Evidence for three-nucleon spin-orbit interaction in nuclear charge radii

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<table>
<thead>
<tr>
<th>Year</th>
<th>Prof. Arima</th>
<th>me</th>
</tr>
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<tbody>
<tr>
<td>1985</td>
<td>Dean (Sci. School, U.Tokyo)</td>
<td>Joined his group in U.Tokyo (as one of his last students)</td>
</tr>
<tr>
<td>1989</td>
<td>President of U.Tokyo</td>
<td>Left U.Tokyo</td>
</tr>
</tbody>
</table>

**Enormous supports!** (teaching physics & basics of research, helps for academic career & personal matters, encouragement, ...)

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
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<tbody>
<tr>
<td>2018</td>
<td>88th birthday</td>
</tr>
<tr>
<td>2018</td>
<td>Almost the age of Prof. Arima when I joined his group</td>
</tr>
<tr>
<td>2030</td>
<td>100th birthday</td>
</tr>
<tr>
<td>2030</td>
<td>Already retired</td>
</tr>
<tr>
<td>2050</td>
<td>120th birthday</td>
</tr>
<tr>
<td>2050</td>
<td>??? (rice age, if alive)</td>
</tr>
</tbody>
</table>

**Congratulations for ‘rice age’,**

**Thank you so much for continuous supports,**

**Take care of yourself & keep guiding us!**
I. Introduction

★ $\ell s$ splitting in s.p. levels

$\leftrightarrow$ shell structure $\leftrightarrow$ magic number (in $Z, N > 20$)

— fundamental concept for nuclear structure

astrophysical importance

$e.q.$ \begin{align*}
& \text{waiting point in } s & \& r\text{-processes} \\
& \text{constraint on EoS} & \leftarrow \text{subtracting shell effects}
\end{align*}

origin? (→ correct prediction of shell structure)

$\ell s$ splitting

$3N$ LS int. (?)

tensor int. (1st order)

tensor int. (2nd order)

T. Terasawa & A. Arima
P.T.P. 23, 87; 115 ('60)

$2N$ LS int.

$\chi$EFT → $3N$ LS int. (→ $\rho$-dep. LS int.)

N. Kaiser, PRC 68, 054001; M. Kohno, PRC 86, 061301(R)

strength? (↔ convergence of $\chi$EFT) exp. evidence?
★ Nuclear charge radius \( \langle r^2 \rangle_c(A) \) [ = \( \langle r^2 \rangle_p(A) + \langle r^2 \rangle_c(p) + (N/Z)\langle r^2 \rangle_c(n) \) ]
\[ \leftarrow (e,e) \text{ etc.} \]

- model-indep. data
- (lowest-order) quantity reflecting proton dist. in nucleus
  — nuclear structure information additional to energy

\[ \leftrightarrow \text{Isotope shift} \cdots \text{atomic X-ray freq. difference among isotopes} \]
\[ \propto \Delta \langle r^2 \rangle_c \left[ \approx \Delta \langle r^2 \rangle_p \right] \text{ for heavy nuclei} \]
- exp. data — high-precision, incl. unstable nuclei
- good indicator to structure change \( e.g. \) spherical to deformed
- \( Z = \text{magic nuclei} \cdots \) (quite likely) spherical \( \rightarrow \text{several puzzles!} \)
• Isotope shifts in Pb nuclei

\[ \Delta \langle r^2 \rangle_c(A_{\text{Pb}}) := \langle r^2 \rangle_c(A_{\text{Pb}}) - \langle r^2 \rangle_c^{(208}\text{Pb}) \]

exp. \( \Rightarrow \text{kink at } N = 126! \)

- not reproduced by Skyrme EDF (up to '95)
- reproduced by RMF due to isospin content of LS int. (not direct rel. effect)
- leading to extension of Skyrme EDF

\[ \text{M.M. Sharma et al., PRL 74, 3744} \]
kink in $\Delta \langle r^2 \rangle_c(^A\text{Pb})$ at $N = 126 \uparrow n0i_{11/2}$ occupation

$p-n$ attraction

\[
\left\{
\begin{array}{l}
\text{larger } \langle r^2 \rangle \\
\text{than neighboring orbits}
\end{array}
\right.
\]

\[
\left\{
\begin{array}{l}
N < 126 \quad \text{unocc.} \\
N > 126 \quad \text{sizable occ. prob.}
\end{array}
\right.
(\because \text{pairing})
\]

However, $\varepsilon_n(0i_{11/2}) \approx \varepsilon_n(1g_{9/2})$ required! ($\leftrightarrow$ equal occ. prob.)

cf. pseudo-spin

on the contrary \ldots

\[
\begin{array}{c}
\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2}) \\
\leftrightarrow \text{isospin content of LS int.}
\end{array}
\]

\[
\begin{array}{c}
0.78 \text{ MeV} \\
\text{Pairing between } 11/2^+ \\
\text{and } 9/2^+ (\approx \Delta_{\text{pair}})
\end{array}
\]

P.-G. Reinhard & H. Flocard,

NPA 584, 467
- Charge radii of $^{40}\text{Ca}$ & $^{48}\text{Ca}$

$^{40}\text{Ca}$ & $^{48}\text{Ca}$ — doubly magic

$\langle r^2 \rangle_c (^{40}\text{Ca}) \approx \langle r^2 \rangle_c (^{48}\text{Ca})$

$\langle r^2 \rangle_c (^{40}\text{Ca}) < \langle r^2 \rangle_c (^{48}\text{Ca})$

in most MF cal.

- exception — RMF

G.A. Lalazissis et al., ADNDT 71, 1

$\cdots$ physics?

cf. $^{42-46}\text{Ca}$ $\cdots$ beyond-MF effects?

*e.g.* excitation across $Z = 20$ shell gap

E. Caurier et al., PLB 522, 240

$(\nabla \rho)$-dep. pairing? S.A. Fayans, JETP Lett. 68, 169
II. Mean-field approaches with semi-realistic interaction

“Semi-realistic” nucleonic interaction \( \leftarrow \) microscopic \( 2N (+3N) \) int.
\[ \uparrow \]
phenomenological modification

\[ \hat{v}_{M3Y} \approx G\text{-}matrix \]
\[ \downarrow \]
\[ \hat{v}_{M3Y-P_n} \]

\[
\begin{cases}
\text{central} & \cdots \ \rho\text{-dep. introduced (} \leftrightarrow \text{saturation, incl. } 3N \text{ effects)} \\
\text{LS} & \cdots \ \text{modification (} \leftrightarrow \ell s \text{ splitting, shown later)} \\
\text{finite-range} & \to T = 0 \& 1 \ channels \\
\text{tensor} & \cdots \ \text{unchanged (} \hat{v}_{M3Y-P_n}^{(TN)} = \hat{v}_{M3Y}^{(TN)} ) \to \text{realistic}
\end{cases}
\]

‘M3Y-P6’ \( \to \) reasonable shell structure \( (e.g. \text{ magic } \#) \)

H.N., PRC 68, 014316

H.N., PRC 87, 014336; H.N. & K. Sugiura, PTEP 2014, 033D02
\[ \leftarrow \]
yardstick
Incorporating $3N$ LS int.

$\ell s$ splitting — more or less fitted in MF cal. (→ shell structure)

⋯ should not be changed seriously

(Do we really need $3N$ LS int. in MF approaches, even if it significantly contributes to $\ell s$ splitting?)

$3N$ LS int. $\leftrightarrow \rho$-dep. LS int. ($\hat{v}^{(LS\rho)}$)  

M. Kohno, PRC 86, 061301(R)

\begin{align*}
\hat{v}^{(LS\rho)} &= 2i D[\rho(R_{ij})] p_{ij} \times \delta(r_{ij}) p_{ij} \cdot (s_i + s_j) ; \\
D[\rho(r)] &= -w_1 \frac{\rho(r)}{1 + d_1 \rho(r)} \left( \approx -w_1 \rho(r) \right) \\
&\propto D[\rho] \cdot \hat{v}^{(LS)}_{\text{Sky}} \approx \rho \cdot \hat{v}^{(LS)}_{\text{Sky}}
\end{align*}

\begin{align*}
\Delta \varepsilon^{\ell s}_{\text{M3Y-P6}} \approx \Delta \varepsilon^{\ell s}_{\text{M3Y-P6a}} &\rightarrow w_1 (> 0) \\
(\Delta \varepsilon^{\ell s}(n0i) \text{ at } ^{208}\text{Pb}) &\rightarrow \text{influence on energies not large}
\end{align*}

M3Y-P6 $\rightarrow \hat{v}^{(LS)}_{\text{M3Y}} \times 2.2$

vs.

M3Y-P6a $\rightarrow \hat{v}^{(LS)}_{\text{M3Y}} + \hat{v}^{(LS\rho)}$
★ Influence on s.p. wave functions:

In the presence of $D[\rho]$, we have

- stronger LS for interior (higher $\rho$)
- weaker LS for exterior (lower $\rho$)

$\Rightarrow$

- $j = \ell + \frac{1}{2}$ orbitals shrink
- $j = \ell - \frac{1}{2}$ orbitals extend

E.g. larger $\langle r^2 \rangle$ for $n0i_{11/2}$

$$\Delta[r \ R_j(r)] := [r \ R_j(r)]_{M3Y-P6a} - [r \ R_j(r)]_{M3Y-P6} @ ^{208}\text{Pb}$$
III. 3N LS interaction & isotope shifts in $Z = $ magic nuclei

Spherical HFB cal. → isotope shifts in $Z = $ magic nuclei

M3Y-P6 vs. M3Y-P6a → effects of 3N LS int.

★ Isotope shifts of Pb nuclei

$$
\Delta \langle r^2 \rangle_c (^{A}\text{Pb}) := \langle r^2 \rangle_c (^{A}\text{Pb}) - \langle r^2 \rangle_c (^{208}\text{Pb})
$$

\[ Z = 82 \]
S.p. energies & occ. prob.

\[ \varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2}) : \begin{cases} \text{exp. @ }^{209}\text{Pb} & \rightarrow 0.78 \text{ MeV} \\ \text{M3Y-P6a} & \rightarrow 0.72 \text{ MeV} \end{cases} \]

occ. prob.

\[ n_{1g_{9/2}} \begin{cases} \text{M3Y-P6} \\ \text{M3Y-P6a} \end{cases} \]

\[ n_{0i_{11/2}} \begin{cases} \text{M3Y-P6a} \\ \text{M3Y-P6} \end{cases} \]

\[ \Rightarrow \text{kink at } N = 126 \text{ reproduced without } n_{1g_{9/2}}-n_{0i_{11/2}} \text{ degeneracy!} \]
★ Isotope shifts of Ca nuclei

\[ \Delta \langle r^2 \rangle_c^{(A \text{Ca})} := \langle r^2 \rangle_c^{(A \text{Ca})} - \langle r^2 \rangle_c^{(40 \text{Ca})} \]

\[ \langle r^2 \rangle_c^{(40 \text{Ca})} \approx \langle r^2 \rangle_c^{(48 \text{Ca})} \text{ reproduced!} \]

\[ \therefore \rho \text{-dep. LS } \rightarrow n0f_{7/2} \text{ shifts inward} \quad (\text{RMF}?) \]

cf. \( \chi \text{EFT + CC / SRG also reproduce } \langle r^2 \rangle_c^{(40 \text{Ca})} \approx \langle r^2 \rangle_c^{(48 \text{Ca})} \)
Isotope shifts of Sn nuclei \( \Delta \langle r^2 \rangle_{c(A\text{Sn})} := \langle r^2 \rangle_{c(A\text{Sn})} - \langle r^2 \rangle_{c(\text{120Sn})} \)

[Graph showing data points and fits for \( \Delta \langle r^2 \rangle_{c} \) vs. \( N \).]

kink predicted at \( N = 82 \)!

\( \therefore \) \( \rho \)-dep. LS \( \rightarrow \) \[
\begin{align*}
\text{n0} h_{11/2} & \text{ shifts inward} \quad \rightarrow \text{slope in } 70 \lesssim N \lesssim 82 \\
\text{n0} h_{9/2} & \text{ shifts outward} \quad \rightarrow \text{slope in } N > 82
\end{align*}
\]

--- in sharp contrast to results w/o \( \rho \)-dep. LS (incl. RMF)

\( \Rightarrow \) data? \( \rightarrow \) seems confirmed in recent exp.!
\( \rho \)-dep. LS \( \rightarrow \) kink generally expected wherever \( N \) is \( jj \)-closed magic

\( \cdots \) consistent with exp. data

Note: deformation will be responsible for a certain part
IV. Summary

1. We have investigated effects of $3N$ LS int. on isotope shifts of nuclei. ← sph. HFB with semi-realistic int. M3Y-P6 & its variant M3Y-P6a

2. With M3Y-P6a which contains $\rho$-dep. LS channel,
   • isotope shifts of the Pb nuclei are described fairly well without fictitious $n_{1g_{9/2}}-n_{0i_{11/2}}$ degeneracy (broken pseudo-spin sym.),
   • almost equal charge radii between $^{40}$Ca and $^{48}$Ca are reproduced,
   • isotope shifts of the Sn nuclei are in agreement with available data, and a kink is predicted at $N = 82$. → seems confirmed!

3. Results may be regarded as evidence for $3N$ LS interaction based on $\chi$EFT, indep. of $\ell s$ splitting.
   — qualitative evidence for $3N$ interaction! (?)
   (cf. RMF, Fayan’s EDF ?)
Collaborators:

for this study

T. Inakura  (Niigata U., Japan)

H.N. & T. Inakura, PRC 91, 021302(R)
H.N., PRC 92, 044307

for preceding studies

K. Sugiura  (Chiba U., Japan)
J. Margueron (IPNL, France)

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M. Kohno  (RCNP, Japan)