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Evidence for three-nucleon spin-orbit interaction in nuclear charge radii

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Contents :

Prof. Arima & me

Year	Prof. Arima	me
1985	Dean (Sci. School, U.Tokyo)	Joined his group in U.Tokyo (as one of his last students)
1989	President of U.Tokyo	Left U.Tokyo

Enormous supports ! (teaching physics & basics of research, helps for academic career & personal matters, encouragement, $\cdots$)

2018	88th birthday	Almost the age of Prof. Arima when I joined his group
2030	100th birthday	Already retired
2050	120th birthday	??? (rice age, if alive)

Congratulations for 'rice age',

Thank you so much for continuous supports,

Take care of yourself & keep guiding us !

I. Introduction

★ ℓs **splitting in s.p. levels**

$\leftrightarrow$ **shell structure** $\leftrightarrow$ **magic number (in $Z, N > 20$)**

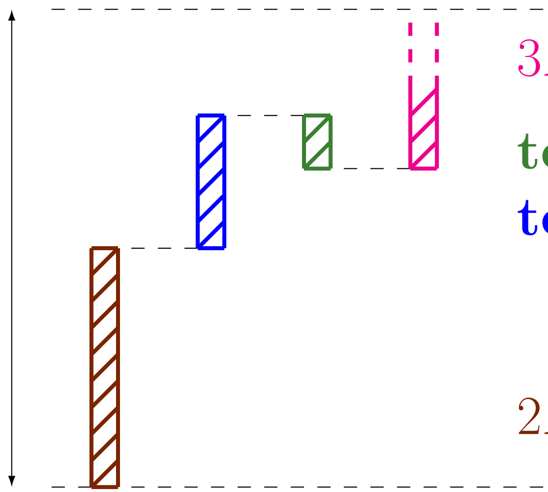
— **fundamental concept for nuclear structure**

astrophysical importance

e.g.
- **waiting point in s- & r-processes**
- **constraint on EoS** $\leftarrow$ **subtracting shell effects**

origin ? (→ **correct prediction of shell structure**)

ℓs splitting



$3N$ LS int. (?)

tensor int. (1st order)

tensor int. (2nd order)

T. Terasawa & A. Arima P.T.P. 23, 87; 115 ('60)

$2N$ LS int.

χEFT → $3N$ LS int. (→ ρ-dep. LS int.)

N. Kaiser, PRC 68, 054001; M. Kohno, PRC 86, 061301(R)

strength ? (↔ convergence of χEFT) **exp. evidence ?**

★ **Nuclear charge radius** $\langle r^2\rangle_c(A)\,\big[=\langle r^2\rangle_p(A)+\langle r^2\rangle_c(p)+(N/Z)\langle r^2\rangle_c(n)\big]$

$\leftarrow (e,e)$ ***etc.***

- **model-indep. data**
- **(lowest-order) quantity reflecting proton dist. in nucleus**

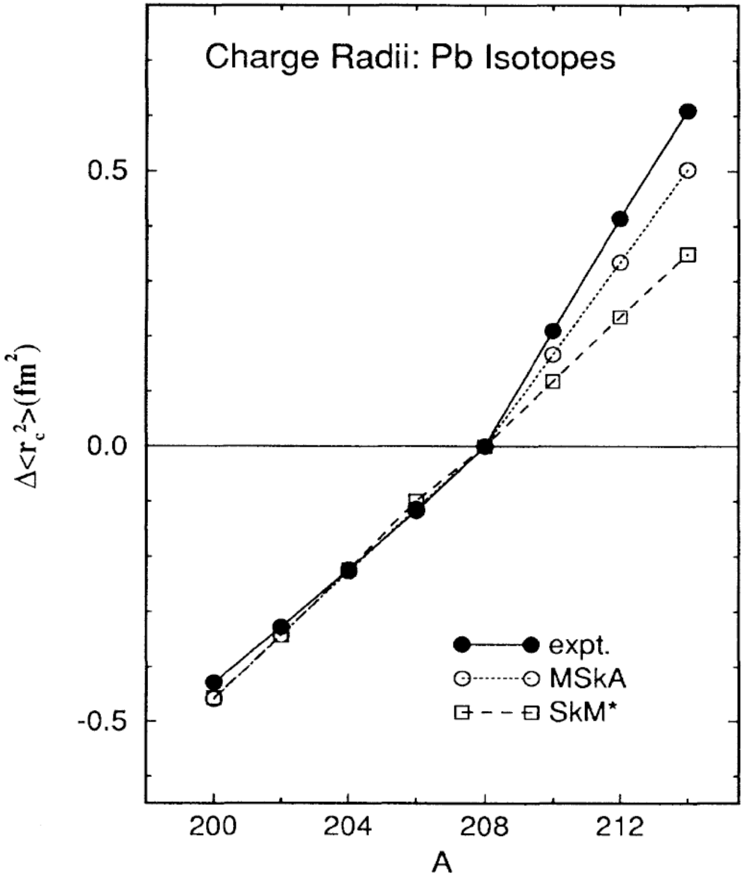
  — nuclear structure information additional to energy

$\hookrightarrow$ **Isotope shift** $\cdots$ **atomic X-ray freq. difference among isotopes**

$\propto \varDelta\langle r^2\rangle_c\,[\approx \varDelta\langle r^2\rangle_p]$ **for heavy nuclei**

- **exp. data — high-precision, incl. unstable nuclei**
- **good indicator to structure change** ***e.g.*** **spherical to deformed**
- $Z=\mathrm{magic}$ **nuclei** $\cdots$ **(quite likely) spherical** $\rightarrow$ **several puzzles !**

• **Isotope shifts in Pb nuclei** $\Delta\langle r^2\rangle_c(^A\mathbf{Pb}) := \langle r^2\rangle_c(^A\mathbf{Pb}) - \langle r^2\rangle_c(^{208}\mathbf{Pb})$



M.M. Sharma *et al.*, PRL 74, 3744

exp. $\Rightarrow$ **kink at** $N = 126$ **!**

∘ **not reproduced by Skyrme EDF (up to '95)**

∘ **reproduced by RMF**
due to isospin content of LS int.
(not direct rel. effect)

∘ **leading to extension of Skyrme EDF**

$\cdots$ **but problem remains !**

kink in $\Delta\langle r^2\rangle_c({}^{A}\mathbf{Pb})$ at $N = 126$ $\longleftarrow$ $n0i_{11/2}$ **occupation**

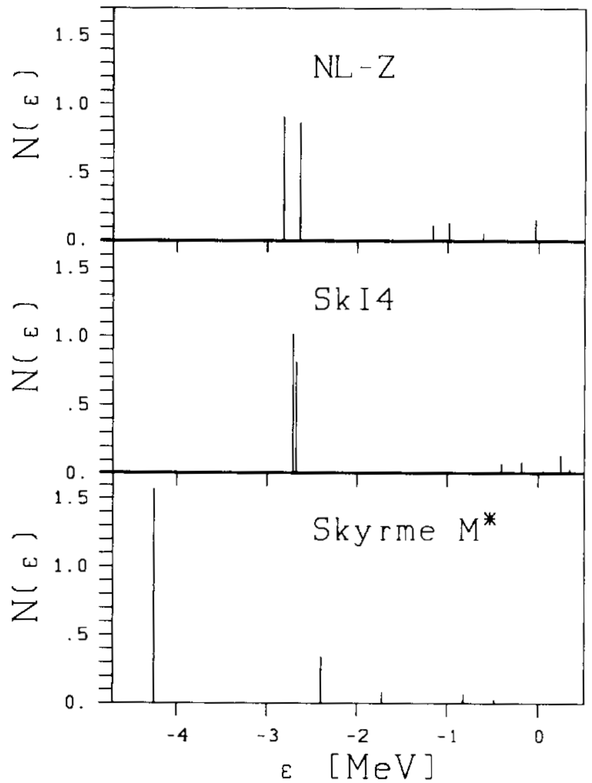
$\Uparrow$

p-n **attraction**

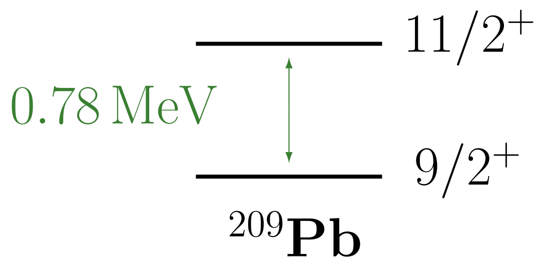
- **larger $\langle r^2\rangle$ than neighboring orbits**
- $N < 126$ — **unocc.**
- $N > 126$ — **sizable occ. prob. ($\because$ pairing)**

However, $\varepsilon_n(0i_{11/2}) \approx \varepsilon_n(1g_{9/2})$ required ! **($\leftrightarrow$ equal occ. prob.)**

cf. pseudo-spin



on the contrary $\cdots$



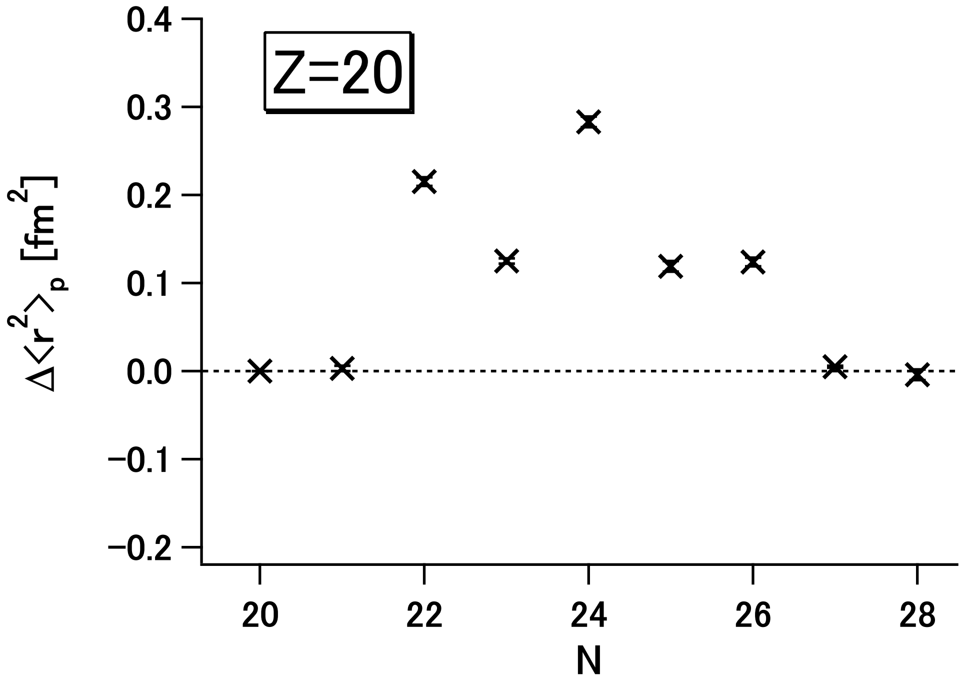
$(\approx \Delta_{\text{pair}})$

$\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2})$

$\leftrightarrow$ **isospin content of LS int.**

P.-G. Reinhard & H. Flocard, NPA 584, 467

- **Charge radii of ^{40}Ca & ^{48}Ca**



(from ADNDT 99, 69)

^{40}**Ca** & ^{48}**Ca** — **doubly magic**
→ **MF cal. good (?)**

exp. ⇒ $\langle r^2 \rangle_c(^{40}\mathbf{Ca}) \approx \langle r^2 \rangle_c(^{48}\mathbf{Ca})$ **!**

∘ $\langle r^2 \rangle_c(^{40}\mathbf{Ca}) < \langle r^2 \rangle_c(^{48}\mathbf{Ca})$
in most MF cal.

∘ **exception — RMF**
G.A. Lalazissis *et al.*, ADNDT 71, 1

⋯ **physics ?**

cf. $^{42-46}$**Ca** ⋯ **beyond-MF effects ?**
e.g. **excitation across** $Z = 20$ **shell gap**
E. Caurier *et al.*, PLB 522, 240

$(\nabla\rho)$**-dep. pairing ?** S.A. Fayans, JETP Lett. 68, 169

II. Mean-field approaches with semi-realistic interaction

"Semi-realistic" nucleonic interaction $\longleftarrow$ **microscopic $2N$ ($+3N$) int.**

$\uparrow$

phenomenological modification

$\hat{v}_{\mathrm{M3Y}} \approx G$**-matrix**

$\downarrow$

$\hat{v}_{\mathrm{M3Y-P}n}$ **H.N., PRC 68, 014316**

- **central** $\cdots$ ρ**-dep. introduced** ($\leftrightarrow$ **saturation, incl.** $3N$ **effects)**
- **LS** $\cdots$ **modification** ($\leftrightarrow$ ℓs **splitting, shown later)**
  - **finite-range** $\rightarrow$ $T = 0$ **&** 1 **channels**
- **tensor** $\cdots$ **unchanged** ($\hat{v}^{(\mathrm{TN})}_{\mathrm{M3Y-P}n} = \hat{v}^{(\mathrm{TN})}_{\mathrm{M3Y}}$) $\rightarrow$ **realistic**

'M3Y-P6' $\rightarrow$ **reasonable shell structure (*e.g.* magic #)**

H.N., PRC 87, 014336 ; H.N. & K. Sugiura, PTEP 2014, 033D02

$\hookrightarrow$ **yardstick**

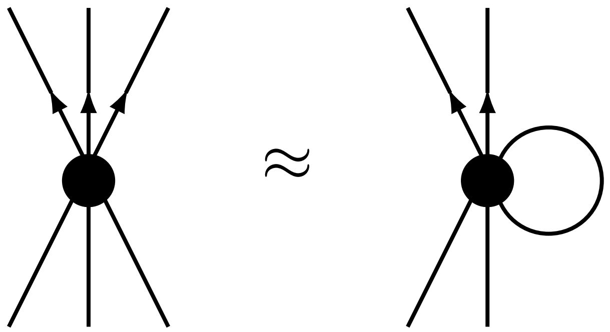
★ **Incorporating $3N$ LS int.**

ℓs **splitting** — **more or less fitted in MF cal.** **($\to$ shell structure)**

$\cdots$ **should not be changed seriously**

$$\left(\begin{array}{l} \textbf{Do we really need } 3N \textbf{ LS int. in MF approaches,} \\ \quad \textbf{even if it siginifcantly contributes to } \ell s \textbf{ splitting ?} \end{array} \right)$$

$3N$ LS int. $\leftrightarrow$ ρ-dep. LS int. ($\hat{v}^{(\mathrm{LS}\rho)}$) **M. Kohno, PRC 86, 061301(R)**



$$v^{(\mathrm{LS}\rho)} = 2i\, D[\rho(\boldsymbol{R}_{ij})]\, \boldsymbol{p}_{ij} \times \delta(\boldsymbol{r}_{ij})\, \boldsymbol{p}_{ij} \cdot (\boldsymbol{s}_i + \boldsymbol{s}_j)\,;$$

$$D[\rho(\boldsymbol{r})] = -w_1 \frac{\rho(\boldsymbol{r})}{1 + d_1 \rho(\boldsymbol{r})} \left(\approx -w_1\, \rho(\boldsymbol{r}) \right)$$

$$\propto D[\rho] \cdot \hat{v}^{(\mathrm{LS})}_{\mathrm{Sky}} \approx \rho \cdot \hat{v}^{(\mathrm{LS})}_{\mathrm{Sky}}$$

$\Downarrow$

$$\left. \begin{array}{lll} \textbf{M3Y-P6} & — & \hat{v}^{(\mathrm{LS})}_{\mathrm{M3Y}} \times 2.2 \\ \textbf{\textit{vs.}} & & \\ \textbf{M3Y-P6a} & — & \hat{v}^{(\mathrm{LS})}_{\mathrm{M3Y}} + \hat{v}^{(\mathrm{LS}\rho)} \end{array} \right\}$$

$\Delta\varepsilon^{\ell s}_{\mathrm{M3Y-P6}} \approx \Delta\varepsilon^{\ell s}_{\mathrm{M3Y-P6a}} \to w_1\,(> 0)$

($\Delta\varepsilon^{\ell s}(n0i)$ at ^{208}Pb)

$\to$ **influence on energies not large**

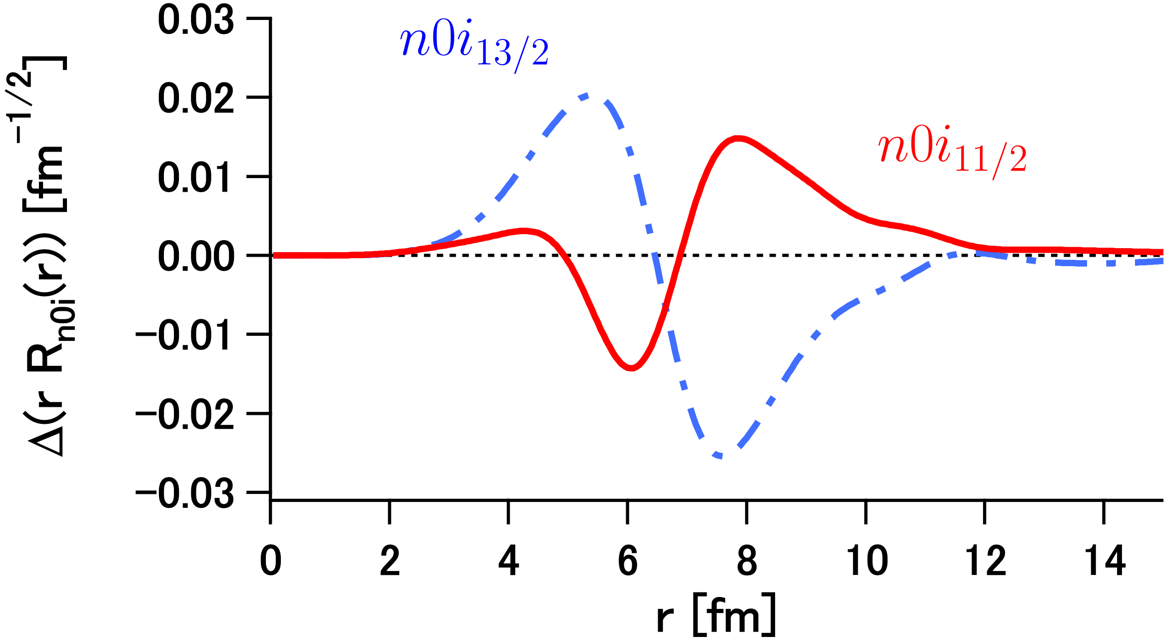
★ **Influence on s.p. wave functions :**

presence of $D[\rho]$ $\rightarrow$ $\begin{cases} \textbf{stronger LS for interior (higher } \rho) \\ \textbf{weaker LS for extetior (lower } \rho) \end{cases}$

$\rightarrow$ $\begin{cases} j = \ell + 1/2 \textbf{ orbitals shrink} \\ j = \ell - 1/2 \textbf{ orbitals extends} \end{cases}$

e.g. **larger** $\langle r^2 \rangle$ **for** $n0i_{11/2}$

$$\Delta[r\,R_j(r)] := [r\,R_j(r)]_{\mathrm{M3Y-P6a}} - [r\,R_j(r)]_{\mathrm{M3Y-P6}} \quad @\ {}^{208}\mathbf{Pb}$$

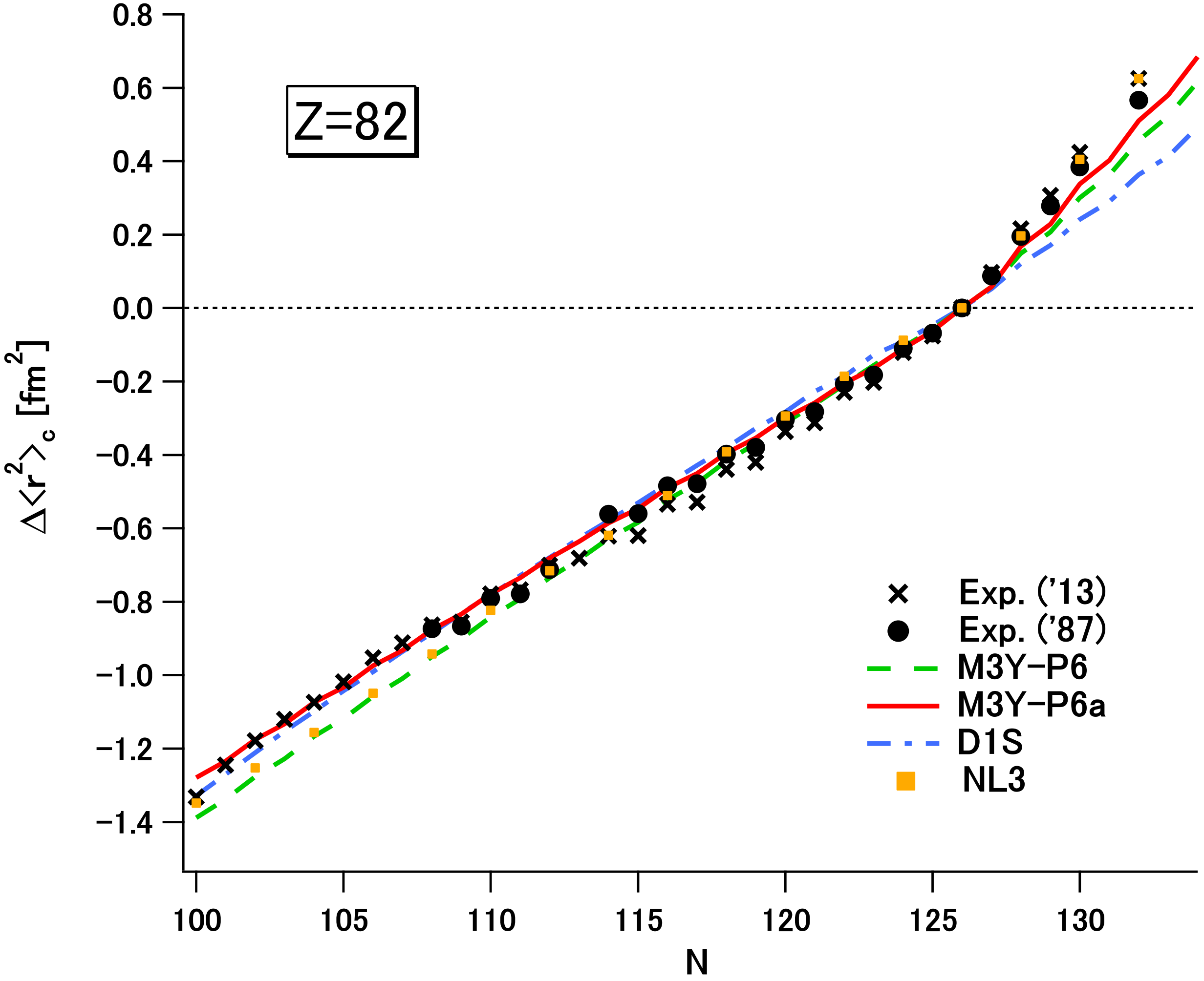


III. $3N$ LS interaction & isotope shifts in $Z=\mathrm{magic}$ nuclei

Spherical HFB cal. $\rightarrow$ **isotope shifts in** $Z=\mathrm{magic}$ **nuclei**

M3Y-P6 *vs.* M3Y-P6a $\rightarrow$ **effects of** $3N$ **LS int.**

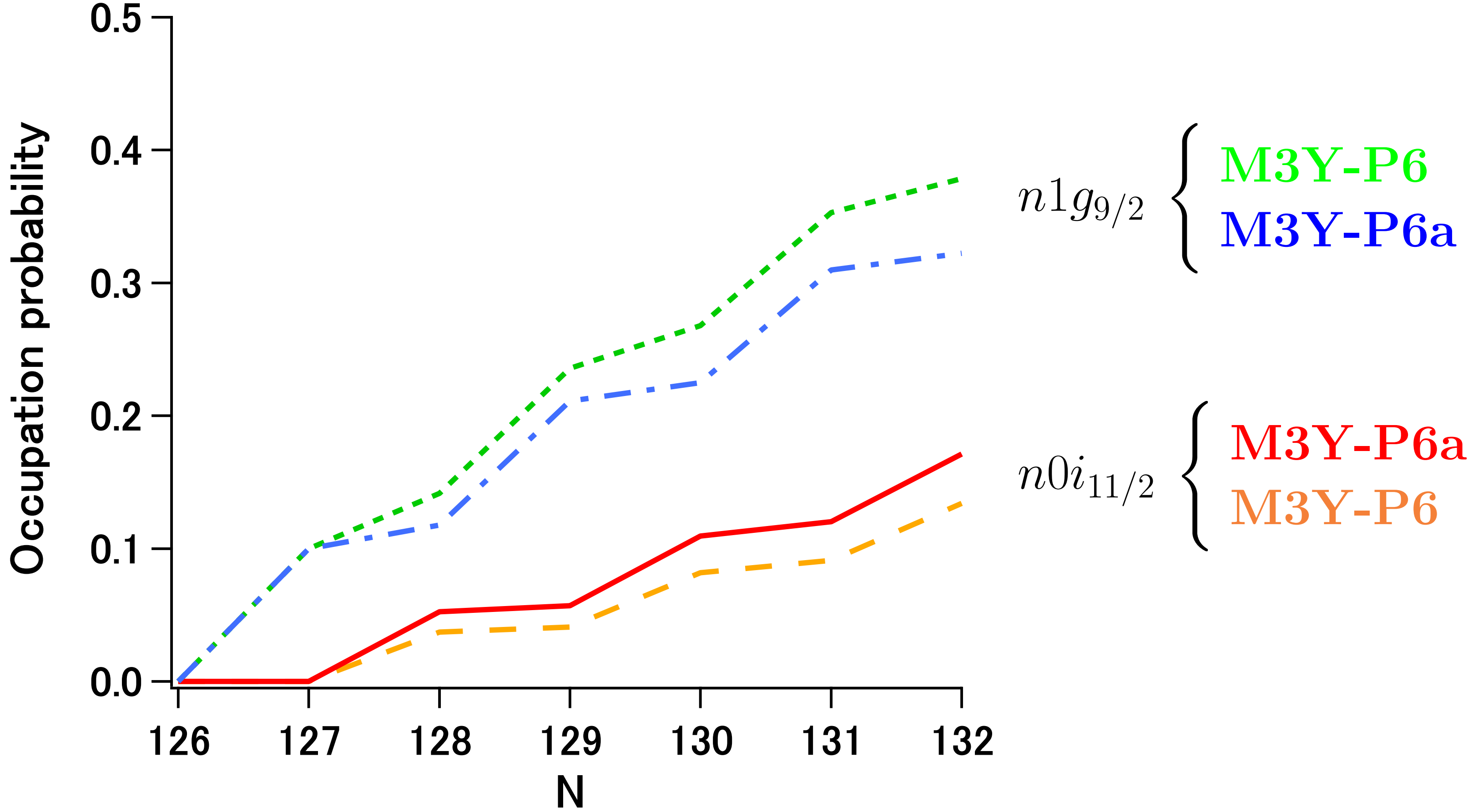
★ **Isotope shifts of Pb nuclei** $\Delta\langle r^2\rangle_c(^{A}\mathbf{Pb}) := \langle r^2\rangle_c(^{A}\mathbf{Pb}) - \langle r^2\rangle_c(^{208}\mathbf{Pb})$



S.p. energies & occ. prob.

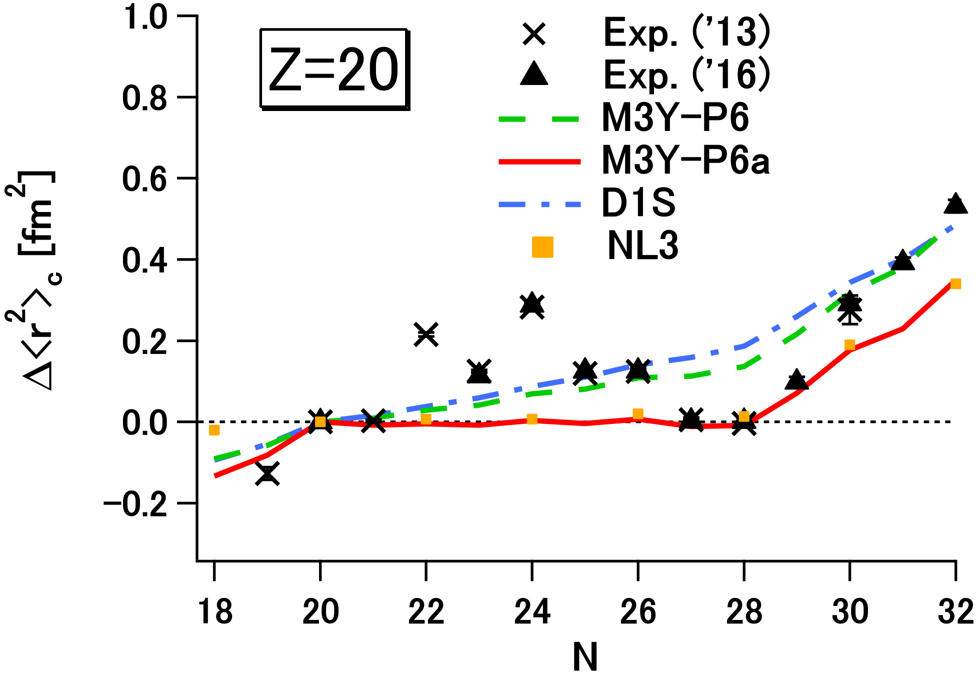
$\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2})$: $\begin{cases} \textbf{exp. @ }{}^{209}\textbf{Pb} \rightarrow 0.78\,\mathrm{MeV} \\ \textbf{M3Y-P6a} \rightarrow 0.72\,\mathrm{MeV} \end{cases}$

occ. prob.



$\Rightarrow$ **kink at $N = 126$ reproduced without $n1g_{9/2}$-$n0i_{11/2}$ degeneracy !**

★ Isotope shifts of Ca nuclei $\Delta\langle r^2\rangle_c(^{A}\mathbf{Ca}) := \langle r^2\rangle_c(^{A}\mathbf{Ca}) - \langle r^2\rangle_c(^{40}\mathbf{Ca})$



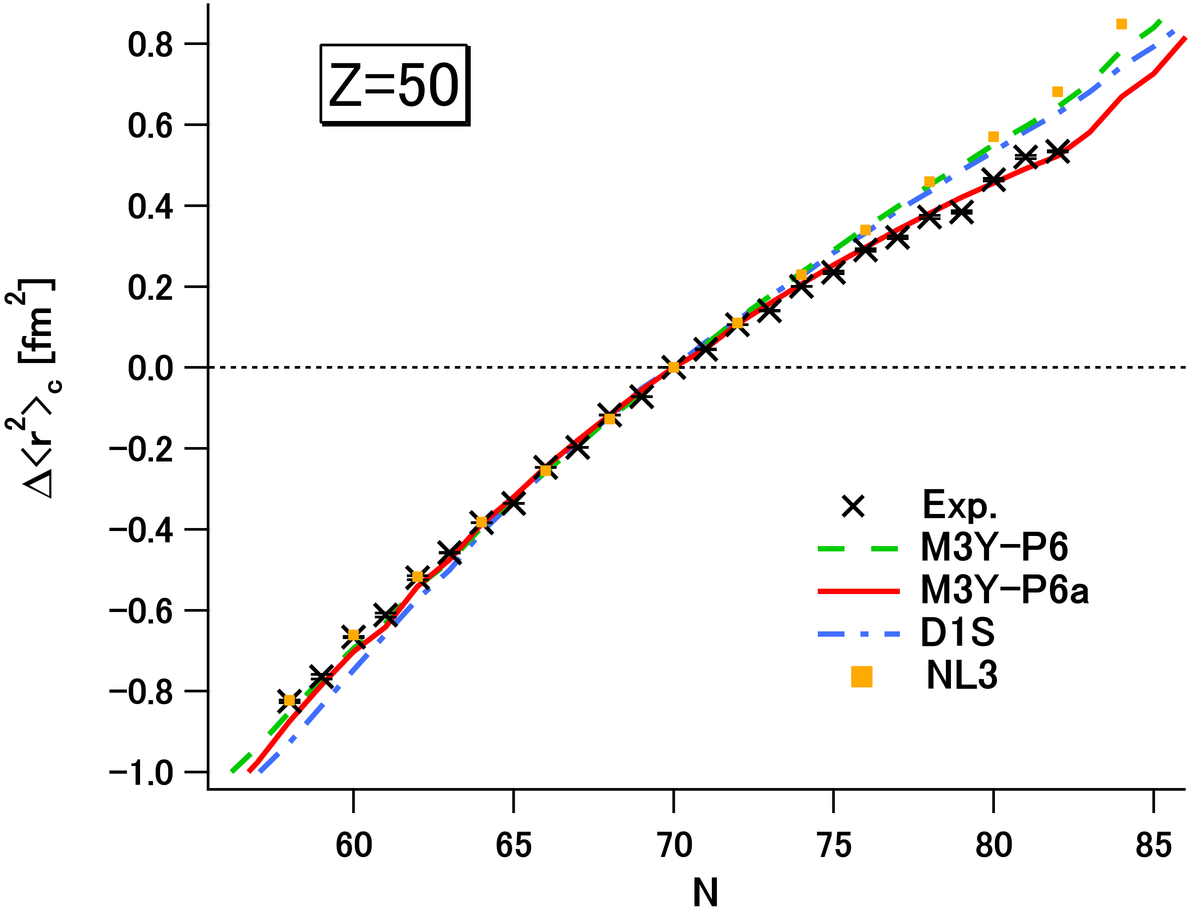
(▲ : R.F. Garcia Ruiz *et al.*, Nat. Phys. 12, 594)

$\langle r^2\rangle_c(^{40}\mathbf{Ca}) \approx \langle r^2\rangle_c(^{48}\mathbf{Ca})$ **reproduced !**

$\because$) ρ**-dep. LS** $\rightarrow n0f_{7/2}$ **shifts inward** (**RMF ?**)

cf. χ**EFT + CC / SRG also reproduce** $\langle r^2\rangle_c(^{40}\mathbf{Ca}) \approx \langle r^2\rangle_c(^{48}\mathbf{Ca})$

★ **Isotope shifts of Sn nuclei** $\Delta\langle r^2\rangle_c(^A\mathbf{Sn}) := \langle r^2\rangle_c(^A\mathbf{Sn}) - \langle r^2\rangle_c(^{120}\mathbf{Sn})$



kink predicted at $N = 82$!

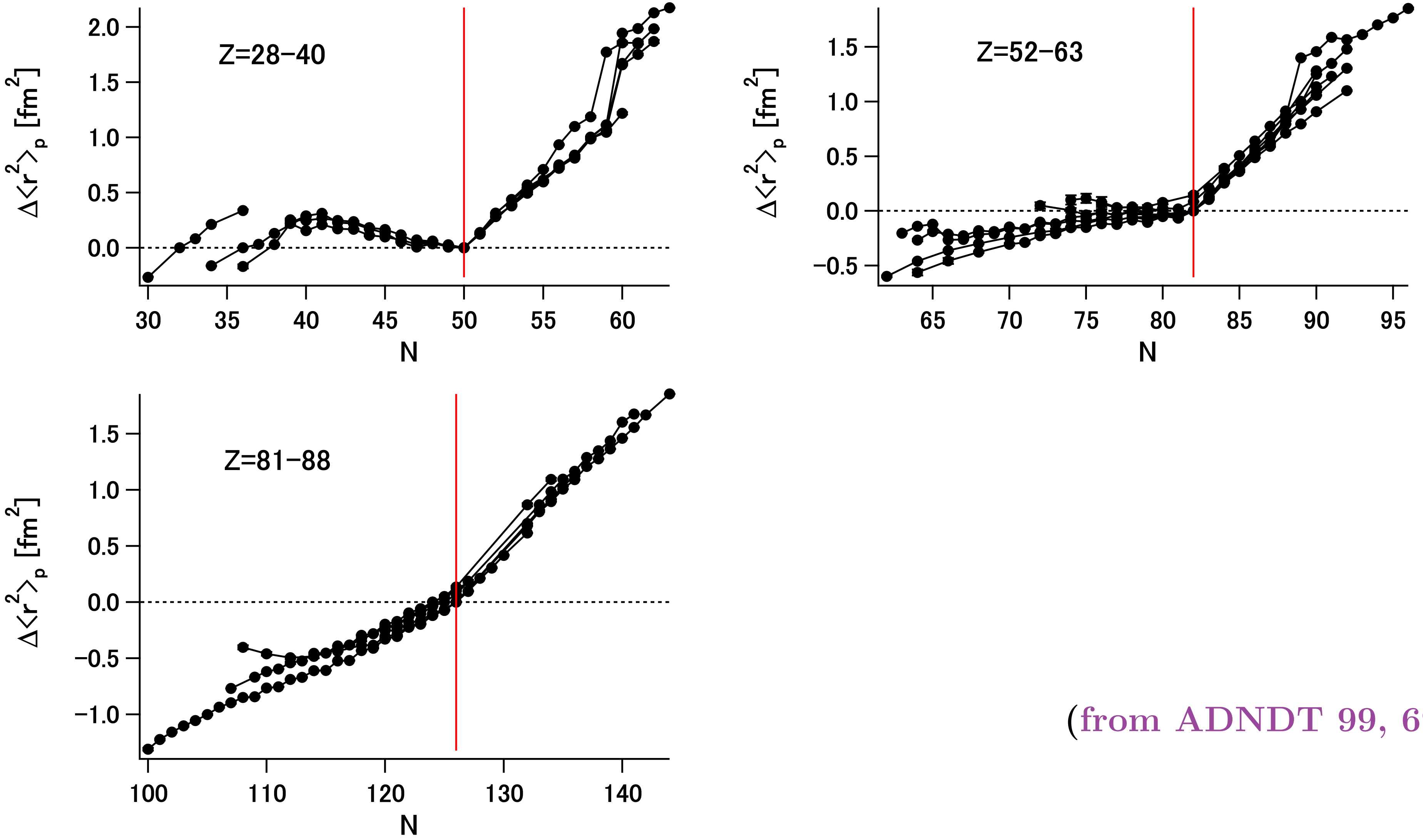
$\because$) ρ-**dep. LS** $\rightarrow$ $\begin{cases} n0h_{11/2} \textbf{ shifts inward} & \rightarrow \textbf{ slope in } 70 \lesssim N \leq 82 \\ n0h_{9/2} \textbf{ shifts outward} & \rightarrow \textbf{ slope in } N > 82 \end{cases}$

— **in sharp contrast to results w/o ρ-dep. LS (incl. RMF)**

$\Rightarrow$ **data?** $\rightarrow$ **seems confirmed in recent exp.!**

★ ρ-dep. LS → kink generally expected wherever N is jj-closed magic

$\cdots$ consistent with exp. data



(from ADNDT 99, 69)

Note : deformation will be responsible for a certain part

IV. Summary

1. **We have investigated effects of $3N$ LS int. on isotope shifts of nuclei.**

   ← sph. HFB with semi-realistic int. M3Y-P6 & its variant M3Y-P6a

2. **With M3Y-P6a which contains ρ-dep. LS channel,**

   - **isotope shifts of the Pb nuclei are described fairly well without fictitious $n1g_{9/2}$-$n0i_{11/2}$ degeneracy (broken pseudo-spin sym.),**
   - **almost equal charge radii between ^{40}Ca and ^{48}Ca are reproduced,**
   - **isotope shifts of the Sn nuclei are in agreement with available data, and a kink is predicted at $N = 82$. → seems confirmed !**

3. **Results may be regarded as evidence for $3N$ LS interaction based on χEFT, indep. of ℓs splitting.**

   — qualitative evidence for $3N$ interaction ! (?)

   (cf. RMF, Fayan's EDF ?)

Collaborators :

for this study

T. Inakura (***Niigata U., Japan***)

H.N. & T. Inakura, PRC 91, 021302(R)
H.N., PRC 92, 044307

for preceding studies

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