The wobbling mode in odd-Z nuclei and the self-consistent constrained HFB equation

For the 88th Birthday of Prof. Akito Arima

Kazuko Sugawara-Tanabe and Kosai Tanabe

(1) The first work to collaborate with Prof.Akito Arima was based on the self-consistent constrained HFB equation

- **The microscopic proof of IBM** (S-T & Arima, Phys. Lett. B110,87 (1982))

$$|\phi_{2n}(I)\rangle = \left(\sum_{JM} \Gamma^{\dagger}_{JM}\right)^{n} |0\rangle .$$

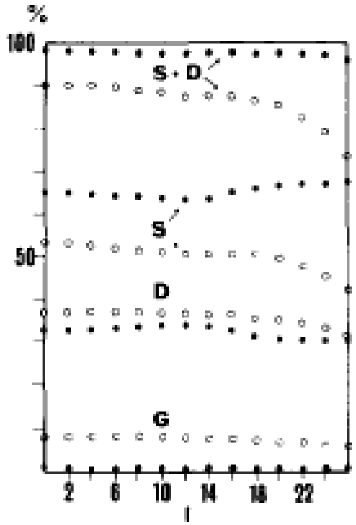


Fig. 2. The probabilities of $\Gamma^{\dagger}_{JM}$ in the one paired state for each of the p_+ and p_- shell versus the angular momentum I. The circles are for the p_- shell and the dots are for the p_+ shell. The summed probabilities of S + D are also shown.

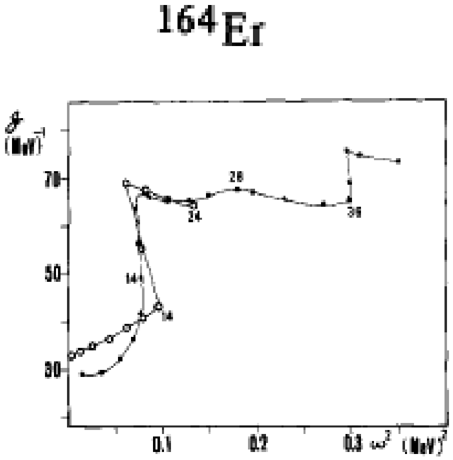


Fig. 1. The moments of inertia estimated from the excitation energies. The circles are experimental values and the dots are theoretical values. The theoretical value starts from the 4$^+$ state.

Self-consistent constrained HFB equation

$$H = \sum_i \epsilon_i c_i^+ c_i + \frac{1}{2} \sum_{\mu=-2}^{2} \sum_{ijkl} \chi_{ijkl} (r^2 Y_{2\mu}^*)_{ik} c_i^+ c_k (r^2 Y_{2\mu})_{jl} \times c_j^+ c_l + \frac{1}{4} \sum_{ij} G_{ij} c_i^+ c_i^+ c_j c_j \,,$$

$$H' = H - \lambda_\pi \hat{N}_\pi - \lambda_\nu \hat{N}_\nu - \omega \hat{J}_x$$

$$\langle \hat{N}_\pi \rangle = N_\pi, \quad \langle \hat{N}_\nu \rangle = N_\nu, \quad \langle \hat{J}_x \rangle = I$$

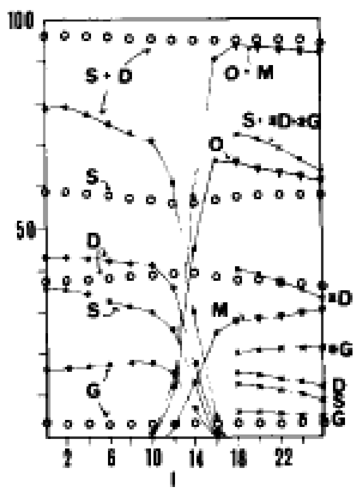
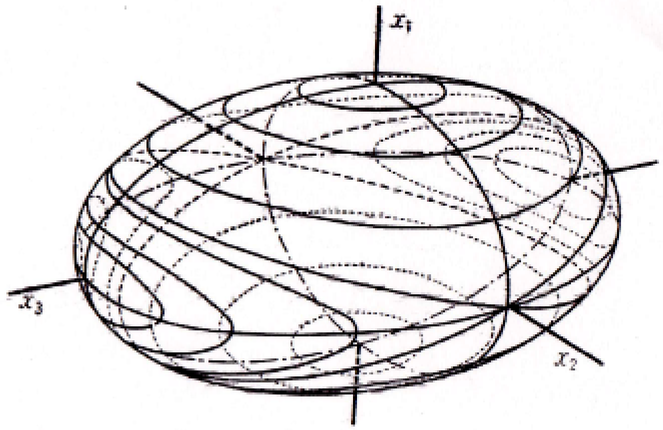


Fig. 3. The same values as fig. 2 for each of the n_+ and n_- shell. The circles are for the n_- shell and the dots are for the n_+ shell. The crosses are for the paired part and the triangles are for the decoupled levels at $I \geq 18$. ΣD and ΣG are the summed values over all the components of D_q and G_q, while D and G denote D_0 and G_0. O and M are also summed values over all components.

Spherical single-particle energy
monopole pairing and
quadrupole –quadrupole interactions

(2) Wobbling motion （Precession)



Landau and Lifschits (1958)：
Goldstein(2002)

$$\mathcal{J}_1 < \mathcal{J}_2 < \mathcal{J}_3$$
$$2E = \frac{L_1^2}{\mathcal{J}_1} + \frac{L_2^2}{\mathcal{J}_2} + \frac{L_3^2}{\mathcal{J}_3} \text{ (ellipsoid)}$$
$$L^2 = L_1^2 + L_2^2 + L_3^2 \text{ (sphere)}$$
$$2E\mathcal{J}_1 < L^2 < 2E\mathcal{J}_3$$

The intersection between ellipsoid and sphere is the orbit for given *L*and *E*. In the plane perpendicular to x_1 or x_3 axis the orbit is ellipse, while to x_2 axis it becomes hyperbola. There is **no stable rotation around the x_2 axis with middle MoI**.

Bohr-Mottelson (1967)

$$\mathcal{J}_1 > \mathcal{J}_2 > \mathcal{J}_3 \qquad A_k = 1/(2\mathcal{J}_k) \ (k = 1, 2, 3 \text{ or } x, y, z)$$
$$I_+ = \sqrt{2I}c^+, \ I_- = \sqrt{2I}c, \ I_1 = I$$
$$H_{\rm rot} = A_1 I(I+1) + (n + \frac{1}{2})\hbar\omega$$
$$\hbar\omega = 2I\sqrt{(A_2 - A_1)(A_3 - A_1)}$$

There is no wobbling around 2-axis with middle MoI, because wobbling energy is imaginary.

Top-on-top model (particle-rotor model) in odd-A nucleus

T & S-T, P.R. C73,034305(2006); C77,064318(2008); C95 064315,(2017)

$H=H_{rot}+H_{sp}$

$$H_{\rm rot} = \sum_{k=x,y,z} A_k(I_k - j_k)^2, \qquad A_k = 1/(2\mathcal{J}_k)$$

- Single-particle H

$$H_{\rm sp} = \frac{V}{j(j+1)}\left[\cos\gamma\left(3j_z^2 - \vec{j}^2\right) - \sqrt{3}\sin\gamma\left(j_x^2 - j_y^2\right)\right],$$

- Rigid-body *MoI*

$$\mathcal{J}_k^{\rm rig} = \frac{\mathcal{J}_0}{1+\left(\frac{5}{16\pi}\right)^{1/2}\beta_2}\left[1-\left(\frac{5}{4\pi}\right)^{1/2}\beta_2\cos\left(\gamma+\frac{2}{3}\pi k\right)\right]$$

- Hydrodynamical *MoI*

$$\mathcal{J}_k^{\rm hyd} = \frac{4}{3}\mathcal{J}_0\sin^2\left(\gamma+\frac{2}{3}\pi k\right)$$

- D2-invariance

$$\left\{\sqrt{\frac{2I+1}{16\pi^2}}\left[\mathcal{D}^I_{MK'}(\theta_i)\phi^j_{\Omega'} + (-1)^{I-j}\mathcal{D}^I_{M-K'}(\theta_i)\phi^j_{-\Omega'}\right];\right.$$

$$\left. |K' - \Omega'| = \text{even}, \quad \Omega' > 0\right\},$$

Wigner-Eckart theorem
(the strength of V is not arbitrary)

K.S.-T., K.T. & N.Yoshinaga , PTEP 2014, 063D01 (2004)

$$H_{sp} = H_0 + H_\delta,$$
$$H_\delta = -\hbar\omega_0(\delta)\beta_2 r^2\left(\cos\gamma Y_{20} - \sin\gamma\frac{1}{2}(Y_{22}+Y_{2-2})\right)$$

$$\langle jm|r^2(\cos\gamma Y_{20} - \sin\gamma\frac{1}{2}(Y_{22}+Y_{2-2})|jm\rangle$$
$$= -\ \langle jm|r^2\frac{1}{8j(j+1)}\sqrt{\frac{5}{\pi}}\left(\cos\gamma(3j_z^2 - \vec{j}^2) - \sqrt{3}\sin\gamma(j_x^2 - j_y^2)\right)|jm\rangle$$

$$V = \frac{1}{8}\sqrt{\frac{5}{\pi}}\hbar\omega_0(\delta)\beta_2\langle r^2\rangle$$

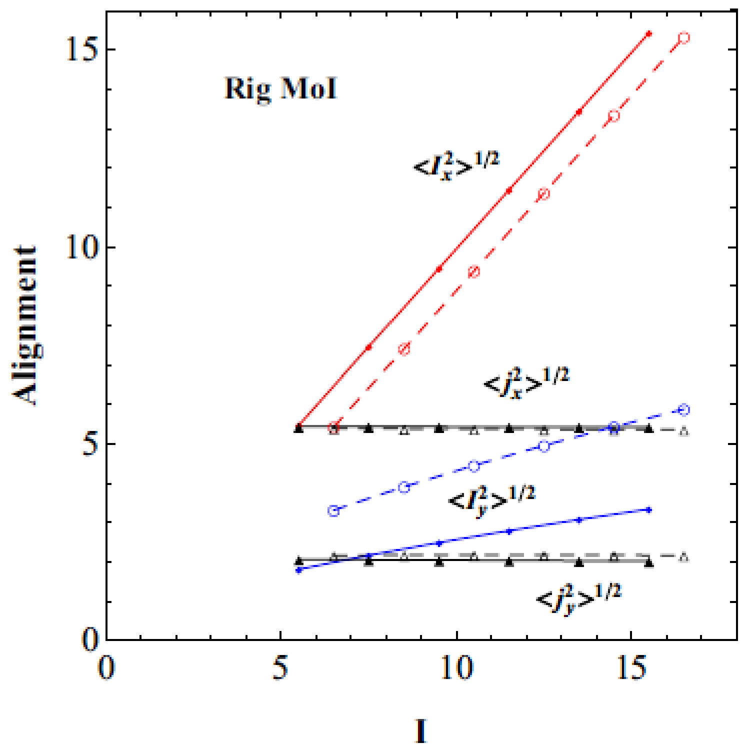
$$\langle r^2\rangle = N + \frac{3}{2}$$

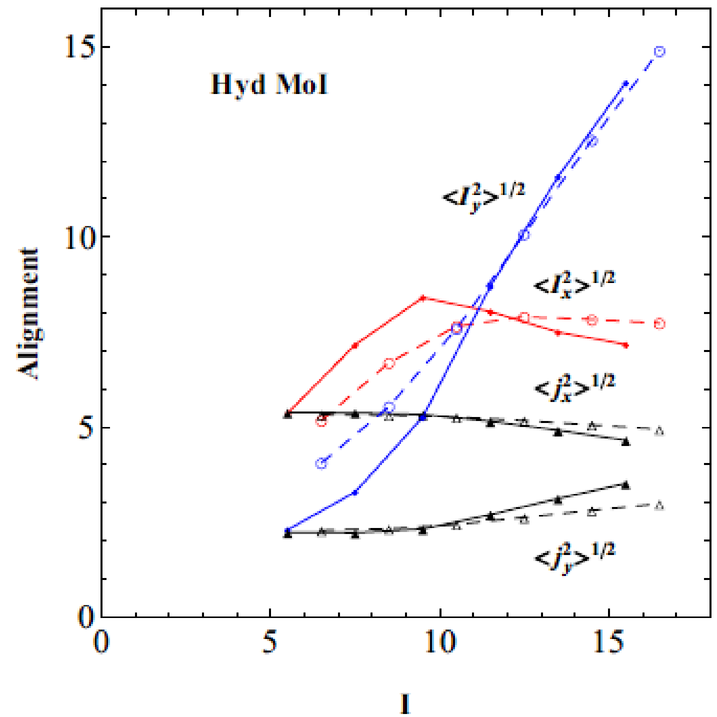
$h_{11/2}$ at $\beta_2 = 0.18 \rightarrow V = 1.5$ MeV

The competition between Coriolis term ***I.j*** and single-particle pot. V/j(j+1)

Comparison of Alignment between rig MoI and hyd MoI with V

1 unit alignment is found in rig MoI, but not in hyd MoI.





-------: I-j=even, - - - - - : I-j=odd

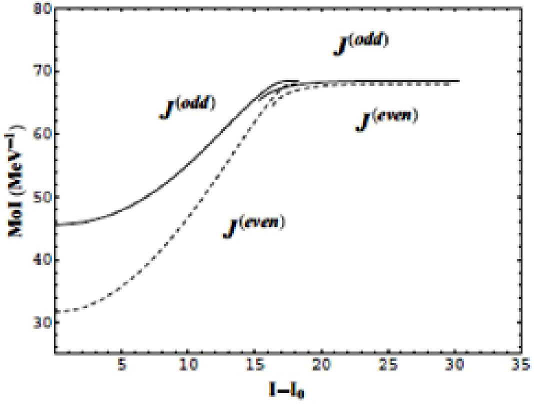
J_0 =25 MeV^{-1} V=1.6MeV γ=26°

Frozen alignment approximation (Frauendorf & Doenau) is very bad!

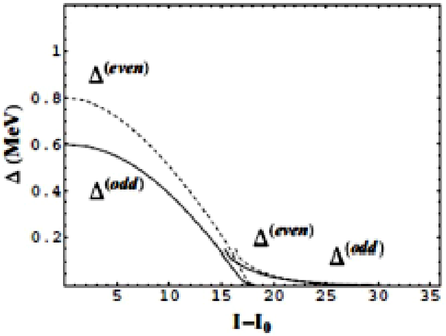
The pairing effect on MoI in perturbation treatment of Coriolis term

T. & S-T, P.R. C91, 034328 (2015).

I-dependence of MoI through Δ



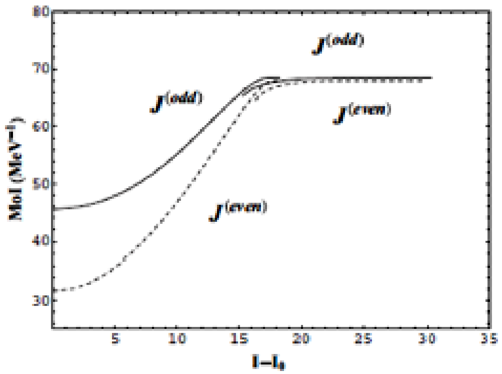
2 kinds of asymptotic series expansion for Δ≥d/2 and Δ<d case. d is the average single-particle level distance.



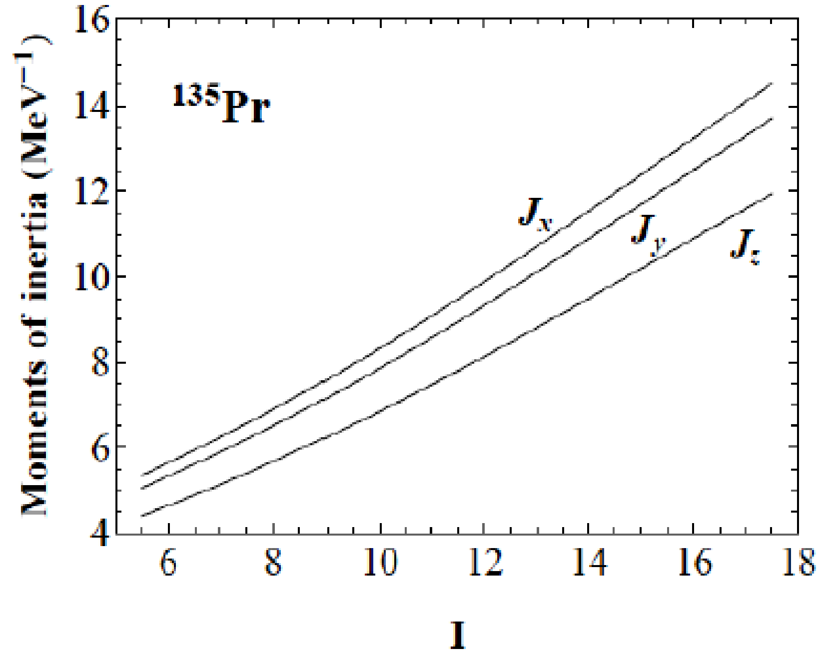
Δ keeps a small but finite values even for high spin states, indicating no sharp phase transition in nucleus.

I-dependence of MoI for low-spin state

T. &S-T, C95, 064315 (2017)



From this figure we assume the *I*-dependence of MoI in low-spin and low-excited levels.



Wood-Saxon type

$$\frac{\mathcal{J}_0}{1+\exp(-(I-\bar{b})/\bar{a})}$$

$\bar{a} = 7.5$ and $\bar{b} = 15.5$ $\gamma = 18^\circ$

$\mathcal{J}_0 = 25$ MeV^{-1}, $V = 1.6$ MeV, β_2 =0.18

I-dependence of MoI for high spin state

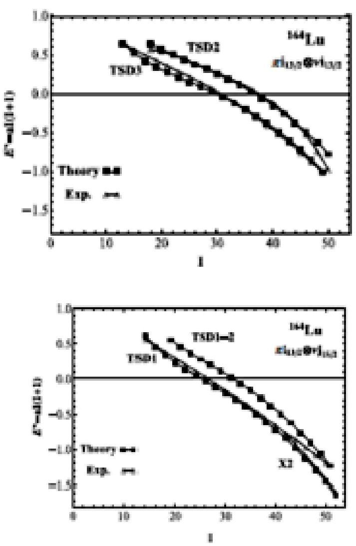
Cf. **High spin highly excited band case**

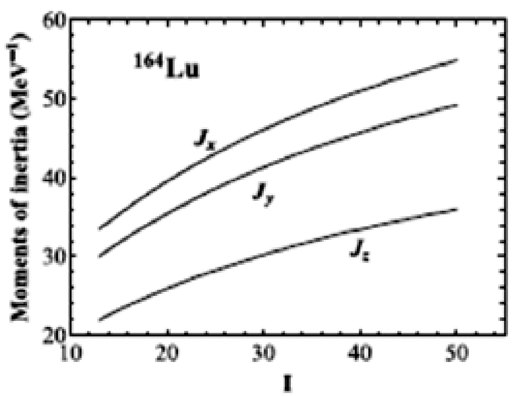
$$\mathcal{J}_0 \rightarrow \mathcal{J}_0 \frac{I - c_1}{I + c_2}.$$

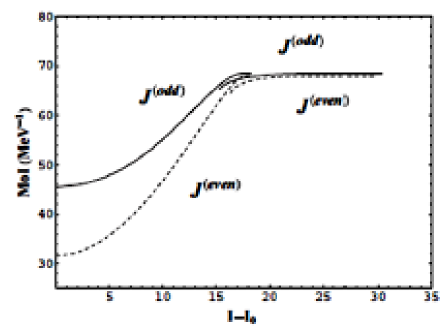
PRC 77, 064318(2008) ^{161,163,165}Lu: c1=0.69, c2=23.5

PRC 82, 051303 (R) (2010) ^{167}Lu,^{167}Ta: c1=4,c2=27.8

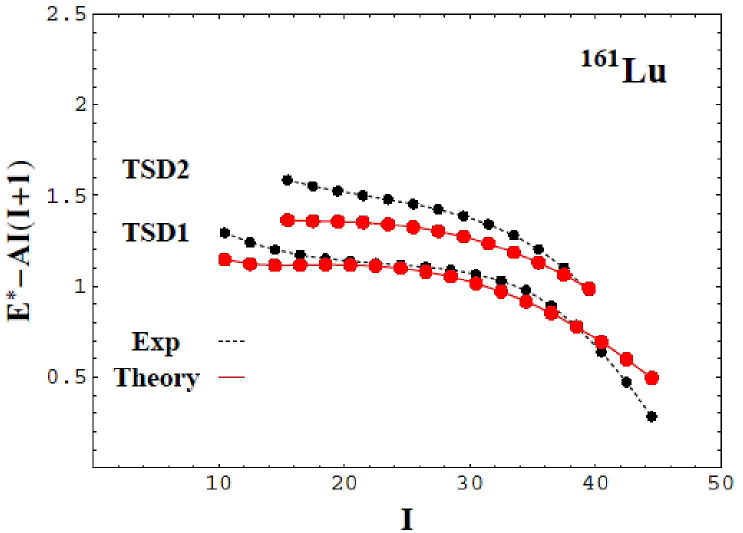
PTEP 2014,063D01 (2014) ^{164}Lu: c1=8,c2=41.0

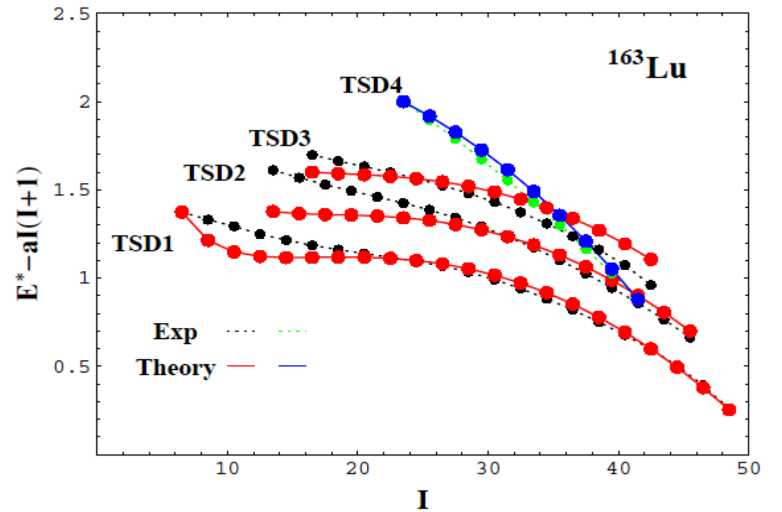


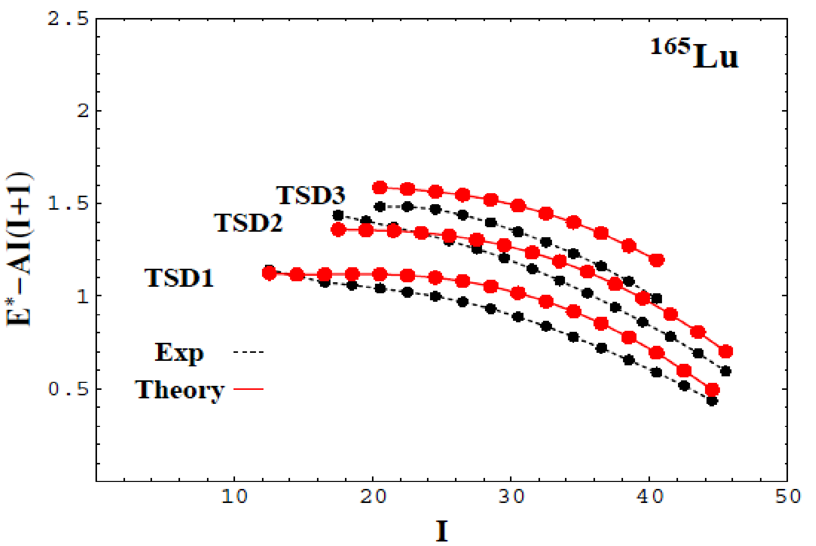


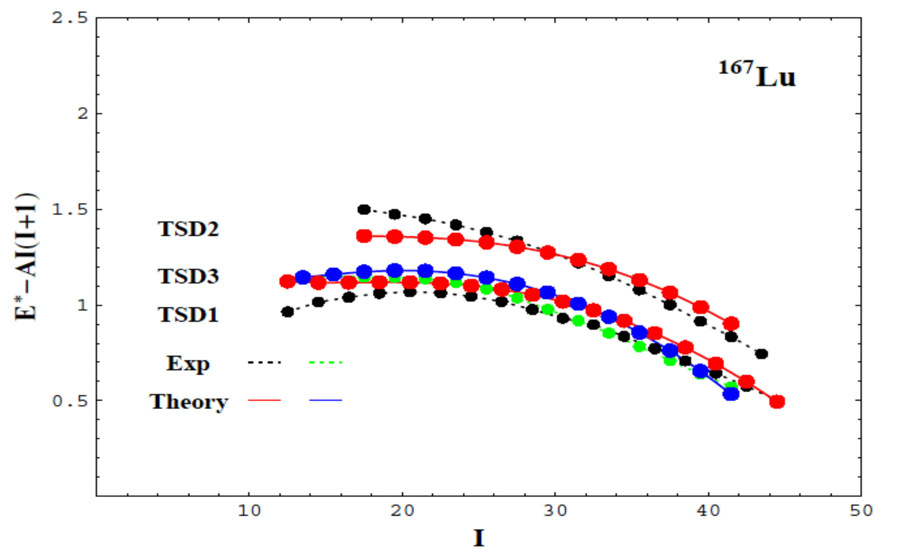


T. & S-T, P.R. C77, 064318(2008)



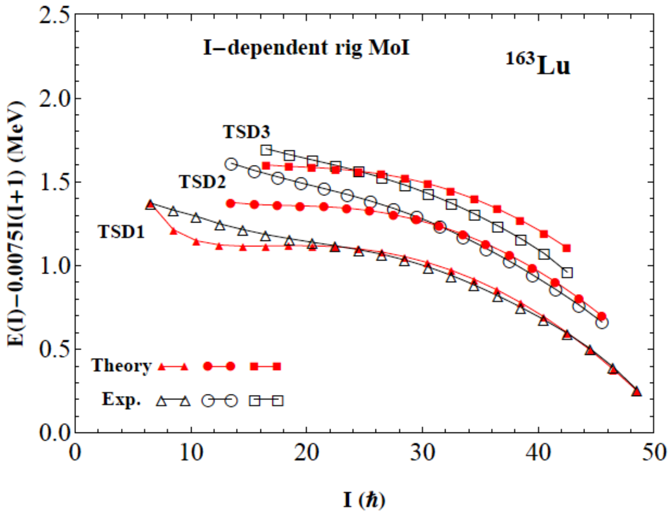




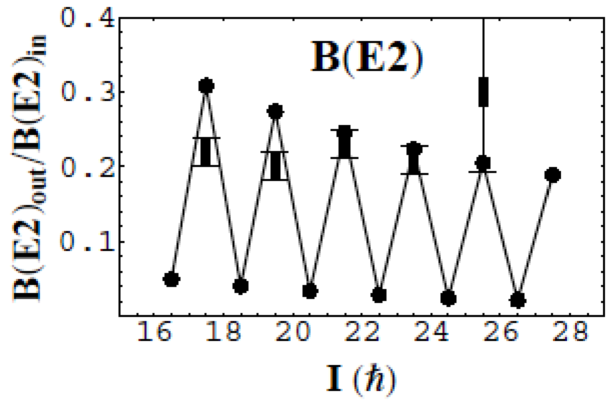


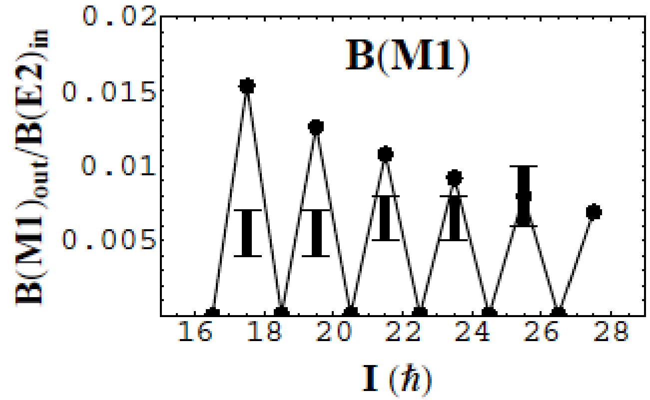
B(E2) and B(M1) transition in addition to energy levels

T. & S-T, P.R. C77, 064318 (2008)



T. &S-T, P.R. C73,034305 (2006)





Exp: Oedegard et al, P.R.L.86, 5866 (2001))

B(M1)$_{out}$ in ^{163}Lu

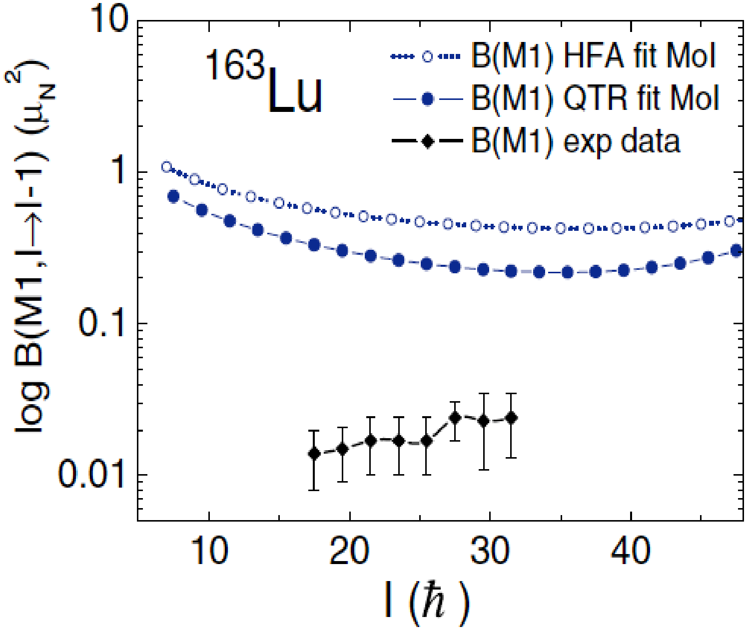
B(M1)$_{out}$ as a function of *I* in (μ_N^2)

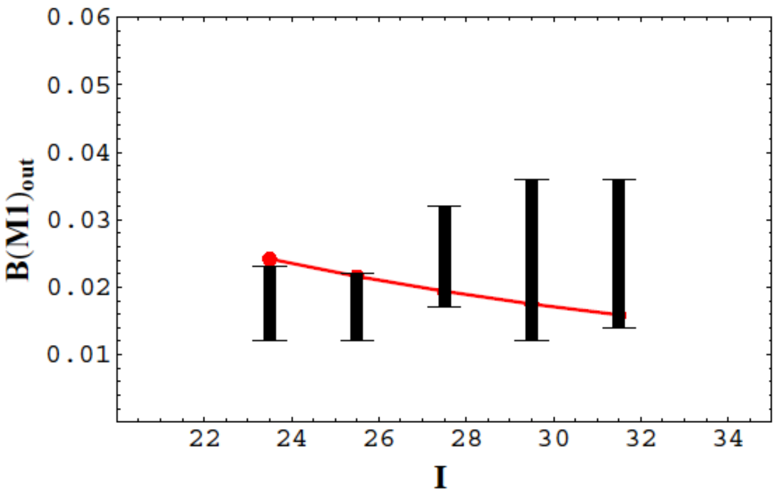
exp: A.Görgen et.al. , Phys. Rev. C69, (2004) 031301(R)

Frauendorf and Doenau

P.R.C.89,014322(2014)

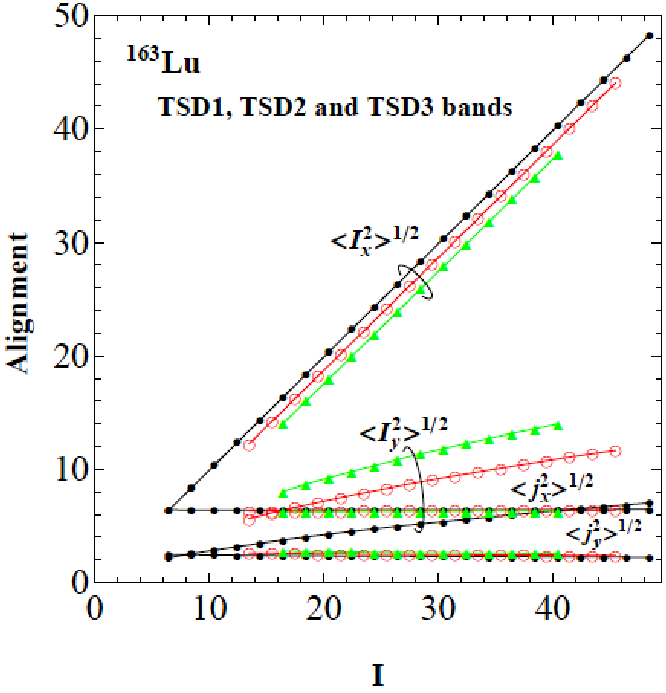
K.T. and K. S.-T., P.R.C77,064318(2008)

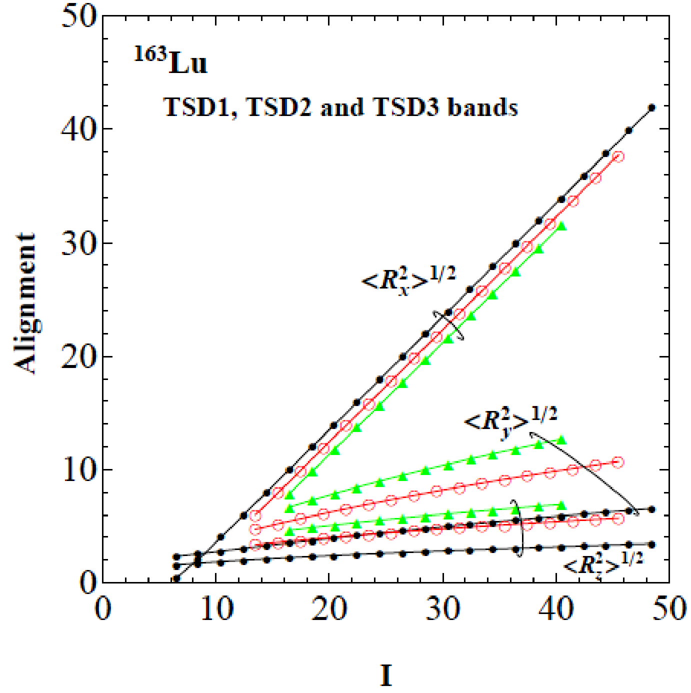




Alignments in ^{163}Lu

S-T & T., to be published in JPSJ for the retirement of Prof. Otsuka (2018)





$\frac{1}{2}(\langle I_x^2\rangle_{I+2}^{1/2} + \langle I_x^2\rangle_I^{1/2}) - \langle I_x^2\rangle_I^{1/2}$ is almost 1 for $I - j$ =even

$$\langle I_x^2\rangle_{I+2}^{1/2} - \langle I_x^2\rangle_I^{1/2} \sim 2$$

$< R_x^2 >_I^{1/2} \sim < R_x^2 >_{I+1}^{1/2} \sim < R_x^2 >_{I+2}^{1/2}$ for $I - j =$ even,

$< R_x^2 >_{I+2}^{1/2} - < R_x^2 >_I^{1/2} \sim 2$ for each TSD band.

$Q_0 = 3$ b for ^{135}Pr, with β_2=0.18 and $\gamma = 18°$
$g_{\rm eff} = 0.414$,
bare value of $g_\ell - g_R + (g_s - g_\ell)/(2j)$ multiplied by the quenching factor 0.5.

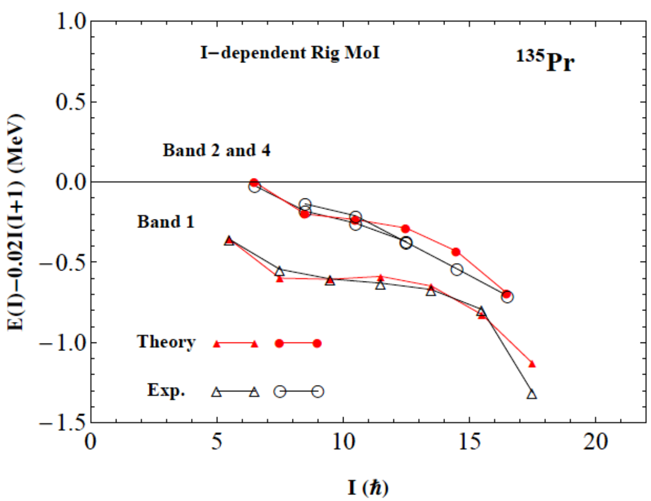
$\delta \propto g_{\rm eff}/Q_0$

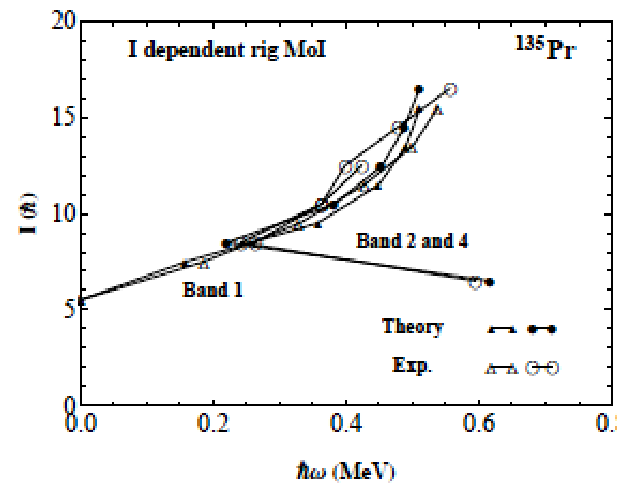
The mixing ratio δ

I	$B(E2)_{\rm out}/B(E2)_{\rm in}$ exp.	$B(E2)_{\rm out}/B(E2)_{\rm in}$ theory	$B(M1)_{\rm out}/B(E2)_{\rm in}$ exp.	$B(M1)_{\rm out}/B(E2)_{\rm in}$ theory	δ exp.	δ theory
17/2	...	0.648	...	0.192	-1.24 ± 0.13	-1.13
21/2	0.843 ± 0.032	0.542	0.164 ± 0.014	0.130	-1.54 ± 0.09	-1.34
25/2	0.500 ± 0.025	0.463	0.035 ± 0.009	0.0987	-2.38 ± 0.37	-1.44
29/2	$\geq 0.261 \pm 0.014$	0.402	$\leq 0.016 \pm 0.004$	0.0791	...	-1.49

$B(M1)_{\rm out}/B(E2)_{\rm in} \propto (g_{\rm eff}/Q_0)^2$

T. &S-T, P.R. C95, 064315 (2017)
Exp.: Matta et al, P.R.L. 114, 082501 (2014)

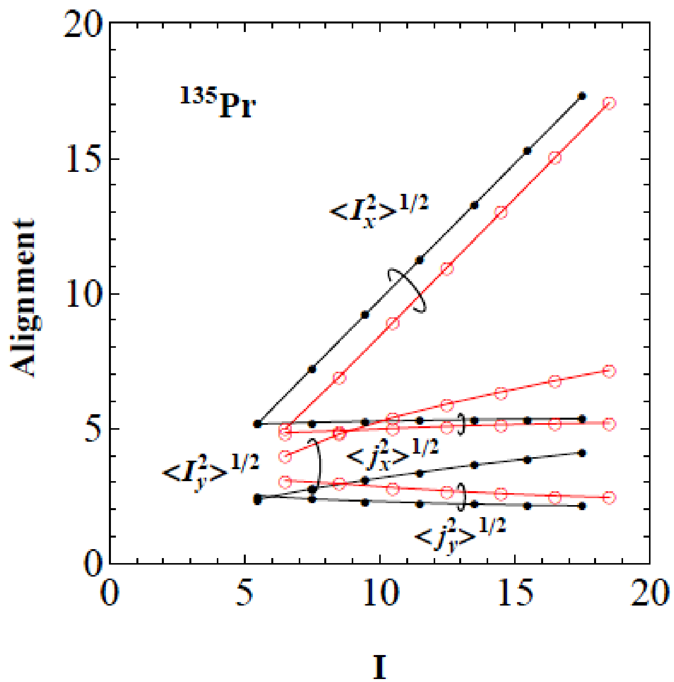


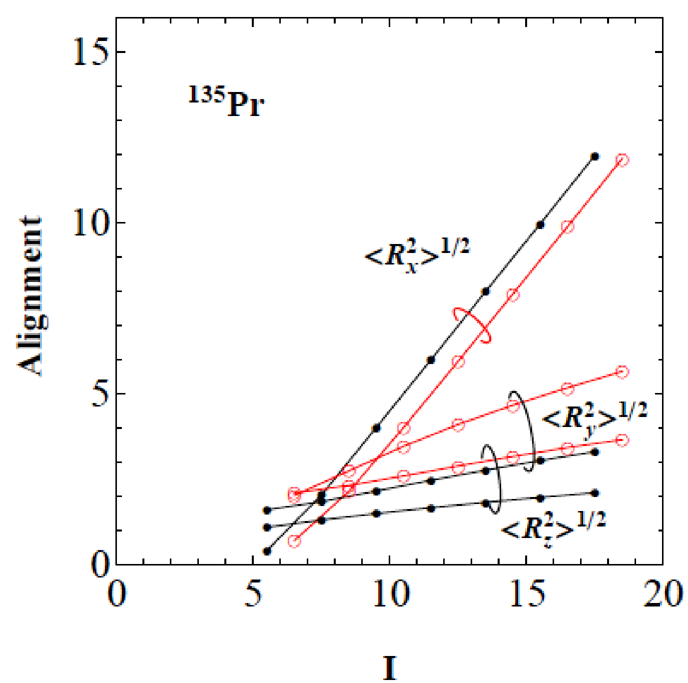


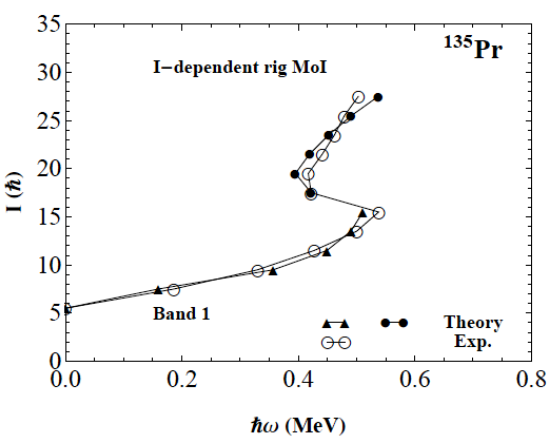
$$Q_0 = \frac{Z}{5}(2R_z^2 - R_x^2 - R_y^2) = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_2 \cos\gamma,$$

$\hbar\omega = E_\gamma/2$
$\hbar\omega$ for the $I = 13/2$ is E_γ to the $I = 11/2$

Alignments in ^{135}Pr case







$\frac{1}{2}(\langle I_x^2\rangle_{I+2}^{1/2} + \langle I_x^2\rangle_I^{1/2}) - \langle I_x^2\rangle_I^{1/2}$ is almost 1 for $I - j$ =even

$$\langle I_x^2\rangle_{I+2}^{1/2} - \langle I_x^2\rangle_I^{1/2} \sim 2$$

Exp: Paul et al, P. R. C84, 047302 (2011)

Backbending plot for Band 1

(3)The self-consistent constrained HFB equation

$$H' = \sum_{\alpha}(\epsilon_\alpha - \lambda)c^\dagger_\alpha c_\alpha - \omega \sum_{\alpha,\beta}(j_x)_{\alpha\beta}\, c^\dagger_\alpha c_\beta - \frac{g}{4}\sum_{\alpha,\beta} c^\dagger_{\bar{\alpha}} c^\dagger_\alpha c_\beta c_{\bar{\beta}},$$

$$-\frac{g'}{4}\sum_{\mu=0}^{4}(\sum_{\alpha,\beta}(y^*_\mu)_{\alpha\,\beta}\, c^\dagger_{\bar{\alpha}} c^\dagger_{\bar{\beta}} \sum_{\gamma,\delta}(y_\mu)_{\gamma\,\delta}\, c_\gamma c_{\bar{\delta}}) \quad - \frac{\chi}{2}\sum_{\mu=0}^{4}(\sum_{\alpha,\beta}(y^*_\mu)_{\alpha\,\beta}\, c^\dagger_\alpha c_\beta \sum_{\gamma,\delta}(y_\mu)_{\gamma\,\delta}\, c^\dagger_\gamma c_\delta).$$

$$\sum_{\alpha}\langle c^\dagger_\alpha c_\alpha\rangle = N,$$

$$\sum_{\alpha,\beta}(j_x)_{\alpha\beta}\langle\, c^\dagger_\alpha c_\beta\rangle = \sqrt{I(I+1)}.$$

$$y_0 = r^2 Y_{20},\ y_1 = \frac{1}{\sqrt{2}}(r^2 Y_{21} + r^2 Y_{2-1}),\ y_2 = \frac{1}{\sqrt{2}}(r^2 Y_{22} + r^2 Y_{2-2})$$

$$y_3 = \frac{1}{\sqrt{2}}(r^2 Y_{21} - r^2 Y_{2-1}),\ y_4 = \frac{1}{\sqrt{2}}(r^2 Y_{22} - r^2 Y_{2-2}).$$

(3-1) Signature invariant base (Goodman-base)

Goodman, N.P. A230, 466 (1974)

$$\begin{aligned}\sqrt{2}C_a^\dagger &= c_\alpha^\dagger + (-)^{\ell_\alpha - m_\alpha + 1/2} c_{\bar\alpha}^\dagger,\\ \sqrt{2}C_{\hat a}^\dagger &= c_{\bar\alpha}^\dagger - (-)^{\ell_\alpha - m_\alpha + 1/2} c_\alpha^\dagger.\end{aligned}$$

$$R_x = \exp(-i\pi J_x),$$

$$R_x C_a^\dagger R_x^\dagger = iC_a^\dagger,\quad R_x C_a R_x^\dagger = -iC_a,\quad R_x C_{\hat a}^\dagger R_x^\dagger = -iC_{\hat a}^\dagger,\quad R_x C_{\hat a} R_x^\dagger = iC_{\hat a}.$$

$$\begin{pmatrix} y_0 & 0 \\ 0 & y_0 \end{pmatrix},\ \begin{pmatrix} y_2 & 0 \\ 0 & y_2 \end{pmatrix},\ \begin{pmatrix} y_1 & 0 \\ 0 & y_1 \end{pmatrix},\ \begin{pmatrix} 0 & y_3 \\ -y_3 & 0 \end{pmatrix},\ \begin{pmatrix} 0 & y_4 \\ y_4 & 0 \end{pmatrix},\begin{pmatrix} J_x & 0 \\ 0 & -J_x \end{pmatrix},\ \begin{pmatrix} 0 & J_y \\ J_y & 0 \end{pmatrix},\ \begin{pmatrix} 0 & J_z \\ J_z & 0 \end{pmatrix}$$

$$(y_{0/2})_{a\,b} = (y_{0/2})_{b\,a},\ (y_1)_{a\,b} = -(y_1)_{b\,a},\ (y_{3/4})_{a\,b} = -(y_{3/4})_{b\,a}.\qquad (J_{x/y/z})_{a\,b} = (J_{x/y/z})_{b\,a}$$

$$\begin{aligned}\alpha_i^\dagger &= \sum_{k>0}(C_k^\dagger A_{ki} + C_{\hat k} B_{ki}),\\ \alpha_{\hat i}^\dagger &= \sum_{k>0}(C_{\hat k}^\dagger \hat A_{ki} + C_k \hat B_{ki}).\end{aligned}$$

$$\begin{aligned}C_i^\dagger &= \sum_{k>0}(A_{ik}^* \alpha_k^\dagger + \hat B_{ik}\alpha_{\hat k}),\\ C_{\hat i}^\dagger &= \sum_{k>0}(\hat A_{ik}^* \alpha_{\hat k}^\dagger + B_{ik}\alpha_k),\end{aligned}$$

$$R_x \alpha_k^\dagger R_x^\dagger = -i\alpha_k^\dagger,\quad R_x \alpha_{\hat k}^\dagger R_x^\dagger = i\alpha_{\hat k}^\dagger$$

Orthonormality relations

$$(A^TA^*+B^TB^*)_{ij}=\delta_{ij},\quad (A^\dagger A+B^\dagger B)_{ij}=\delta_{ij},$$
$$(\hat{A}^T\hat{A}^*+\hat{B}^T\hat{B}^*)_{ij}=\delta_{ij},\quad (\hat{A}^\dagger\hat{A}+\hat{B}^\dagger\hat{B})_{ij}=\delta_{ij},$$
$$(A^T\hat{B}+B^T\hat{A})_{ij}=0,\quad (\hat{B}^TA+\hat{A}^TB)_{ij}=0 \text{ and h. c..}$$

$$(A^*A^T+\hat{B}\hat{B}^\dagger)_{ij}=(AA^\dagger+\hat{B}^*\hat{B}^T)_{ij}=\delta_{ij},$$
$$(\hat{A}^*\hat{A}^T+BB^\dagger)_{ij}=(\hat{A}\hat{A}^\dagger+B^*B^T)_{ij}=\delta_{ij},$$
$$(AB^\dagger+\hat{B}^*\hat{A}^T)_{ij}=(B^*A^T+\hat{A}\hat{B}^\dagger)_{ij}=0$$
$$(\hat{A}^*\hat{B}^T+BA^\dagger)=(A^*B^T+\hat{B}\hat{A}^\dagger)_{ij}=0$$

$$\begin{pmatrix} h^1-\omega J_x & \Delta^T \\ \Delta & -h^2-\omega J_x \end{pmatrix}\begin{pmatrix} \hat{B} & A \\ \hat{A} & B \end{pmatrix}=\begin{pmatrix} \hat{B} & A \\ \hat{A} & B \end{pmatrix}\begin{pmatrix} -\hat{\Lambda} & 0 \\ 0 & \Lambda \end{pmatrix}$$

CHFB equation

$$\delta\langle H'\rangle=0,\quad \delta^2\langle H\rangle\geq 0.$$

$$H'^{11}=\sum_{i>0}(\Lambda_i\alpha_i^\dagger\alpha_i+\hat{\Lambda}_i\alpha_{\hat{i}}^\dagger\alpha_{\hat{i}}),$$
$$H'^{20}=\sum_{i,j>0}\Big(\hat{B}^T(h^1A+\Delta^TB)-\hat{A}^T(h^2B-\Delta A)\Big)_{ij}\alpha_{\hat{i}}\alpha_j+h.c.=0.$$

$$\rho^1_{kk'}=\langle C^\dagger_{k'}C_k\rangle=(\hat{B}\hat{B}^\dagger)_{k'k},\quad \rho^2_{\hat{k}\hat{k}'}=\langle C^\dagger_{\hat{k}'}C_{\hat{k}}\rangle=(BB^\dagger)_{k'k},$$

$$\kappa_{\hat{k}'k}=\langle C_kC_{\hat{k}'}\rangle=(B^*A^T)_{k'k}=-\kappa_{k\hat{k}'},$$

$$h^1_{ij}=(\epsilon-\lambda)\delta_{ij}+\Gamma^1_{ij},\quad h^2_{ij}=(\epsilon-\lambda)\delta_{ij}+\Gamma^2_{ij},$$

$$\Gamma^1_{ij}=\chi\sum_{m,n>0}\Big(\sum_{\mu=0,2}(y_\mu)_{ij}(y_\mu)_{mn}(\rho^1_{mn}+\rho^2_{\hat{m}\hat{n}})-(\sum_{\mu=0}^{2}(y^*_\mu)_{in}\rho^1_{nm}(y_\mu)_{mj}$$
$$+\sum_{\mu=3}^{4}(y_\mu)_{i\hat{n}}\rho^2_{\hat{n}\hat{m}}(y_\mu)_{\hat{m}j})\Big)+g\rho^2_{\hat{i}\hat{j}},$$

$$\Gamma^2_{ij}=\chi\sum_{m,n>0}\Big(\sum_{\mu=0,2}(y_\mu)_{ij}(y_\mu)_{mn}(\rho^1_{mn}+\rho^2_{\hat{m}\hat{n}})-(\sum_{\mu=0}^{2}(y^*_\mu)_{\hat{i}\hat{n}}\rho^2_{\hat{n}\hat{m}}(y_\mu)_{\hat{m}\hat{j}}$$
$$+\sum_{\mu=3}^{4}(y_\mu)_{\hat{i}n}\rho^1_{nm}(y_\mu)_{m\hat{j}})\Big)+g\rho^1_{ij},$$

$$\Delta_{\hat{i}j}=g\sum_k\kappa_{\hat{k},k}\delta_{\hat{i}j}+\sum_{m,n>0}\Big(g'\sum_{\mu=0}^{2}(y^*_\mu)_{\hat{i}\hat{m}}\kappa_{\hat{m}n}(y_\mu)_{nj}$$
$$+\chi(\sum_{\mu=0}^{2}(y^*_\mu)_{\hat{i}\hat{n}}\kappa_{\hat{n}m}(y_\mu)_{mj}-\sum_{\mu=3}^{4}(y_\mu)_{\hat{i}n}\kappa_{\hat{m}n}(y_\mu)_{\hat{m}j})\Big).$$

(3-2) Gradient method (Steepest descent method)

Mang, Samadi & Ring, Z.P. A279, 323 (1976).

$$\delta H'^{20} = \sum_{i,j>0} H'^{20}_{ij} D_{ij}, \quad \delta N = \sum_{i,j>0} N^{20}_{ij} D_{ij}, \quad \delta J = \sum_{i,j>0} J^{20}_{ij} D_{ij}.$$

$$D_{ij} = (-\phi H'^{20} + \phi_N N^{20} + \phi_I J^{20})_{ij}.$$

ϕ is chosen between 0.25 and 2.0

condition

$$\sum_{i,j>0} H'^{20}_{ij} N^{20}_{ij} = 0, \quad \sum_{i,j>0} H'^{20}_{ij} J^{20}_{ij} = 0$$

$$\begin{pmatrix} N^{20}N^{20} & N^{20}J^{20} \\ N^{20}J^{20} & J^{20}J^{20} \end{pmatrix} \begin{pmatrix} \lambda & \phi_N \\ \omega & \phi_I \end{pmatrix} = \begin{pmatrix} H^{20}N^{20} & \delta N \\ H^{20}J^{20} & \delta J \end{pmatrix}.$$

$$\delta J = \sum_{i,j>0} (j_x)_{ij} (\rho^1_{ij} - \rho^2_{\bar{i}\bar{j}}) - \sqrt{I(I+1)}.$$

$$\delta N = \sum_{i>0} (\rho^1_{ii} + \rho^2_{\bar{i}\bar{i}}) - N,$$

Next step

$$A_{ij} \to A_{ij} + \sum_{k>0} \hat{B}_{ik} D_{kj}, \quad B_{ij} \to B_{ij} + \sum_{k>0} \hat{A}_{ik} D_{kj},$$

$$\hat{A}_{ij} \to \hat{A}_{ij} - \sum_{k>0} B_{ik} D_{kj}, \quad \hat{B}_{ij} \to \hat{B}_{ij} - \sum_{k>0} A_{ik} D_{kj}.$$

(3-3) CHFB equation for odd-A case

Projection of angular momentum

$$\delta E^{J} \equiv \delta \frac{\langle\Phi|P^J_M H|\Phi\rangle}{\langle\Phi|P^J_M|\Phi\rangle},$$

$$P^J_M = \sum_K \int d\Omega D^{*J}_{MK}(\Omega) R(\Omega).$$

$$R(\Omega) = e^{i\alpha J_z} e^{i\beta J_y} e^{i\gamma J_z}$$

Beck, Mang & Ring, Z.Physik 231,26(1970)
Ring, Mang & Banerjee, N.P.A225, 141 (1974)

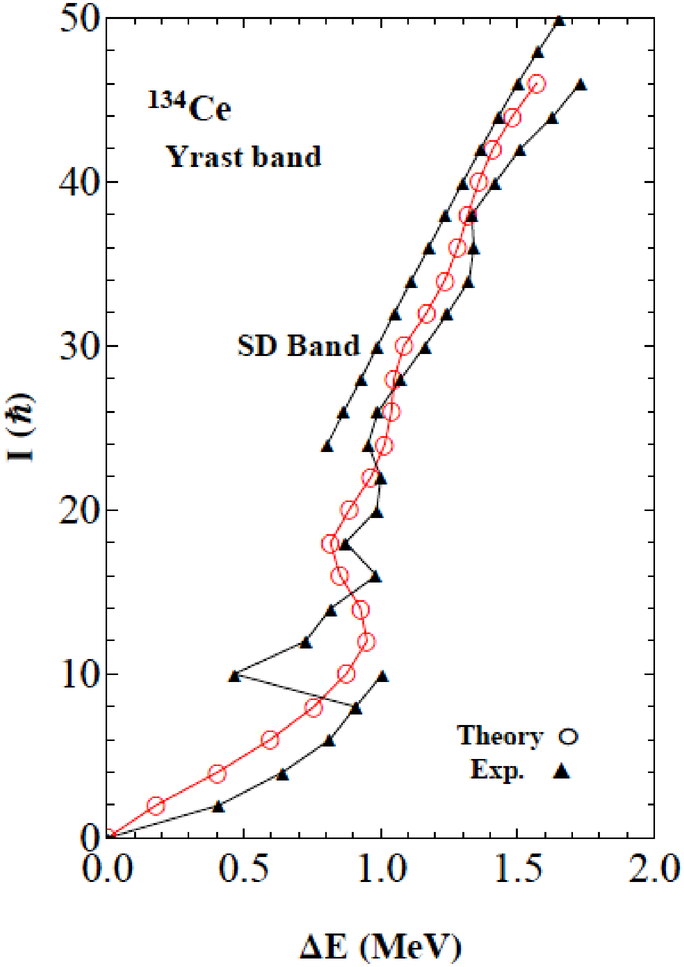
(1) Kamler's expansion (2) Almost axially symmetric

$$\frac{\langle\Phi|J_x P^J + P^J J_x|\Phi\rangle}{\langle\Phi|P^J|\Phi\rangle} \sim \sqrt{J(J+1) - K^2}, \quad \frac{\langle\Phi|J_z^2 P^J + P^J J_z^2|\Phi\rangle}{\langle\Phi|P^J|\Phi\rangle} \sim K^2.$$

$$\delta\langle\Phi|H - \omega J_x|\Phi\rangle = 0, \quad \langle\Phi|J_x|\Phi\rangle = \sqrt{J(J+1) - \langle\Phi|J_z^2|\Phi\rangle}$$

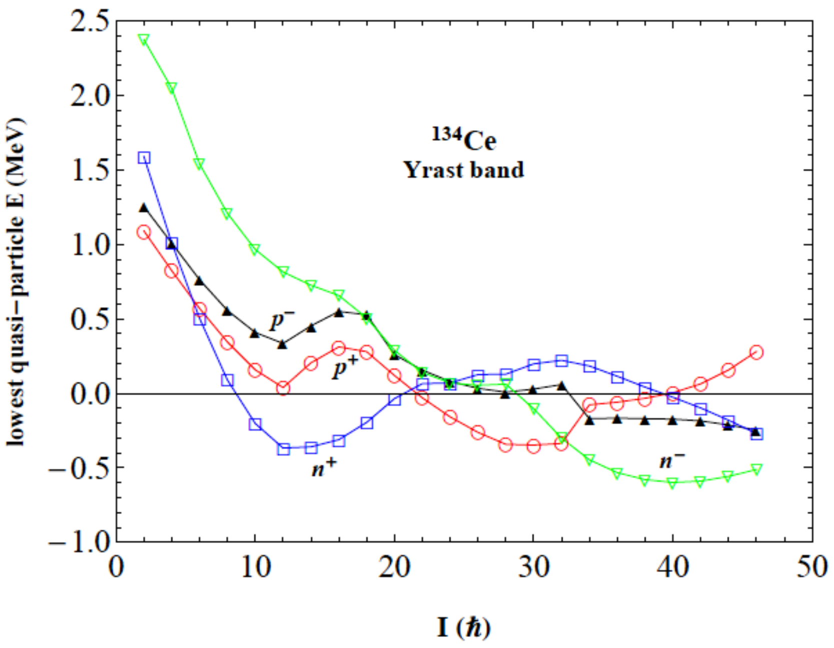
Preliminary results for neighboring even-even nucleus

exp: Paul et al, P.R. C84, 047302 (2011)

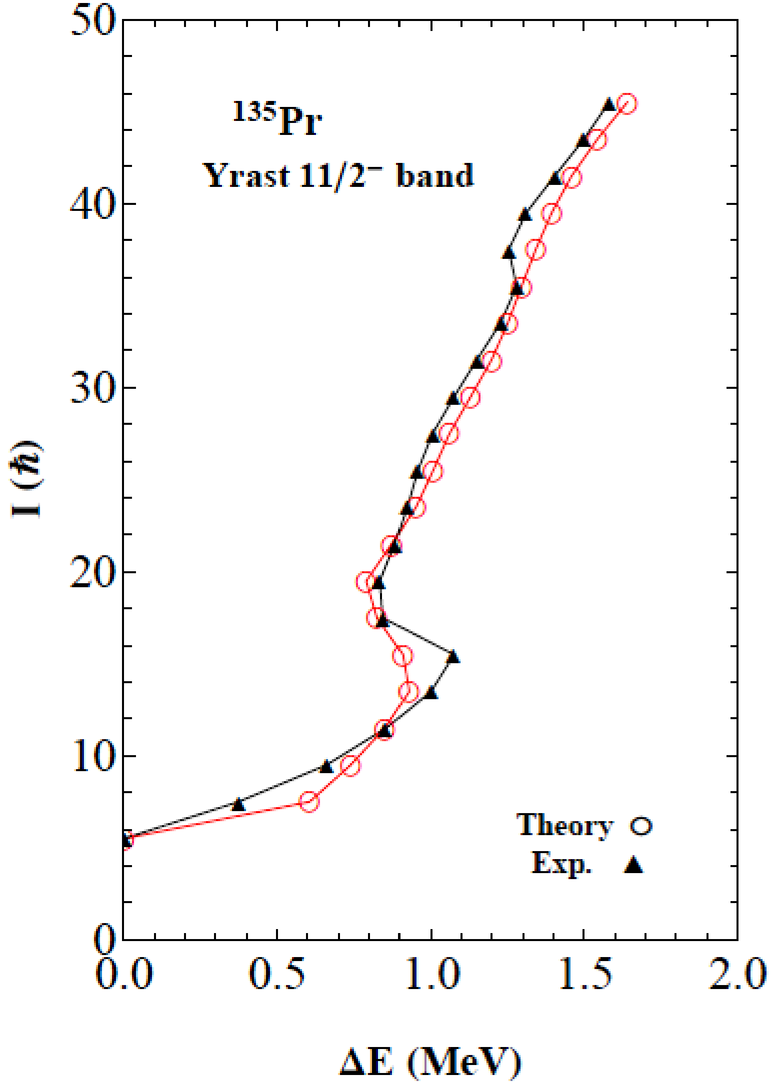


γ is small between -1~2 degrees both for ^{134}Ce and ^{135}Pr.

$$\tan\gamma = -\frac{\sum_{i,j>0}(y_2)ij(\rho^1_{ji}+\rho^2_{ji})}{\sum_{k,l>0}(y_0)_{kl}(\rho^1_{lk}+\rho^2_{lk})}.$$

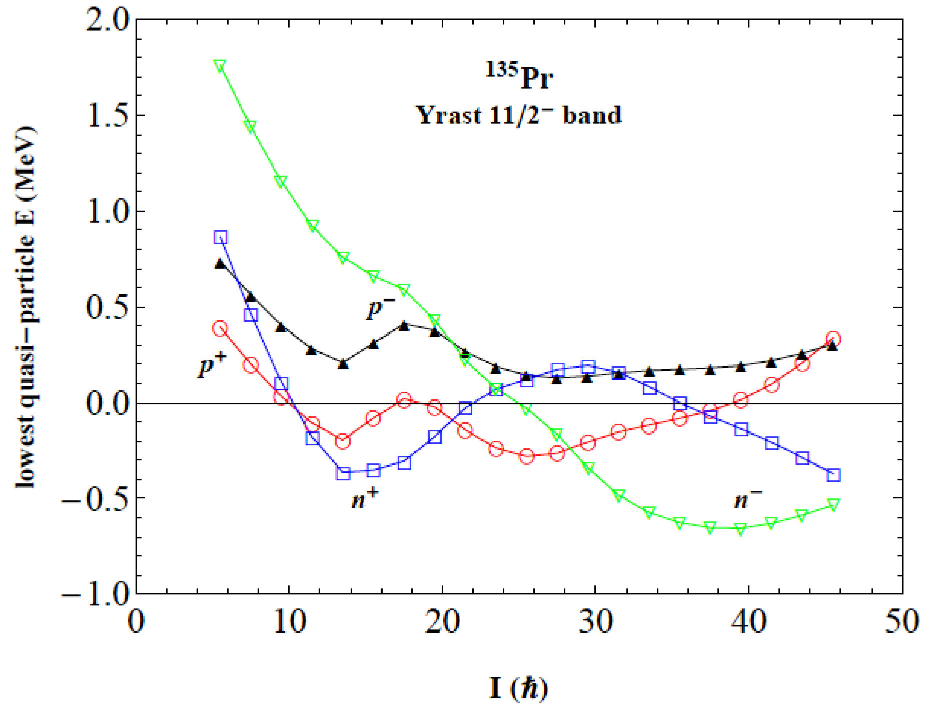


Preliminary results



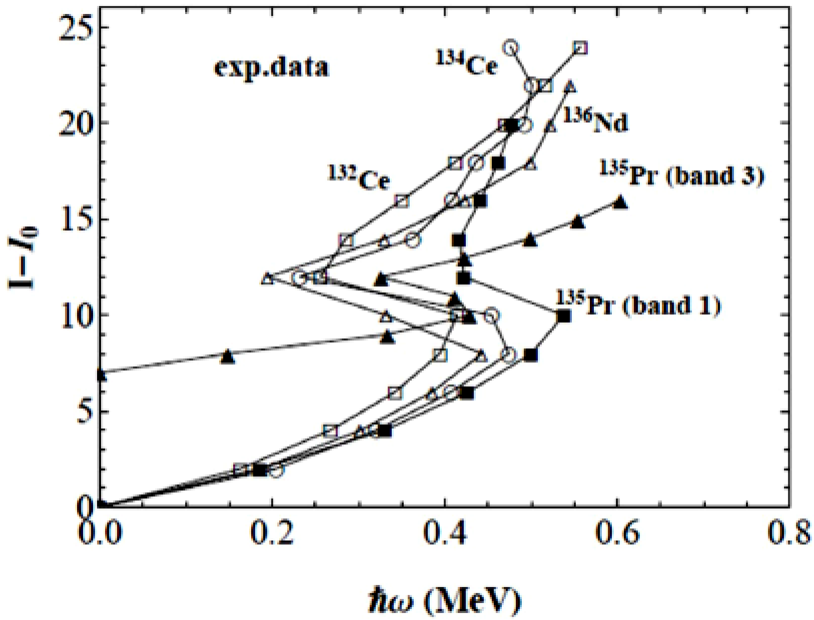
Exp: Paul et al, P.R. C84,047303 (2011)

Jz ; p+:1/2, p-:5/2, n+:3/2, n-:1/2



$\Delta E = E_I - E_{I-2} = E\gamma$

Backbending plot of exp. data



All data show a sharp backbending lump around I-I_0 ~ 12, indicating neutron i13/2 level pair is melted.

Parameters

Table I. Single-particle levels for the valence shell and interaction strengths adopted in the calculation. Single-particle energies are given by the spherical Nilsson levels with $\kappa=0.0637$ and $\mu=0.6$ (0.42) both for positive and negative parity proton (neutron) shells, $\pi\pm$ ($\nu\pm$).

(a) Spherical single-particle levels

$\pi+$: $1g_{9/2}$, $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $1i_{13/2}$

$\pi-$: $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1h_{11/2}$, $1h_{9/2}$, $2f_{7/2}$

$\nu+$: $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, $1i_{13/2}$, $2g_{9/2}$, $1i_{11/2}$

$\nu-$: $1h_{11/2}$, $2f_{7/2}$, $1h_{9/2}$, $3p_{3/2}$, $1j_{15/2}$

Almost the same as ^{132}Ce, and ^{134}Nb case

T. &S-T, P. L. B259, 12(1991)

(b) Parameters for interactions

Monopole-pairing force strength in MeV :

g_n=0.23, g_p=0.22 for Pr and 0.24 for Ce

Quadrupole-pairing force strength in MeV/b^4 :

Monopole-pairing × 10%

Quadrupole-quadrupole force strength in MeV/b^4 :

χ_p=0.03, χ_n=0.032, $\chi_{p\text{-}n}$=0.1

Oscillator strength and length :

$\hbar\omega_0=41.2A^{-1/3}$(MeV), $b=(\hbar/M\omega_0)^{1/2}(\hbar c/\text{MeV})$

(4) Summary

- The wobbling bands in odd-Z nuclei is well explained by the particle-rotor model with I-dependent rigid MoI, which is caused by the Coriolis-anti-pairing effect . This model well reproduces both energies , and B(E2), B(M1) transition rates, together with the mixing ratio δ.
- In order to clarify the wobbling bands from the microscopic point of view, we have developed self-consistent constrained HFB equation both for even and odd mass nucleus.
- The preliminary results in the lowest order approximation to the angular momentum projected HFB treatment, shows not so bad results, but γ is small. The angular momentum projection is necessary.
- The single-particle levels for proton-shell must include the Coulomb effect, because the wobbling mode is observed only in odd-Z nuclei.

Happy Birthday to Prof. Akito Arima

I wish I can answer your question, about pseudospin symmetry on your 99th birthday, if I were still alive.

白寿