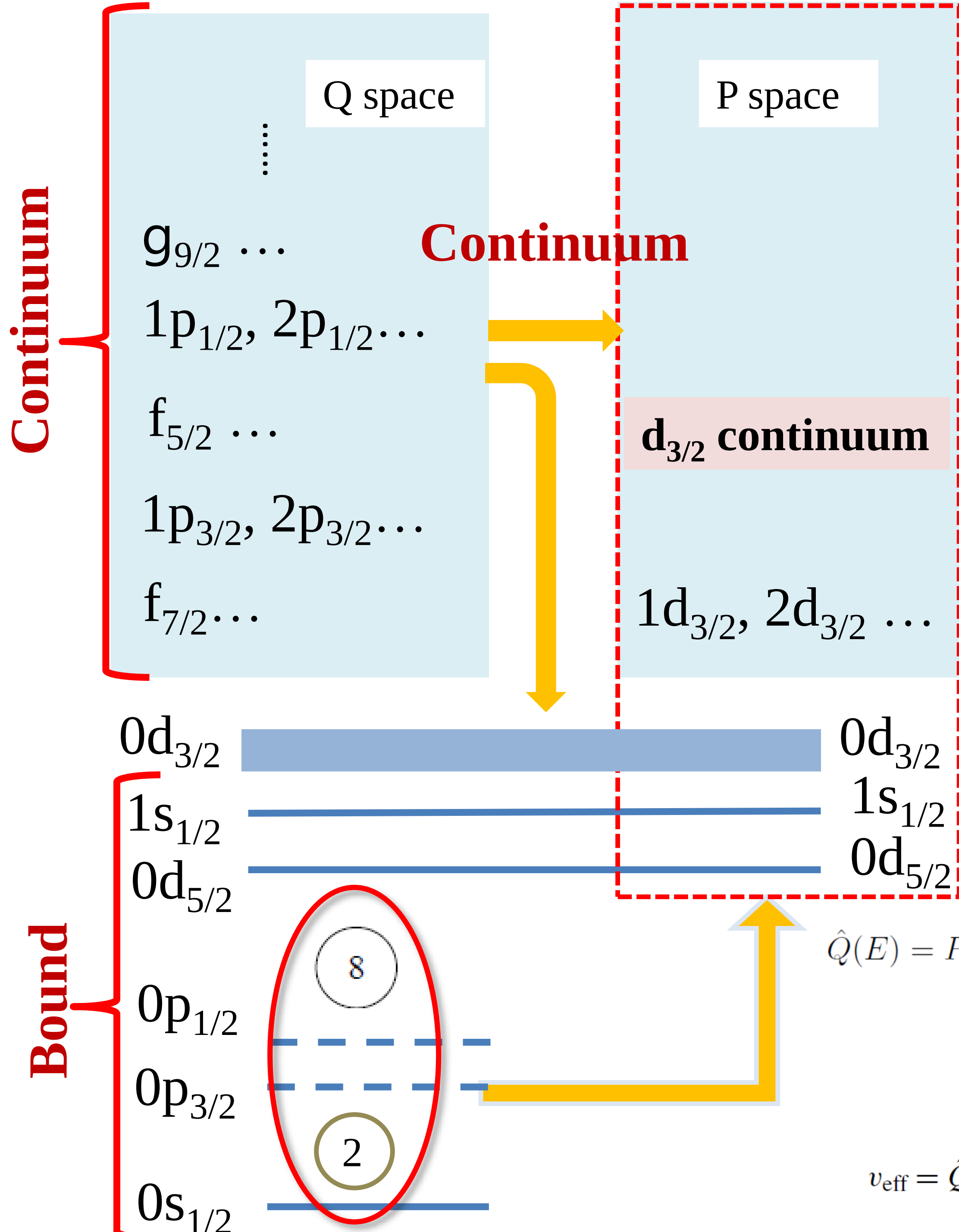


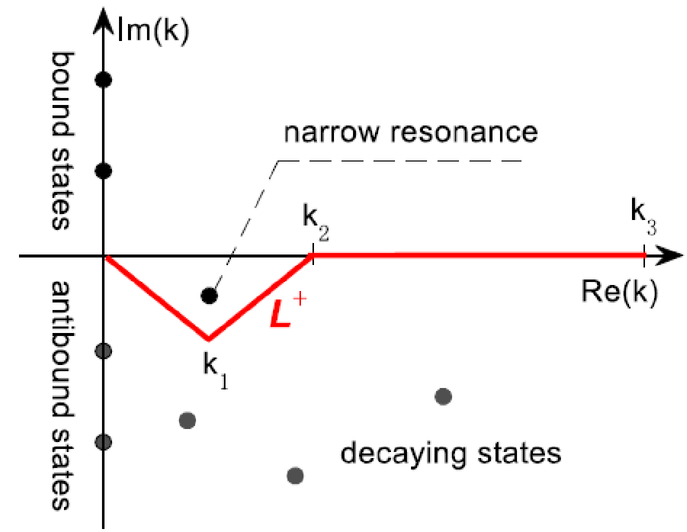


$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP,$$

Non-degenerate extended Kuo-Krenciglowa folded-diagram method (EKK) by Takayanagi, NPA 852, 61 (2011);

Model space

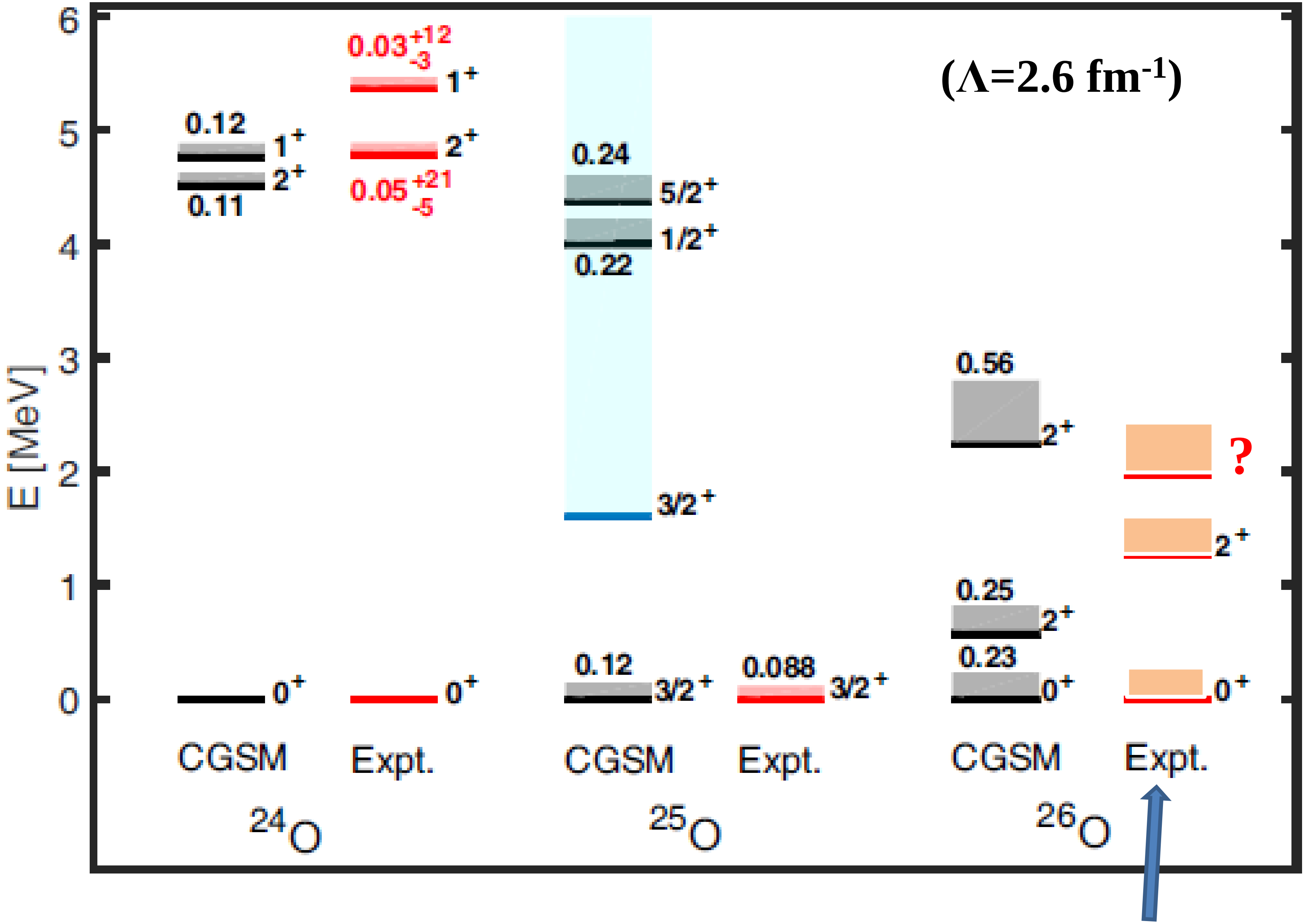




Q-box folded diagrams in complex-k space

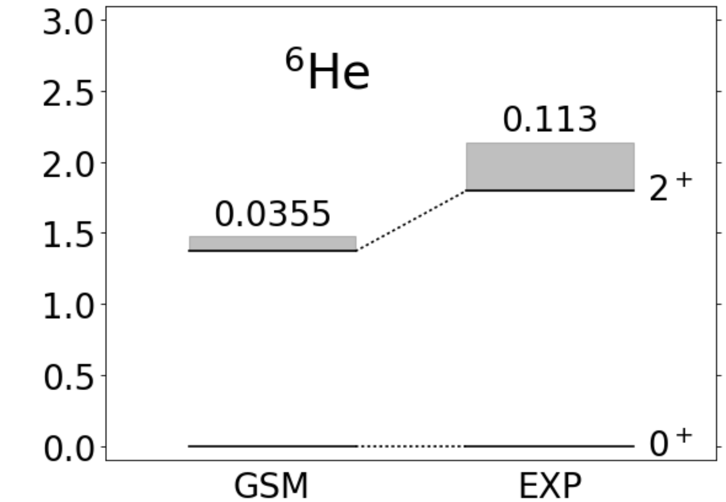
$$\hat{Q}(E) = PH_1P + PH_1Q\frac{1}{E - QH_0Q}QH_1P + PH_1Q\frac{1}{E - QH_0Q}H_1\frac{1}{E - QH_0Q}QH_1P + \cdots .$$

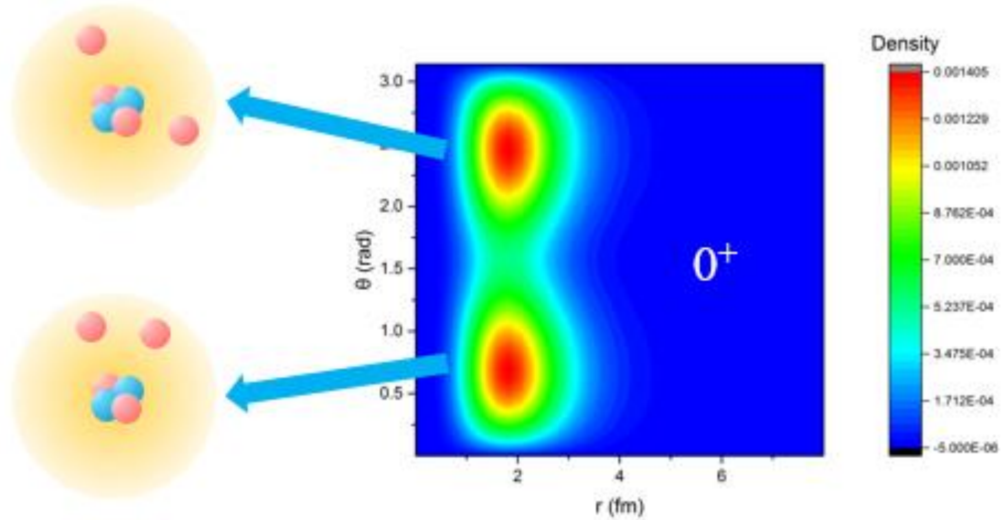
$$v_{\mathrm{eff}} = \hat{Q} - \hat{Q}'\int\hat{Q} + \hat{Q}'\int\hat{Q}\int\hat{Q} - \cdots$$

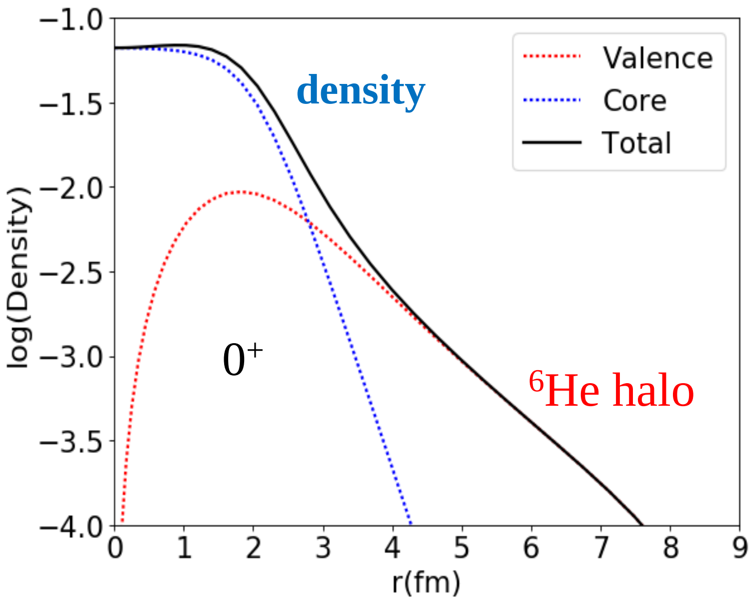


Y. Kondo *et al.*, PRL 116, 102503 (2016)

^{6}He halo



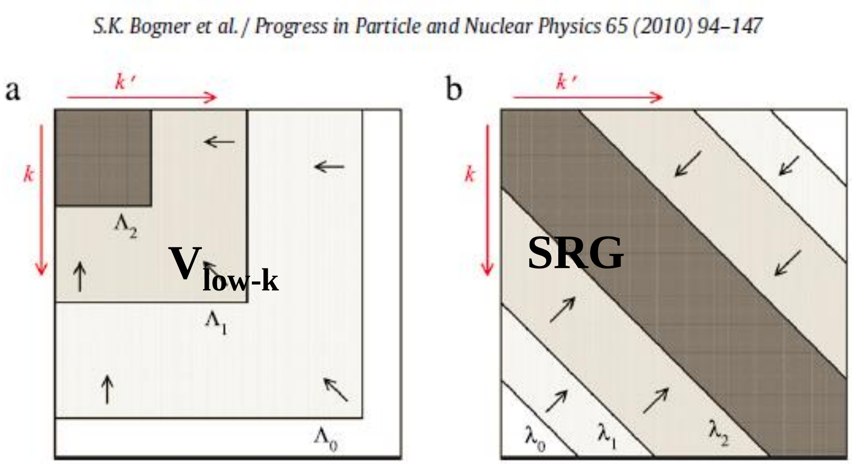


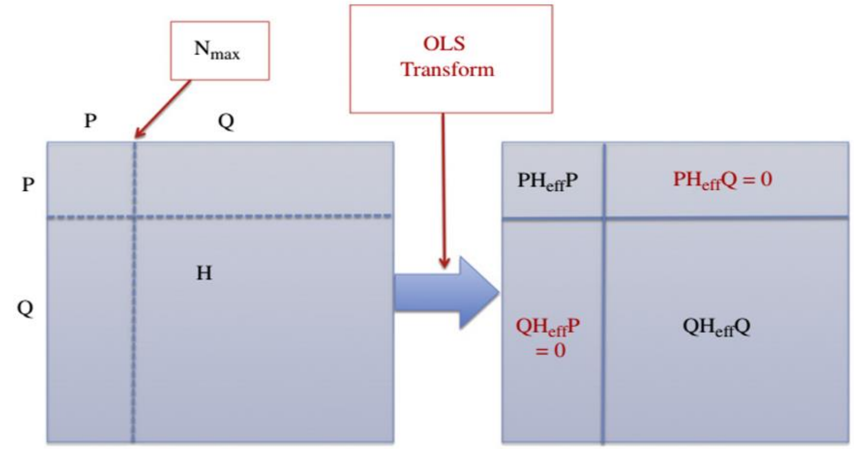


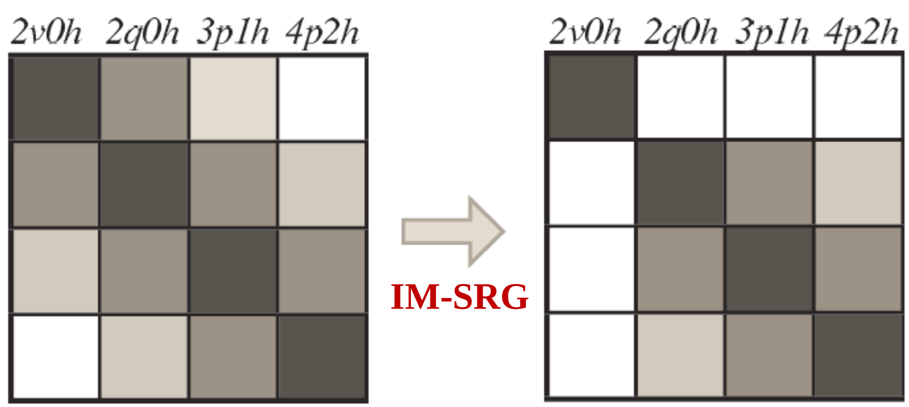
^{6}He correlated density distribution

$$\rho(r,\theta) = \langle \Psi | \delta(r_1 - r)\delta(r_2 - r)\delta(\theta_{12} - \theta) | \Psi \rangle$$

Gamow In-Medium Similarity Renormalization Group with resonance and continuum







e.g., valence-space IM-SRG

We have exploited IM-SRG into the complex-k space to deal with loosely-bound and unbound nuclei: Gamow IM-SRG

Chiral NNLO$_{\text{opt}}$: A. Ekstrom *et al*., Phys. Rev. Lett. 110, 192502 (2013)

Gamow Hartree-Fock :

$$\langle k|h^{\mathrm{HF}}|k'\rangle = (1-\frac{1}{A})\frac{\hbar^2}{2m}k^2\delta_{kk'} + \sum_{pq}\langle p|U_{\mathrm{HF}}|q\rangle\langle k|p\rangle\langle q|k'\rangle$$

Bound, resonant and continuum HF basis states are obtained by diagonalizing the complex-energy HF Hamiltonian

SRG

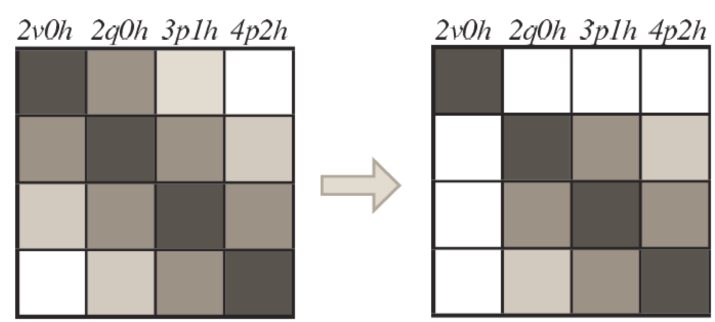
S. D. Glazek and K. G. Wilson, Phys. Rev. D 48, 5863 (1993) (quantum field theory);
F. Wegner, Ann. Phys. (Leipzig) 506, 77 (1994) (for condensed matter)

IM-SRG

K. Tsukiyama, S. K. Bogner, and A. Schwenk, Phys. Rev. Lett. 106, 222502 (2011).
H. Hergert, S. Bogner, T. Morris, A. Schwenk, and K. Tsukiyama, Physics Reports 621, 165 (2016)

$$H(s) = U(s)H(0)U^{-1}(s)$$

Continuous similarity transformation $U(s)\cdot U^{-1}(s) = 1$



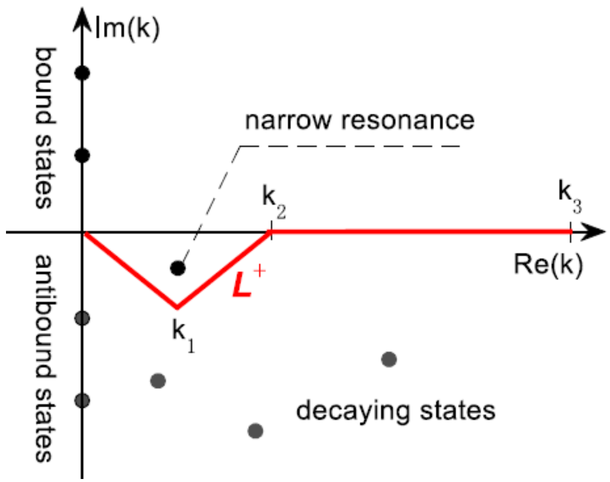
Gamow IM-SRG:

Advantage: include more many-body correlations

Gamow IM-SRG:

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$ **Flow equation**

$$\eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s)$$

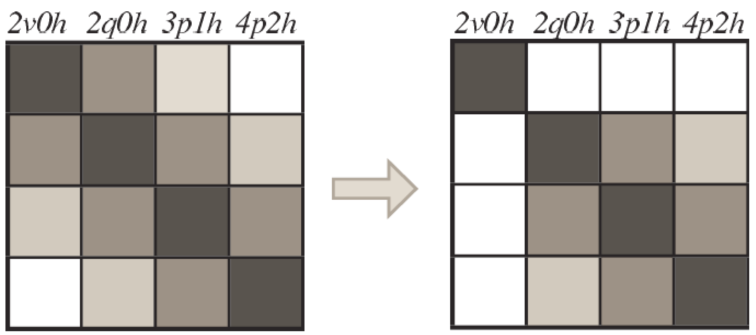


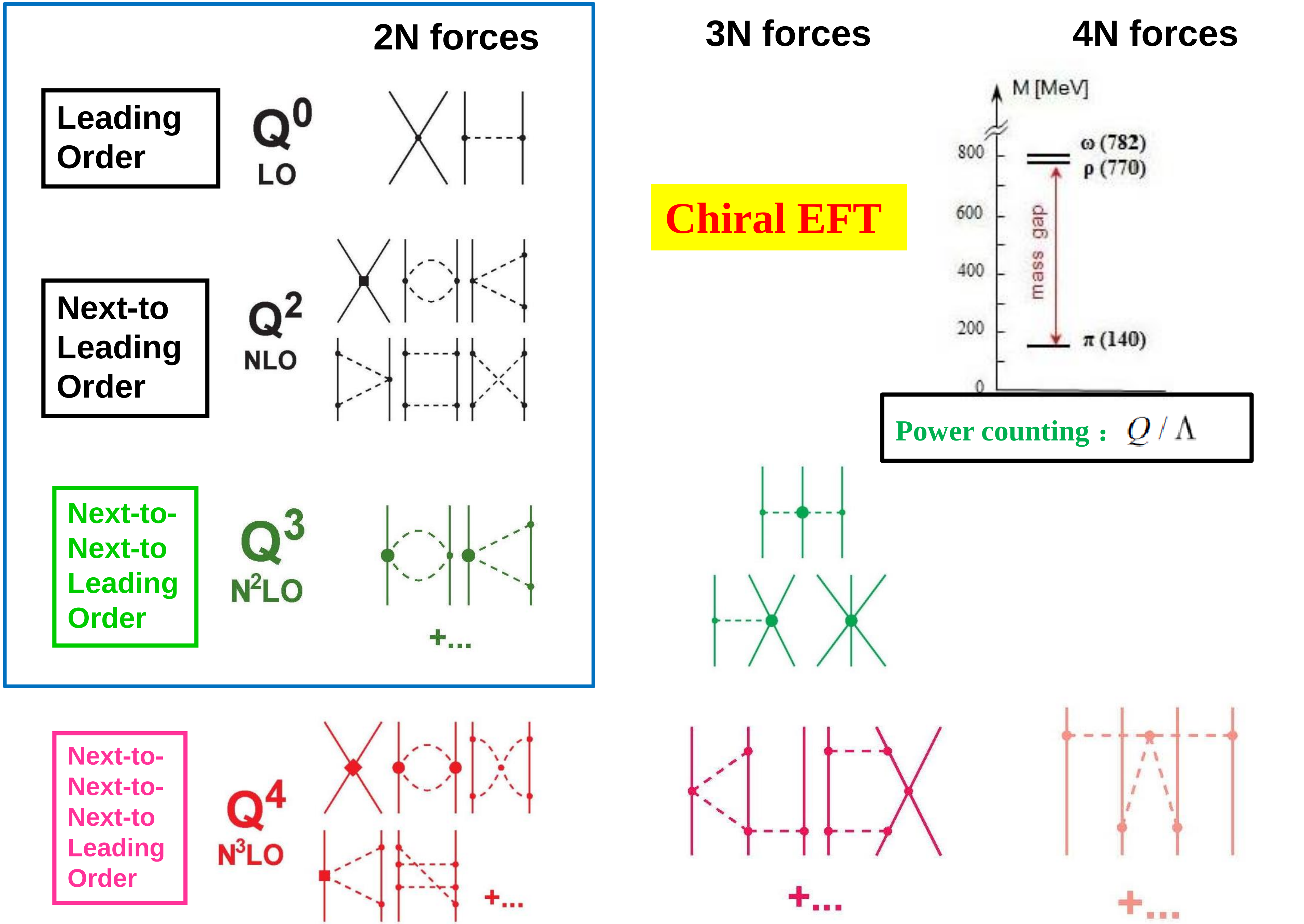
- ✓ **Directly gives bound energies of close-shell nuclei.**
- ✓ **For open shell or excited states, we use the Equation of Motion (EOM) :**

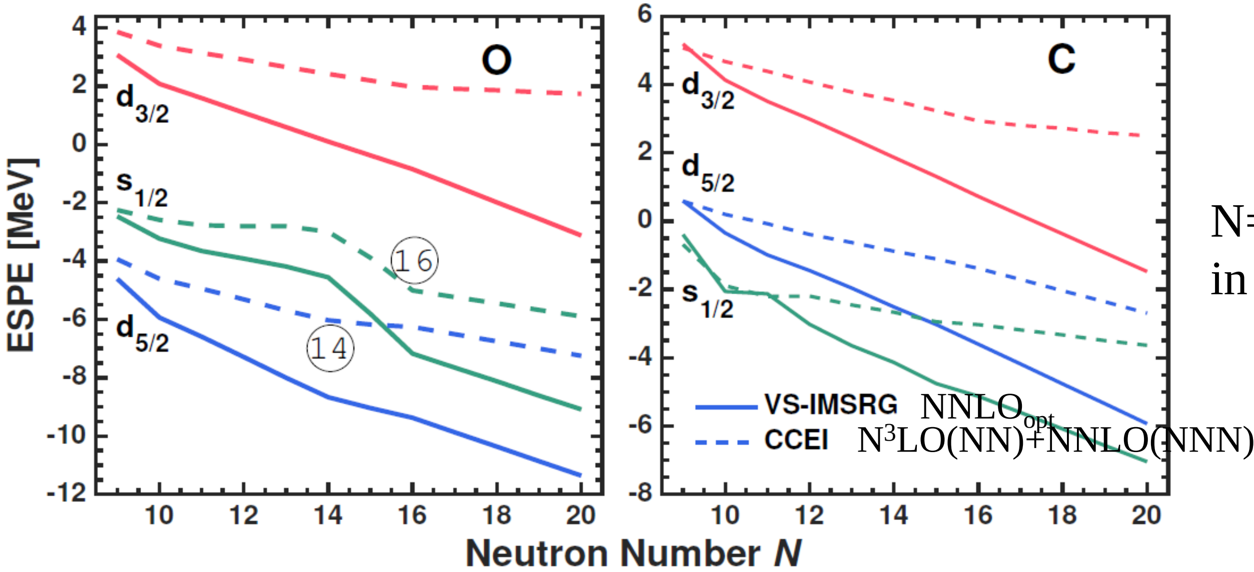
EOM + Gamow IM-SRG:

i) Gamow IM-SRG to decouple for the core;

ii) Use EOM to obtain excited states

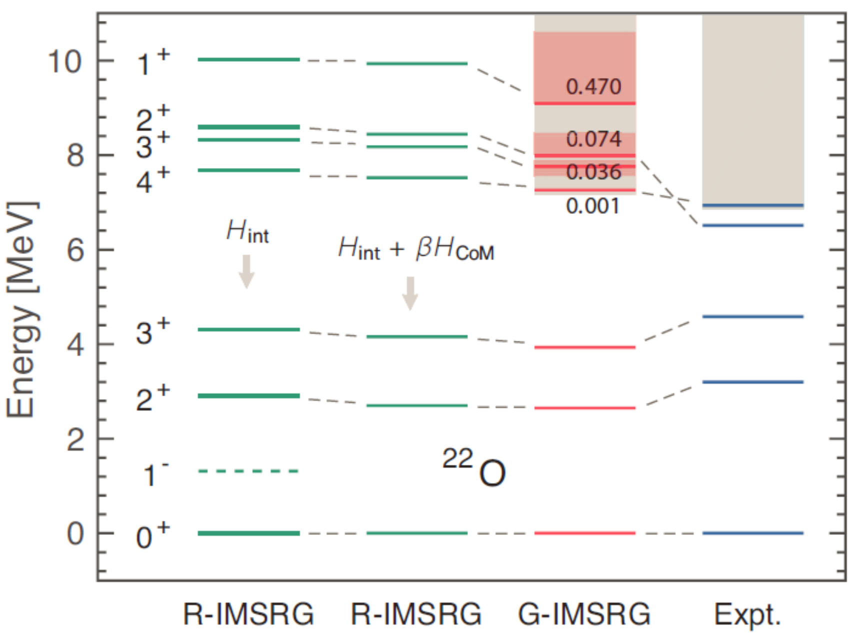


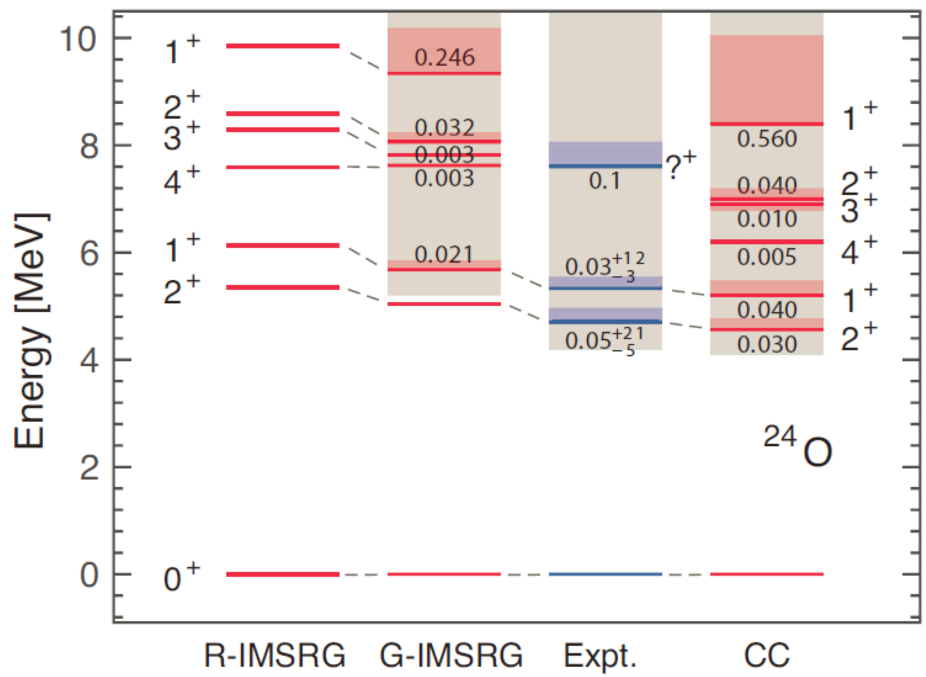

2N forces
3N forces
4N forces
Leading Order
Q^0
LO
Next-to Leading Order
Q^2
NLO
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
+...
Chiral EFT
M [MeV]
800
600
400
200
0
ω (782)
ρ (770)
mass gap
π (140)
Power counting : Q / Λ

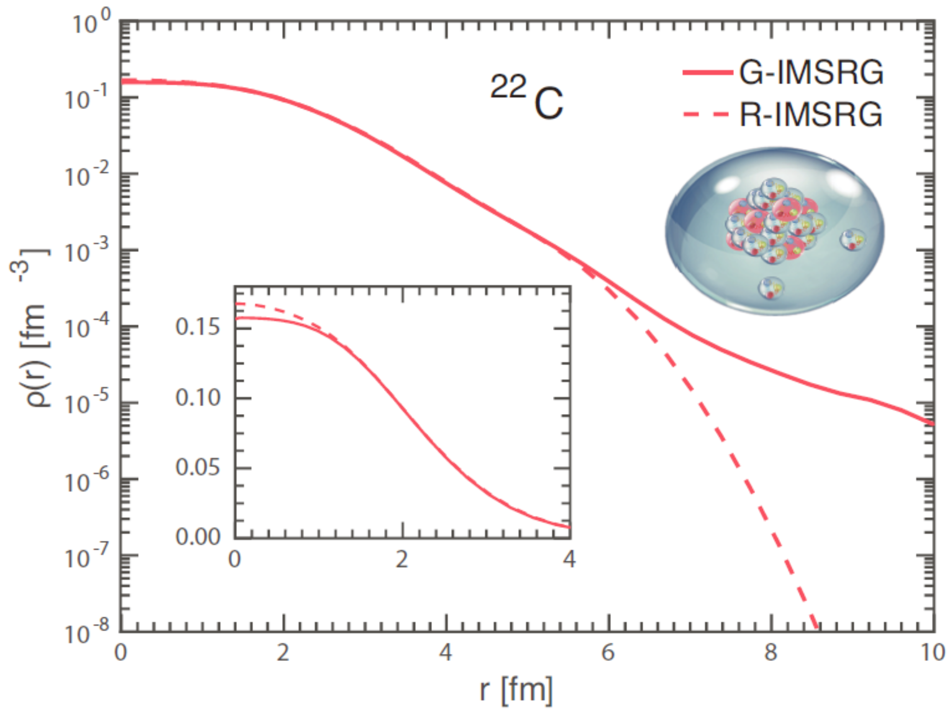


N=14 shell closure disappears in the Carbon chain

Spurious center-of-mass excitation







1. Gamow IMSRG with discrete HF *s*-waves: 2.79 fm
2. Gamow IMSRG with continuum *s*-waves: 3.06 fm
3. Experimentally estimated matter radius: 3.44 ± 0.08 [Y. Togano *et al*., PLB **761**, 412 (2016)]

NNLO$_{opt}$ itself underestimates the radii of carbon isotopes

R. Kanungo *et al*., Phys. Rev. Lett. 117, 102501 (2016)

TABLE I. Excited states for ^{22}C predicted by the Gamow EOM-IMSRG with the chiral NNLO$_{\text{opt}}$ interaction. Energies and widths are in MeV.

J^π	2_1^+	4_1^+	3_1^+	1_1^+	2_2^+	2_3^+	1_2^+
E_{IMSRG}	1.05	3.26	3.36	3.62	3.93	4.94	5.27
Γ_{IMSRG}	0.000	0.245	0.262	0.135	0.266	0.597	0.604

g.s. and 2_1^+ are bound

unbound resonances

LQCD + MBPT

LQCD nuclear force provided by Aoki and Inoue

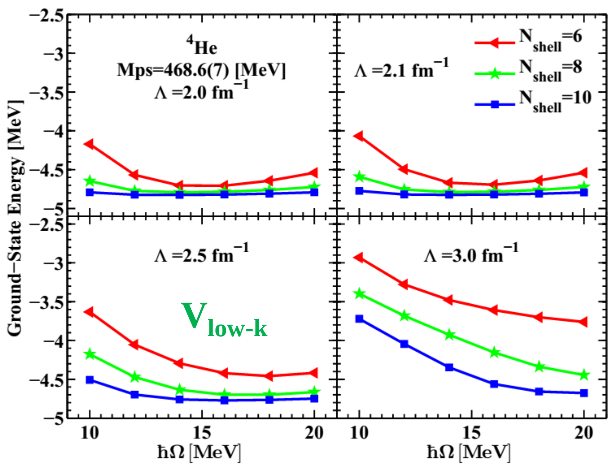


FIG. 6. Ground-state energy of ^4He in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different V_{lowk} interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

E_{expt} = -28.3 MeV

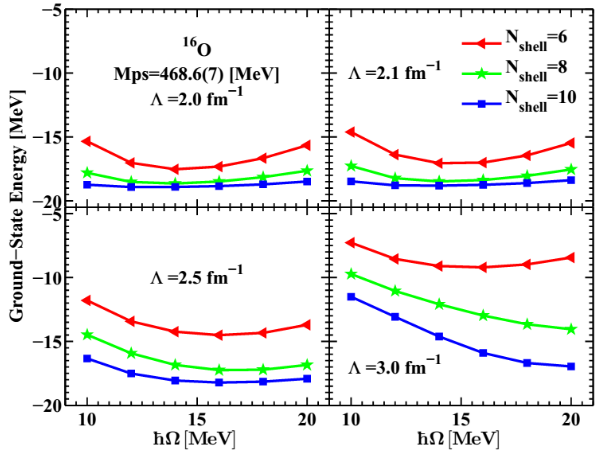


FIG. 7. Ground-state energy of ^{16}O in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different V_{lowk} interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

E^{expt} = -127.6 MeV

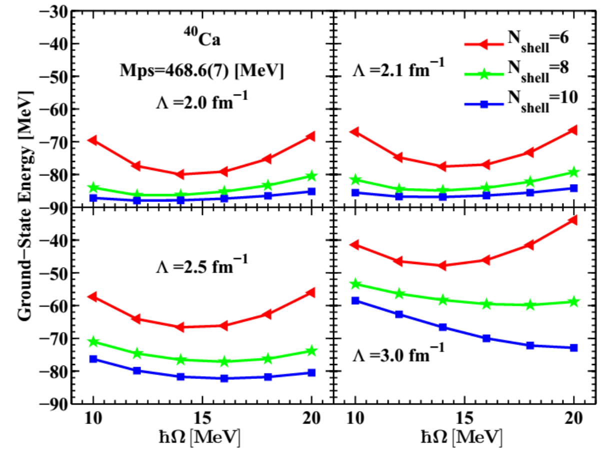
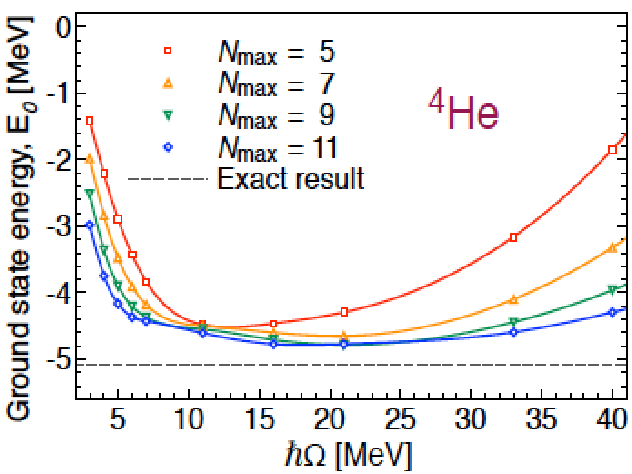


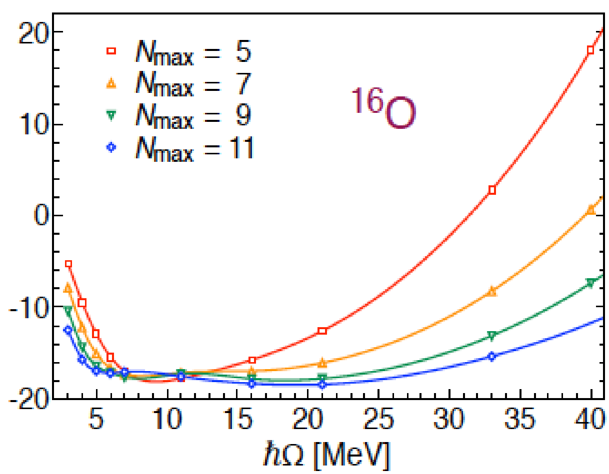
FIG. 8. Ground-state energy of ^{40}Ca in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different V_{lowk} interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

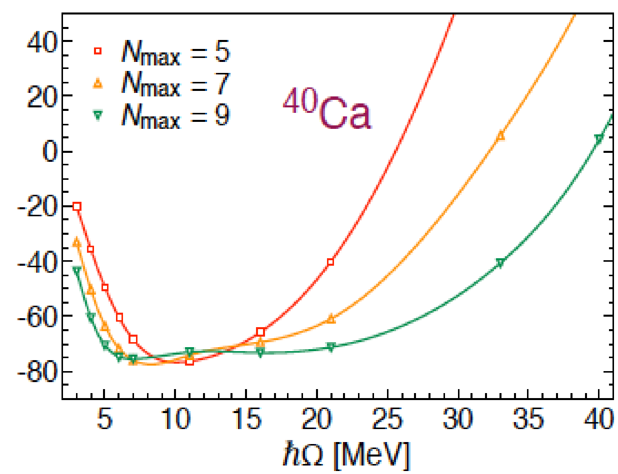
E^{expt} = -342.0 MeV

C. McIlroy et al PRC 2018 [HAL QCD potentials] find B/A ≈ 1–2 MeV in ^{16}O and ^{40}Ca at m_π ≈ 470 MeV.

Self-consistent Green's functions







Summary

Realistic nuclear forces (CD Bonn)

Renormalization by $V_{low\text{-}k}$

Many-body solutions by CGSM

Full Q-box folded diagrams in nondegenerate complex-k space, which includes contributions from core polarization and excided space.

- ✓ **Successfully applied to excitation spectra of weakly-bound or unbound oxygen isotopes.**

- ✓ **Gamow IM-SRG for nuclear halo (resonance and continuum)**

^{22}C halo structure, resonant excited states

Thank you for your attention

Group members: Baishan Hu, Zhonghao Sun, Qiang Wu, Sijie Dai, Furong Xu

Happy Birthday to Prof. Arima, Shanghai Sept. 28, 2018