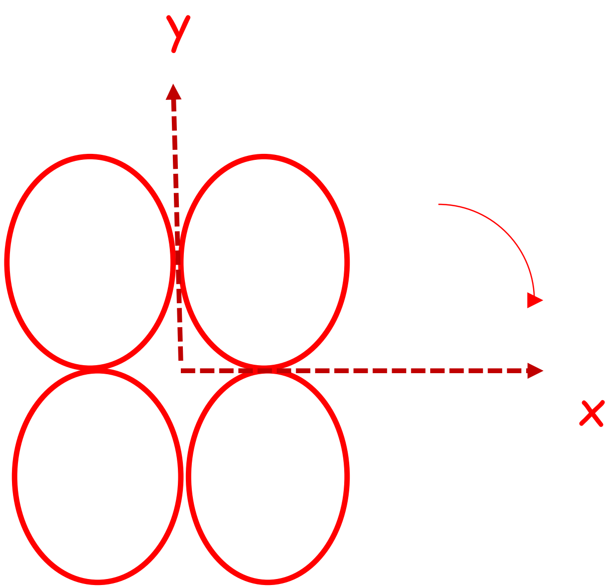
The symmetry of 88

1. 88 is invariant under rotation by π around x, y, z axis.

2. All nonequivalent combinations of the three successive rotations generate the Q8 group, of which the D2 is a special case.

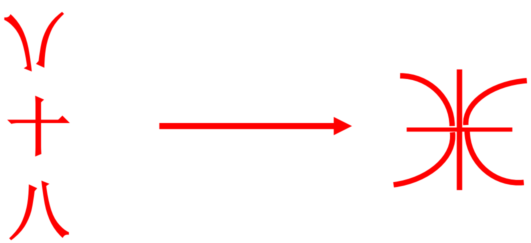
3. The D2 is also known as the intrinsic Vierergruppe (named after Felix Christian Klein in 1884) of asymmetric rotor with integer spins, which has applied to manifest rotational spectra of nuclei, for which Prof. Arima contributed a lot.



88 may be written in Chinese as 八十八.

After rotating the first 八 on the plane around x axis by π:

which may be put together with 十 and another 八 as



which is 米 (rice)

The Q8 symmetry applies to 米 as well.
Therefore, 88 or rice is the most symmetric, rich, and fruitful age.

Multi-particle-Multi-hole Configuration Mixing , Intruder States, and the exact treatment in the Interacting Boson Model

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ICSSBAN2018

Shanghai, 09/26/2018

MP-MH configuration mixing

Beyond mean field approach to the many-body problem

is aimed to provide

- Unified description of correlations beyond the HF approximation
- Treatment on the same footing of even-even, odd-A and odd-odd nuclei
- Description of both ground and excited states

Formalism in MP-MH configuration

Trial wave function: Superposition of Slater Determinants

$$|\psi\rangle = A^{0p0h}_{\pi\nu}|\phi_\pi\phi_\nu\rangle_{0p0h} + \sum_{\alpha_\pi\alpha_\nu} A^{1p1h}_{\alpha_\pi\alpha_\nu}|\phi_{\alpha_\pi}\phi_{\alpha_\nu}\rangle_{1p1h} + \sum_{\alpha_\pi\alpha_\nu} A^{2p2h}_{\alpha_\pi\alpha_\nu}|\phi_{\alpha_\pi}\phi_{\alpha_\nu}\rangle_{2p2h} + \ldots$$

$$|\phi_\tau\rangle = \prod_{i=1}^{N} a_i^+ |-\rangle \qquad |\phi_{\alpha_\tau}\rangle = \prod_{(kl)=1}^{m} (a_k^+ a_l)|\phi_\tau\rangle_{0p0h}$$

O Mixing coefficients

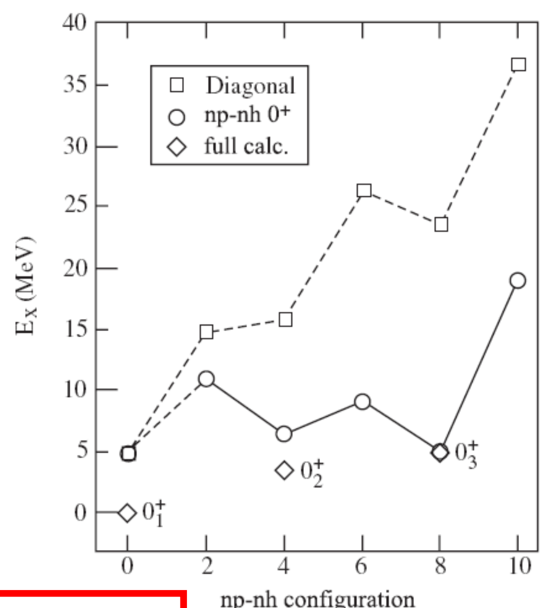
O Single particle orbitals

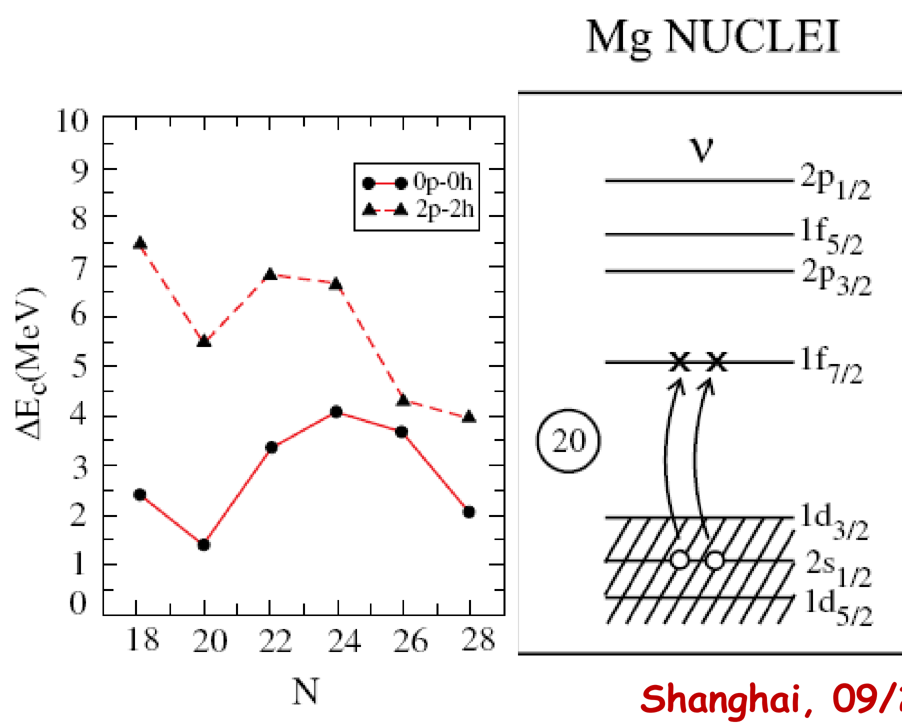
Kris Heyde, Basic Ideas and Concepts in Nuclear Physics, IOP Publishing 1994

Shape coexistence in various regions

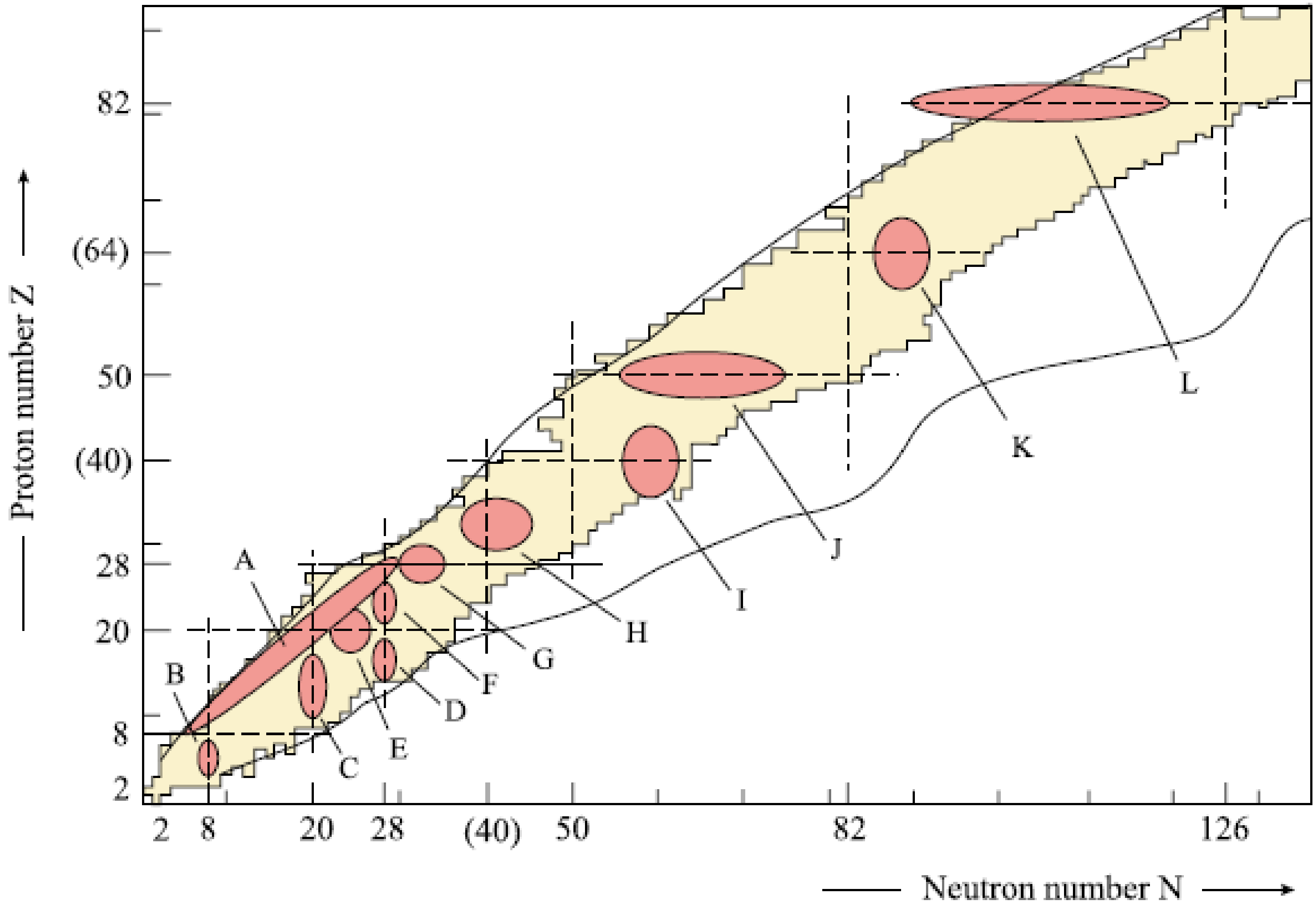
Kris Heyde, John L. Wood, RMP 83 (2011) 1467

- 1. Doubly closed shell nuclei: stable shell closure against MPH.
- 2. Single-closed shell nuclei: N =20 and N=28

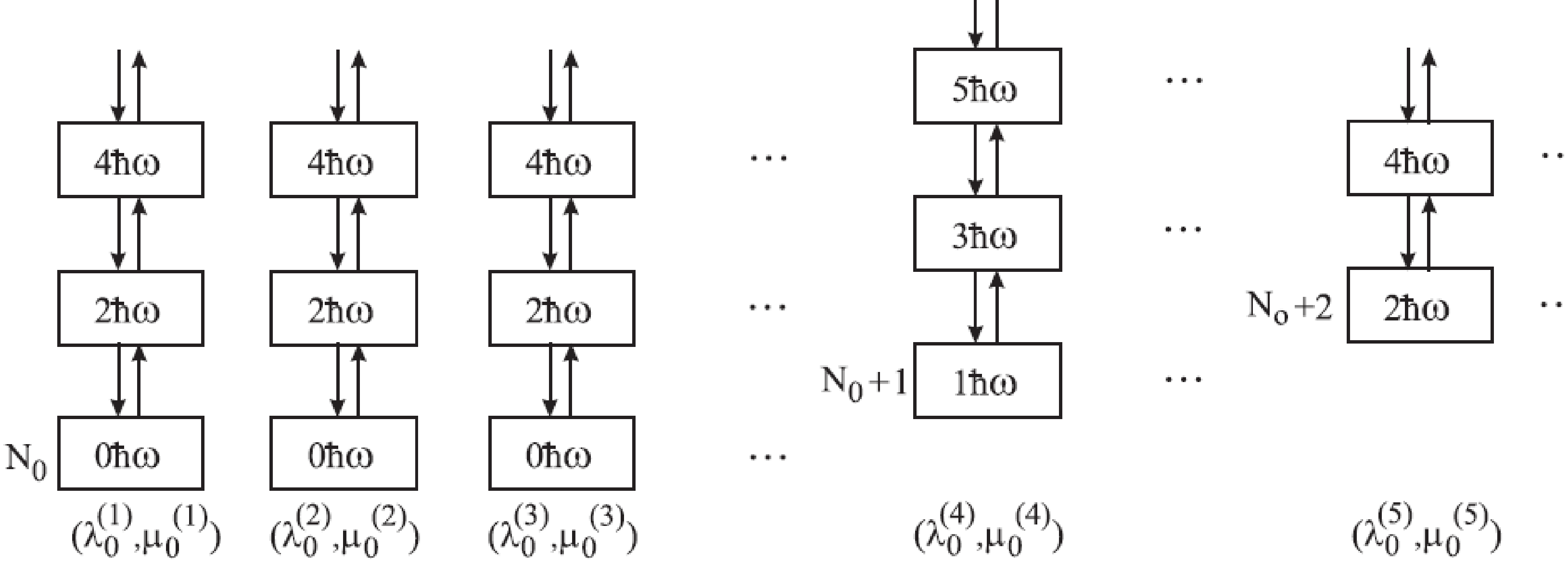




- Heavy nuclei: The Sn (Z =50) and Pb (Z=82) regions



Kris Heyde, John L. Wood, RMP 83 (2011) 1467



Shell-model calculations in taking $2n\hbar\omega$ excitations:

[1] G. Rosensteel and D. J. Rowe, PRL **38**, 10 (1977).

[2] T. Dytrych, K. D. Sviratcheva, C. Bahri, J. P. Draayer, and J. P. Vary, PRL **98**, 162503 (2007).

Intruder 0+2 in 80Ge, PRL 116, 182501 (2016)

Signature of Shape Coexistence near 78Ni, PRL 116, 182502 (2016)

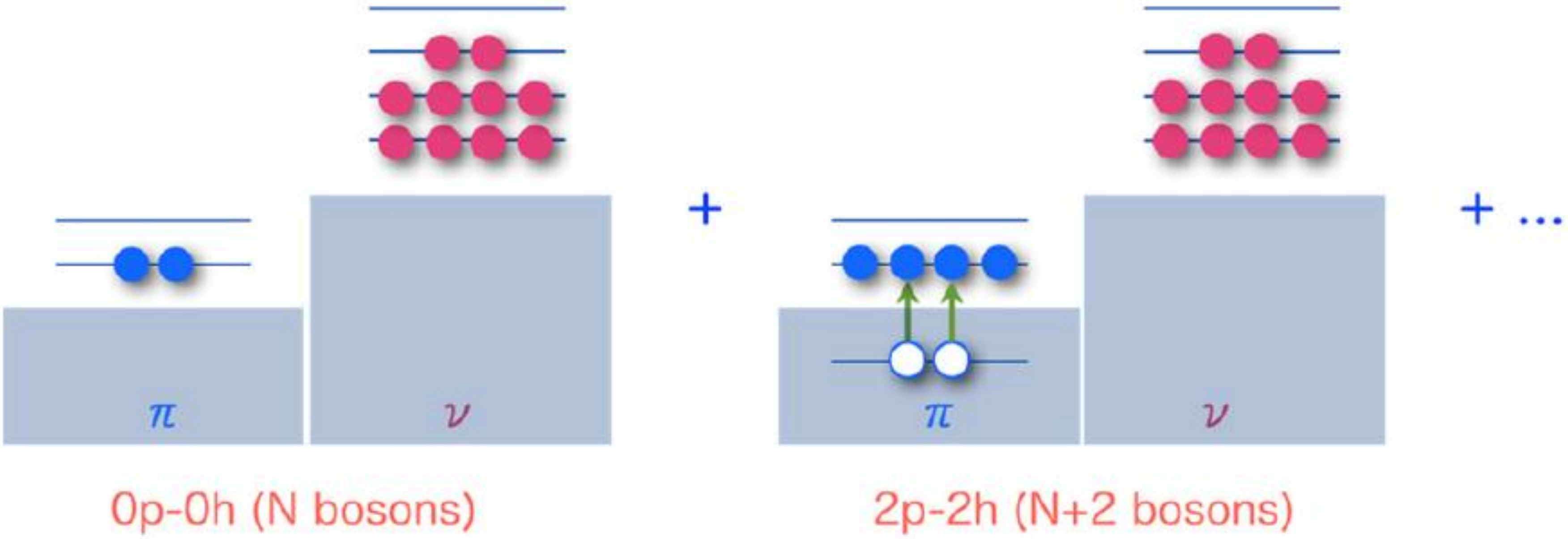


Figure 1. Illustration of the configuration mixing in the IBM framework.

K Nomura, T Otsuka, and P Van Isacker,

J. Phys. G: Nucl. Part. Phys. **43** (2016) 024008

In the IBM, CM calculations were initiated by Duval and Barrett:

PLB **100**, 223 (1981); NPA **376**, 213 (1982).

Then, studied extensively by

K. Heyde, P. Van Isacker, and their collaborators:

1. to elucidate low-lying Intruder states

2. Introducing the shape coexistence concept in the IBM.

The formalism

$$\hat{H} = \hat{H}_{\rm reg} + \hat{H}_{\rm 2p2h} + \hat{H}_{\rm 4p4h} + \hat{V}_{\rm mix}.$$

$$\begin{aligned}
\hat{H}_{\rm reg} &= \epsilon_{\rm reg}\hat{n}_d + \kappa_{\rm reg}\hat{Q}_{\rm reg}\cdot\hat{Q}_{\rm reg}, \\
\hat{H}_{\rm 2p2h} &= \epsilon_{\rm 2p2h}\hat{n}_d + \kappa_{\rm 2p2h}\hat{Q}_{\rm 2p2h}\cdot\hat{Q}_{\rm 2p2h} + \Delta_{\rm 2p2h}, \\
\hat{H}_{\rm 4p4h} &= \epsilon_{\rm 4p4h}\hat{n}_d + \kappa_{\rm 4p4h}\hat{Q}_{\rm 4p4h}\cdot\hat{Q}_{\rm 4p4h} + \Delta_{\rm 4p4h},
\end{aligned}$$

$$\hat{V}_{{\rm mix},i} = \alpha_i\left(s^\dagger s^\dagger\right) + \beta_i\left(d^\dagger d^\dagger\right)^{(0)} + \text{hermitian conjugate},$$

$$\hat{Q}_i = \left(s^\dagger\tilde{d} + d^\dagger\tilde{s}\right)^{(2)} + \chi_i\left(d^\dagger\tilde{d}\right)^{(2)}$$

Diagonalized in [N] ⊕[N+2] ⊕[N+4] ⊕ … configurations of U(6).

$$\hat{H} = \hat{P}\left(\Delta\,\hat{n}_s + \Delta\,\hat{n}_d + \hat{H}_0 + g(S^+ + S^-)\right)\hat{P},$$

$$\hat{H}_0 = \epsilon_d \hat{n}_d + c\,C_2(O(5)) + f\,\hat{L}\cdot\hat{L}$$

$$\hat{P}|N', n_d, \nu, \rho, L, M\rangle = \begin{cases} |N', n_d, \nu, \rho, L, M\rangle & \text{if } N' \geqslant N \\ 0 & \text{otherwise,} \end{cases}$$

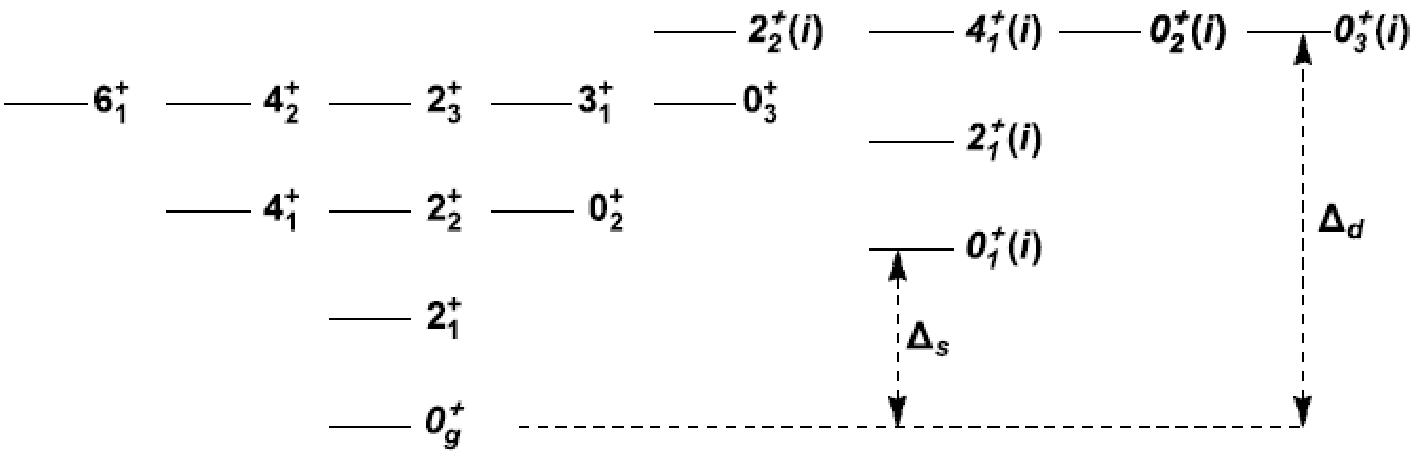
$$|k_s, k_d, \omega\rangle = (1 + a_{n_d}\tilde{S}_s^+)^{k_s}(1 + b_{n_d}\tilde{S}_d^+)^{k_d}\, e^{\alpha\tilde{S}_s^+ + \beta\tilde{S}_d^+}|\omega\rangle$$

the SU(1,1) coherent state

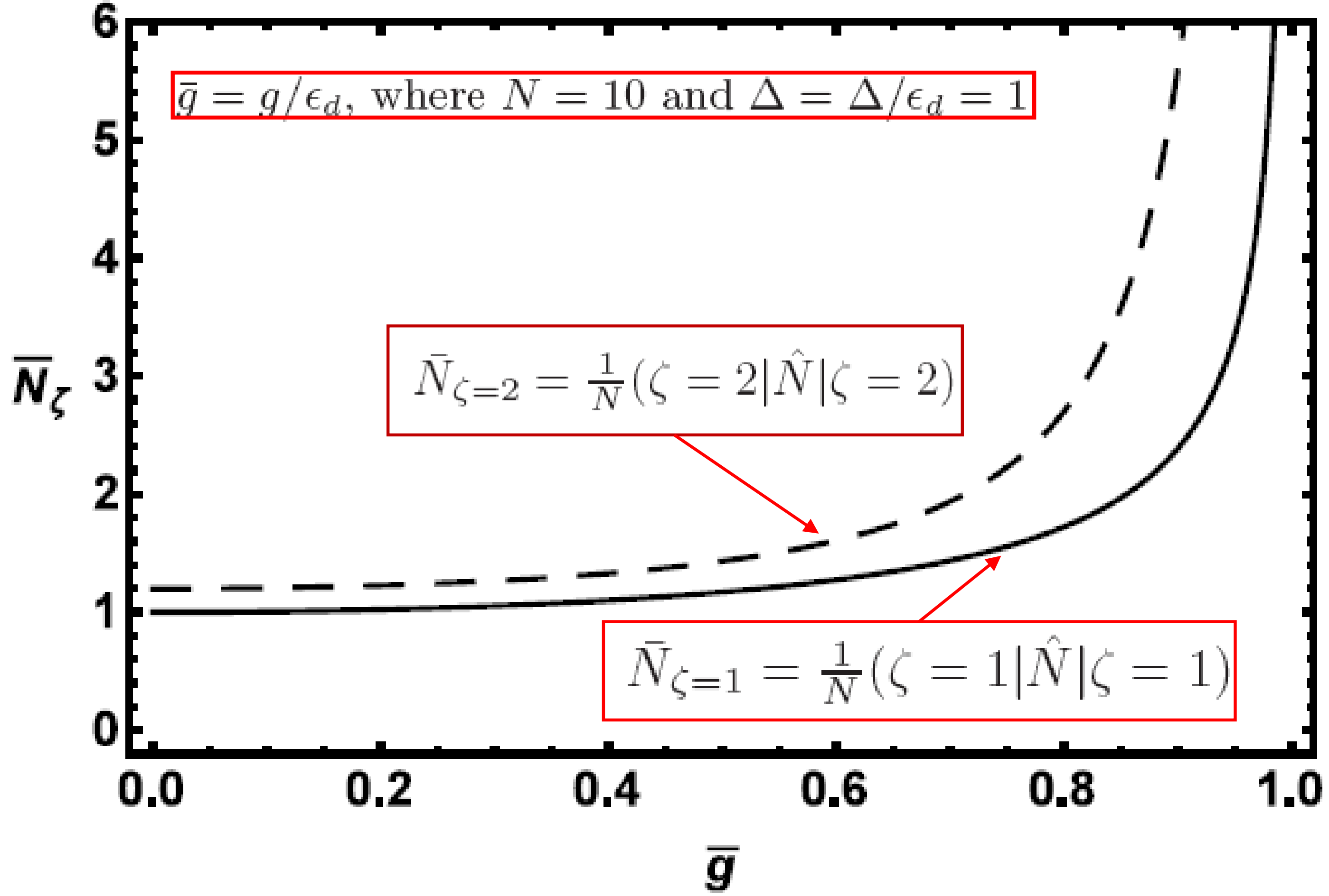
$$E^{(\zeta=1)}_{N,n_d,\nu,L} = \Delta N + \left(\sqrt{\Delta^2 - g^2} - \Delta\right)(N - n_d) + \left(\sqrt{(\Delta + \epsilon_d)^2 - g^2} - \Delta\right) n_d + c\,\nu(\nu + 3) + f\,L(L + 1)$$

$$E^{(k_s,k_d)}_{N,n_d,\nu,L} = E^{(\zeta=1)}_{N,n_d,\nu,L} + 2k_d\sqrt{(\Delta + \epsilon_d)^2 - g^2} + 2k_s\sqrt{\Delta^2 - g^2}.$$

Configuration mixing in the U(5) limit case



Low-lying level pattern of the solvable configuration mixing U(5) model, where the lower left 10 levels belong to $\zeta = 1$ family, which are the same as those generated from the U(5) limit Hamiltonian of the IBM H0 only, but with the modified single d-boson energy, the levels labeled with (i) are intruder levels of the model.



Phys. Rev. C **97**, 034316 (2018)

$$T_\mu(E2) = q_2 \hat{P}_N (d_\mu^\dagger s + s^\dagger \tilde{d}_\mu) \hat{P}_N + q_2' \hat{P} (d_\mu^\dagger s + s^\dagger \tilde{d}_\mu) \hat{P}$$

Some nonzero $B(E2; L_i \to L_f)$ values for the transitions among low-lying states of the model, where $\lambda = q_2/q_2'$.

Transition	$B(E2; L_i \to L_f)/(q_2')^2$
$2_1^+ \to 0_g^+$	$N\left(\frac{\mathcal{N}_1}{\mathcal{N}_0}\right)^2\left(1 - \frac{2(\alpha+\beta)}{5b_0} + \lambda\mathcal{N}_0^2\right)^2$
$4_1^+ \to 2_1^+$	$2(N-1)\left(\frac{\mathcal{N}_2}{\mathcal{N}_1}\right)^2\left(1 - \frac{2(\alpha+\beta)}{7b_1} + \lambda\mathcal{N}_1^2\right)^2$
$2_1^+(i) \to 0_1^+(i)$	$N\left(\frac{\mathcal{N}_1}{\mathcal{N}_0}\right)^2 \frac{a_1}{a_0}\left(1 - \frac{2(\alpha+\beta)}{5b_0} + \mathcal{N}_0^2\alpha(\alpha^2-1)\frac{\lambda}{a_1}\right)^2$
$4_1^+(i) \to 2_1^+(i)$	$2(N-1)\left(\frac{\mathcal{N}_2}{\mathcal{N}_1}\right)^2 \frac{a_2}{a_1}\left(1 - \frac{2(\alpha+\beta)}{7b_1} + \mathcal{N}_1^2\alpha(\alpha^2-1)\frac{\lambda}{a_2}\right)^2$
$0_1^+(i) \to 2_1^+$	$5N\frac{\alpha(\alpha^2-1)}{a_0}\left[\frac{\mathcal{N}_1}{\mathcal{N}_0}\left(1 - \frac{2(\alpha+\beta)}{5b_0}\right)\left(1 - \frac{a_1}{a_0}\right) + \lambda\mathcal{N}_0\mathcal{N}_1\right]^2$
$2_1^+(i) \to 0_g^+$	$N\frac{\alpha(\alpha^2-1)}{a_1}\left\{\frac{\mathcal{N}_0}{\mathcal{N}_1}\left[1 - \frac{a_0}{a_1} - \frac{1+\beta/\alpha}{a_1 N}\left(1 - \frac{\alpha}{2a_0}\right)\right] + \lambda\mathcal{N}_0\mathcal{N}_1\right\}^2$
$4_1^+(i) \to 2_1^+$	$2(N-1)\frac{\alpha(\alpha^2-1)}{a_2}\left\{\frac{\mathcal{N}_1}{\mathcal{N}_2}\left[1 - \frac{a_1}{a_2} - \frac{1+\beta/\alpha}{a_2(N-1)}\left(1 - \frac{\alpha}{2a_1}\right)\right] + \lambda\mathcal{N}_1\mathcal{N}_2\right\}^2$

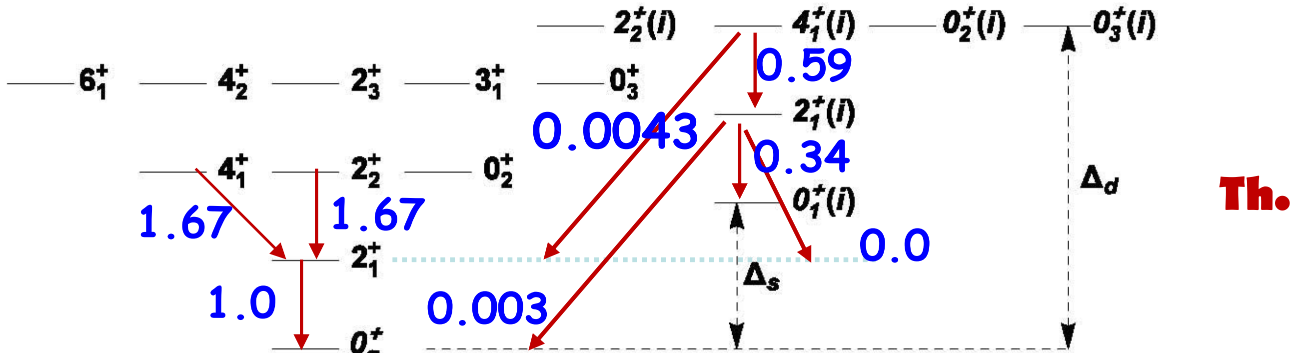
$$a_{n_d} = \frac{2\Delta + 2g\alpha^\pm}{2g\Lambda_s} = \pm\frac{\sqrt{\Delta^2 - g^2}}{g\Lambda_s} \qquad \mathcal{N}_{\zeta=2,n_d} = \sqrt{\frac{\alpha(\alpha^2-1)}{a_{n_d}}}\mathcal{N}_{n_d}$$

$$\Lambda_s = \frac{1}{2}(N - n_d + \frac{1}{2})$$

$$\alpha = \frac{1}{g}(-\Delta + \sqrt{\Delta^2 - g^2}) \; \beta = \frac{1}{g}(-\Delta - \epsilon_d + \sqrt{(\Delta + \epsilon_d)^2 - g^2})$$

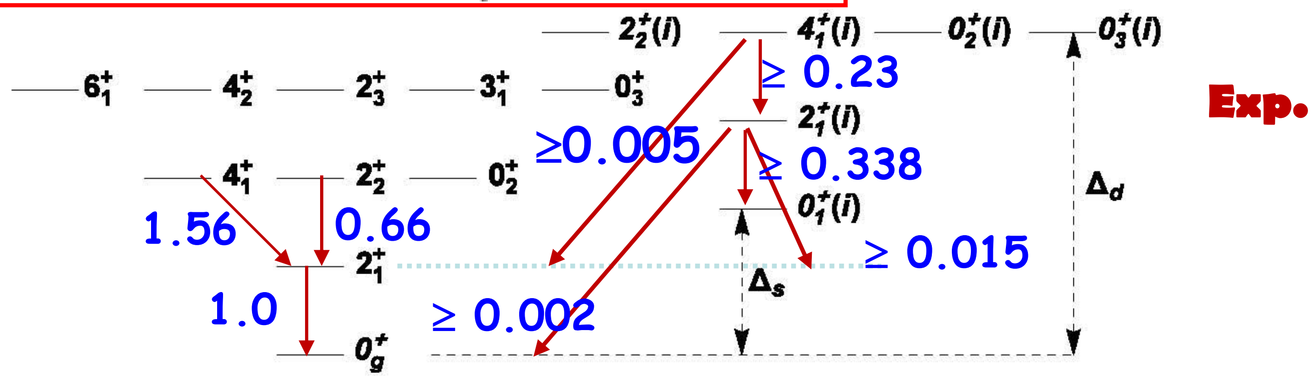
$$T_\mu(E2) = q_2 \hat{P}_N \left(d^\dagger_\mu s + s^\dagger \tilde{d}_\mu\right) \hat{P}_N + q'_2 \hat{P} \left(d^\dagger_\mu s + s^\dagger \tilde{d}_\mu\right) \hat{P}$$

$q_2/q'_2 = 0.89$



$\Delta = 0.8635$ MeV, $g = 0.0782$ MeV

$\epsilon_d = 0.6305$ MeV, $c = 0.0200$ MeV, $f = 0.0010$ MeV



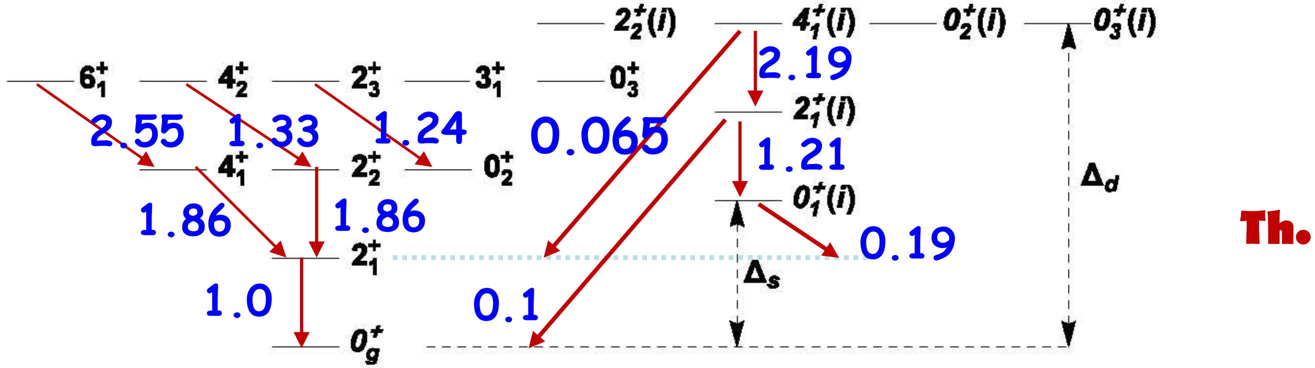
$B(E2; L_i \rightarrow L_f) / B(E2; 2^+_1 \rightarrow 0^+_g)$ ^{108}Cd

$$\hat{H} = \hat{P}\left(\Delta\, \hat{n}_s + \Delta\, \hat{n}_d + \hat{H}_0 + g(S^+ + S^-)\right)\hat{P},$$

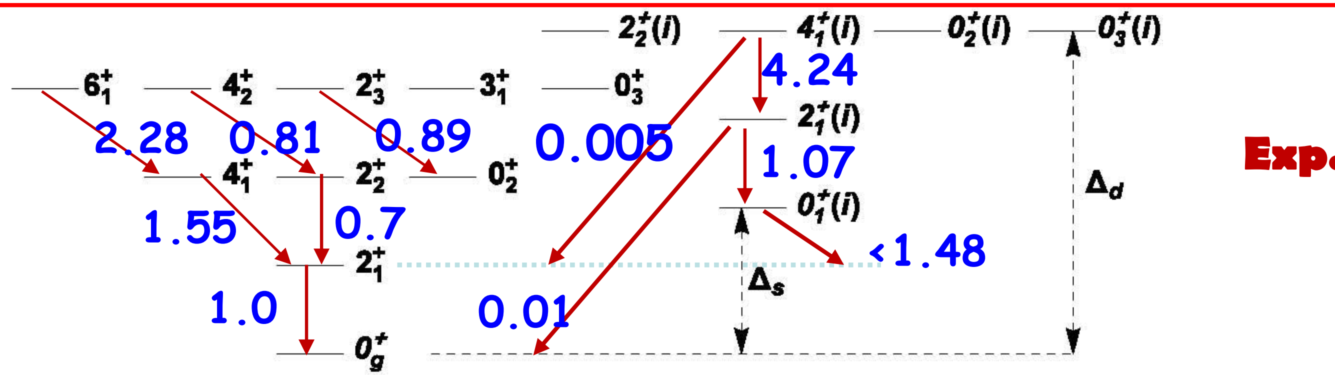
$$\hat{H}_0 = \epsilon_d \hat{n}_d + c\, C_2(O(5)) + f\, \hat{L}\cdot\hat{L}$$

$$T_\mu(E2) = q_2 \hat{P}_N (d^\dagger_\mu s + s^\dagger \tilde{d}_\mu) \hat{P}_N + q'_2 \hat{P} (d^\dagger_\mu s + s^\dagger \tilde{d}_\mu) \hat{P}$$

$q_2/q'_2 = 1.99$



$\Delta = 0.7850$ MeV, $g = 0.5493$ MeV, $\epsilon_d = 0.5800$ MeV, $c = -0.0080$ MeV, $f = 0.0096$ MeV



$B(E2; L_i \rightarrow L_f) / B(E2; 2^+_1 \rightarrow 0^+_g)$ ^{110}Cd

$$\hat{H} = \hat{P}\left(\Delta\,\hat{n}_s + \Delta\,\hat{n}_d + \hat{H}_0 + g(S^+ + S^-)\right)\hat{P},$$

$$\hat{H}_0 = \epsilon_d \hat{n}_d + c\,C_2(O(5)) + f\,\hat{L}\cdot\hat{L}$$

$$\hat{H} = \hat{P}(2\Delta S^0 + \hat{H}_0 + g(S^+ + S^-))\hat{P}$$

$$S^0 = \frac{1}{2}(\hat{N} + 3)$$

$$\hat{H}_0 = -\kappa_0 \, e^{\xi \, \hat{Q}\cdot\hat{Q}} \hat{Q}\cdot\hat{Q} + a\,\hat{L}^2 + b\,X_3 + d\,X_4$$

$$X_3 = \frac{\sqrt{30}}{6} \, (\hat{L}\times\hat{Q}\times\hat{L})^{(0)}$$

$$X_4 = -\frac{5\sqrt{3}}{18} \, ((\hat{L}\times\hat{Q})^{(1)}\times(\hat{L}\times\hat{Q})^{(1)})^{(0)}$$

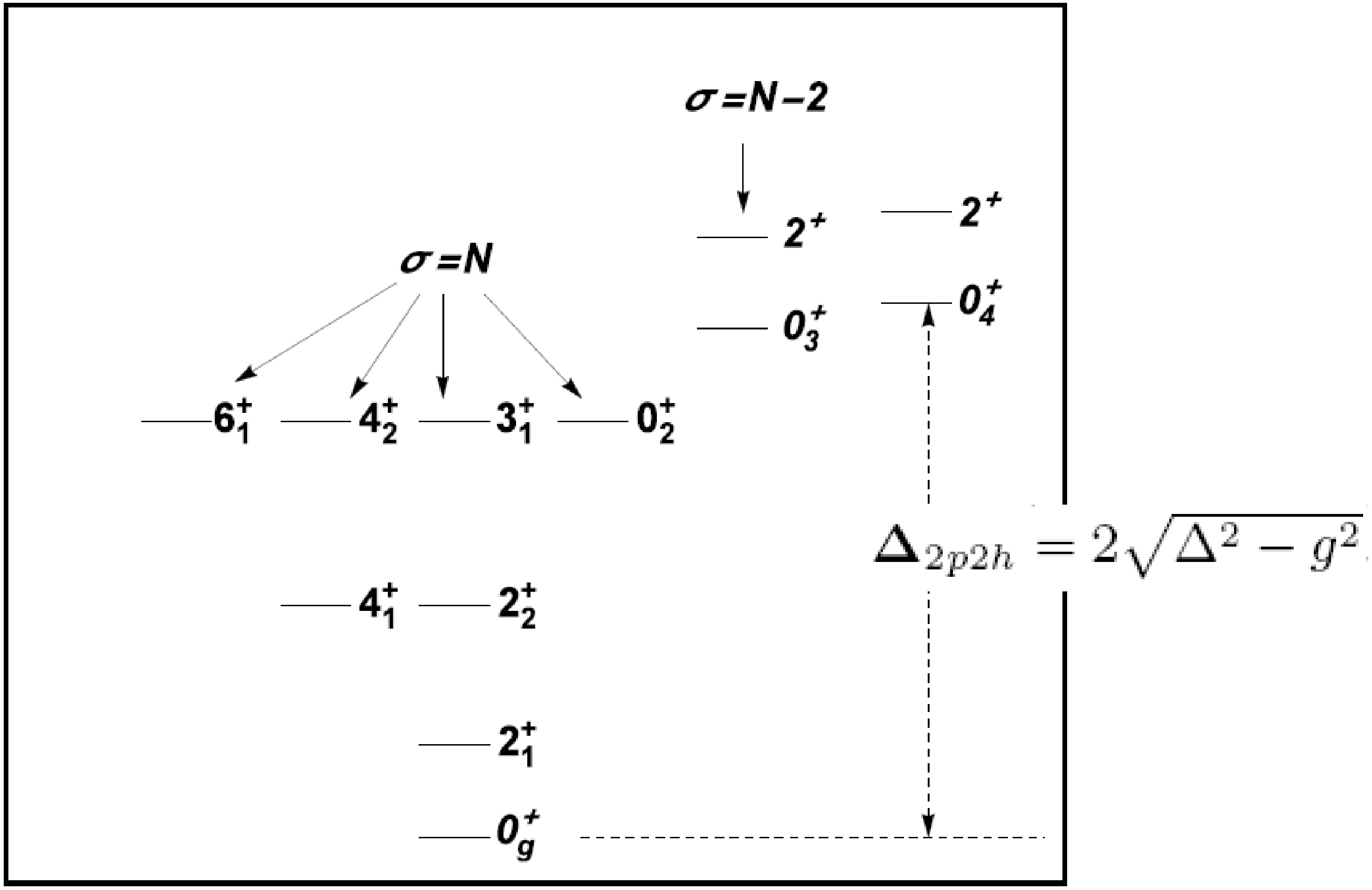
$$|\zeta = 1, \omega\rangle = \mathcal{N}_{\zeta=1} \, e^{\alpha S^+} \, |N, \sigma, \eta, L, M\rangle$$

$$E^{(\zeta=1)}_{N,\sigma,\eta,L} = (\pm)(N+3)\sqrt{\Delta^2 - g^2} + E_0(\sigma, \eta, L)$$

$$|\zeta = 2, \omega\rangle = \mathcal{N}_{\zeta=2} \, (1 + c\,S^+)e^{\alpha S^+} \, |\omega\rangle.$$

$$E^{(\zeta=2)}_{N,\sigma,\eta,L} = E^{(\zeta=1)}_{N,\sigma,\eta,L} \pm 2\sqrt{\Delta^2 - g^2}$$

Configuration mixing in the O(6) limit case



Low-lying level pattern of the solvable configuration mixing O(6) model

$$T_\mu(E2) = q_2 \hat{P}_N (d_\mu^\dagger s + s^\dagger \tilde{d}_\mu) \hat{P}_N + q_2' \hat{P} (d_\mu^\dagger s + s^\dagger \tilde{d}_\mu) \hat{P}$$

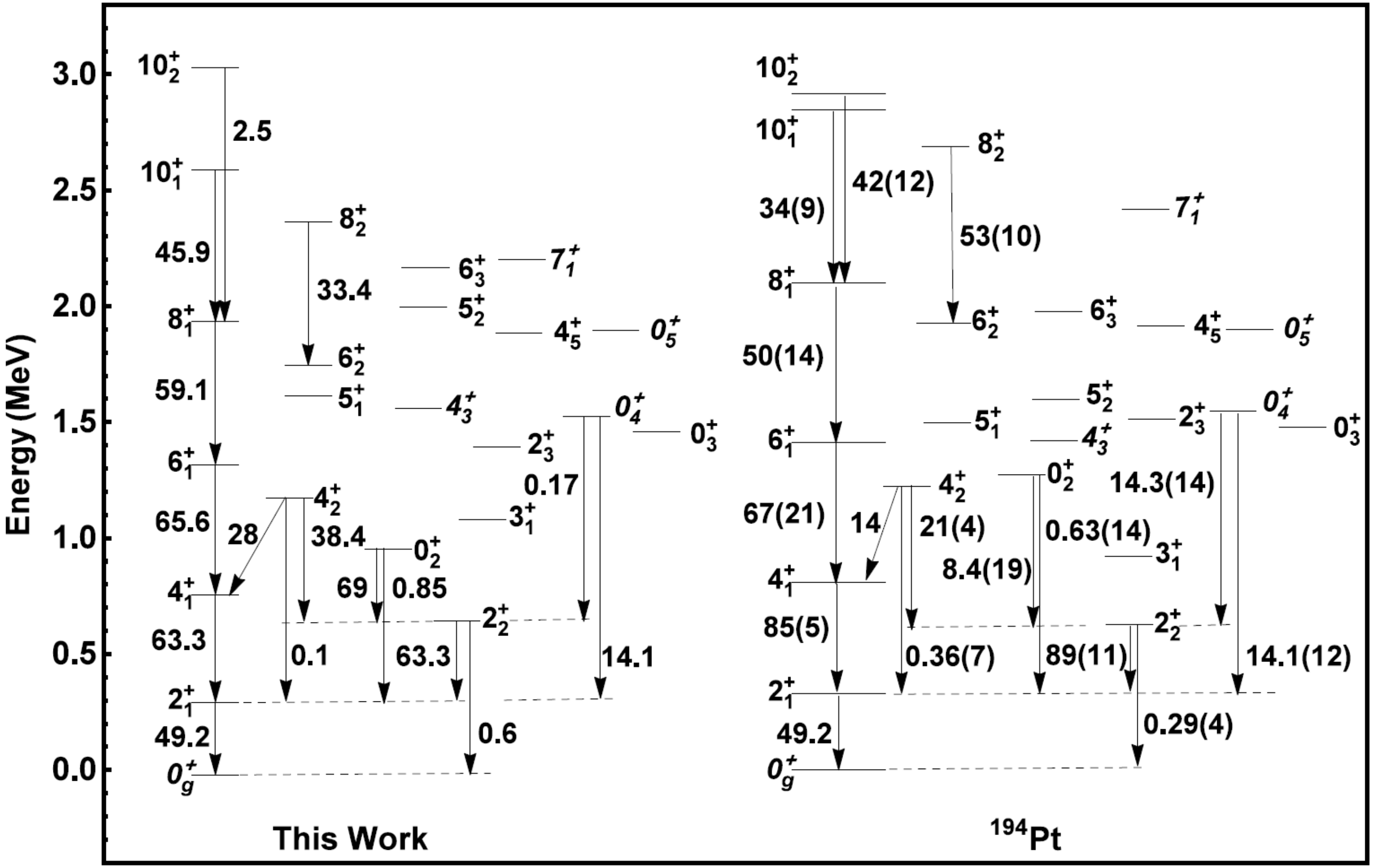
$$|N,\sigma,\eta,L,M\rangle = \sum_{\nu,\rho} C_{\nu,\rho}^{(\eta)} |N,\sigma,\nu,\rho,L,M\rangle \qquad \lambda = q_2'/q_2 \qquad \lambda = 0.5$$

Electric-quadrupole moments of some low-lying states (in e b)

	Experiment	This work	IBM-CM	ECQF
$Q(2_1^+)$	$+0.409\ (^{+62}_{-43})$	0.2895	0	−0.288
$Q(4_1^+)$	$+0.752\ (^{+92}_{-105})$	0.5841	0	−0.308
$Q(6_1^+)$	$+0.195\ (^{+85}_{-188})$	1.0798	0	−0.284
$Q(8_1^+)$	− 0.06, 0.28	0.9357	0	−0.26
$Q(2_2^+)$	$-0.303\ (^{+93}_{-37})$	−0.2897	0	0.259
$Q(4_2^+)$	− 0.06 (11)	−0.0350	0	0.09
$Q(6_2^+)$	$+0.286\ (^{+181}_{-153})$	0.1258		

$$\hat{H}_0 = -\kappa_0 \, e^{\xi \, \hat{Q}\cdot\hat{Q}} \hat{Q}\cdot\hat{Q} + a\,\hat{L}^2 + b\,X_3 + d\,X_4$$

$$X_3 = \frac{\sqrt{30}}{6} (\hat{L} \times \hat{Q} \times \hat{L})^{(0)}$$



Detailed comparison of the most of the excited level energies up to 3MeV with known absolute B(E2) values in W. u. obtained in this work to the experimental results for 194Pt. PHYSICAL REVIEW C **97**, 034316 (2018)

ICSSBAN2018

Shanghai, 09/26/2018

Summary and Perpectives

It is shown that the intruder configuration mixing with 2n-particle and 2n-hole Up to infinity n in both U(5) and O(6) limit of the IBM-1 is analytically solvable based on the SU(1,1) bosonic pairing coherent states.
A complete set of the SU(1,1) coherent states built on those of the U(5) or O(6) of the IBM may be used to study more complicated transitional regions of the model with shape coexistence.

- Self-consistent mp-mh approach in the IBM-1,-2, and IBFM

  -unifies the description of important correlations beyond mean field in nuclei
  -suitable for studying even-even, odd-A, and odd-odd medium-heavy and well-deformed nuclei
  Physics Reports 215(1992) 101
- Applications to odd-A and odd-odd nuclei are quite encouraging
  Physics Reports 102 (1983) 291.
- Systematic study on shape coexistence and shape evolution of both even-even and odd-A nuclei is helpful.

Happy Rice Age to Professor Arima!

Thank you!