Nuclear Mass Relations

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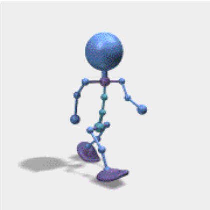
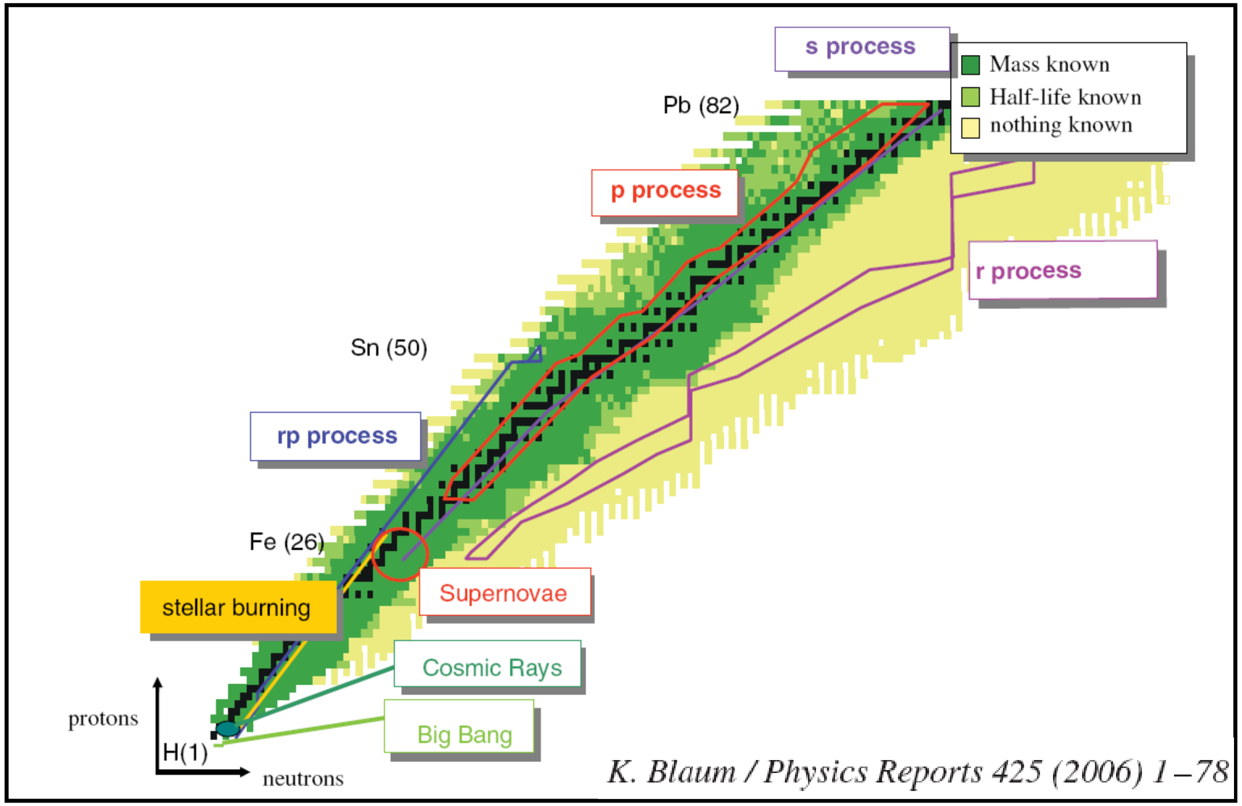
OUTLINE



- Introduction
- Our recent results based on G-K
- Summary and prospect

Nuclear masses or nuclear binding energies



about 3450 nuclear masses have been observed

One needs to rely on theoretical models for nuclear masses.

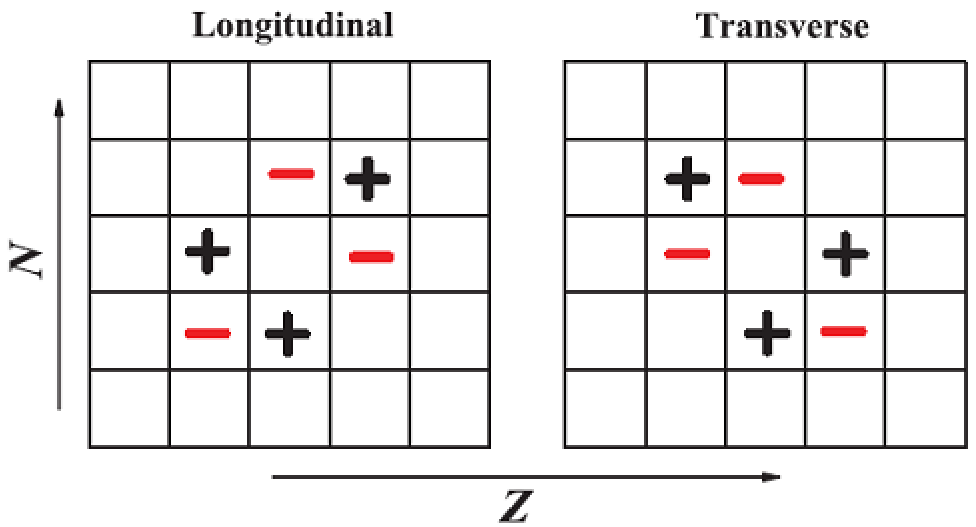
Theoretical models for nuclear masses

- **Global mass models**
- **Local mass models**

Simple algebraic formulas expressing the mass of any given unknown nucleus are written down in terms of masses of known neighboring nuclei.

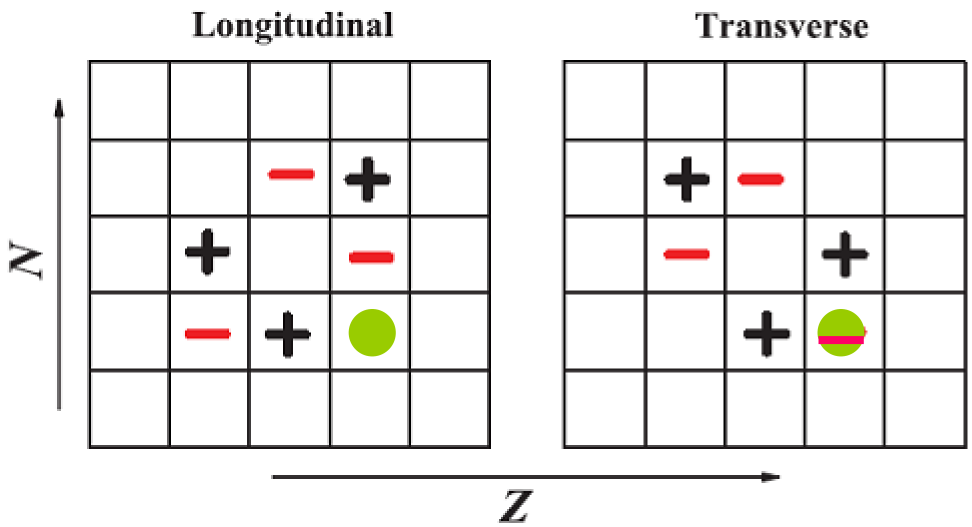
- **Audi-Wapstra extrapolations**
- **Garvey-Kelson mass relations**
- **Mass relations associated with proton-neutron interactions**

Garvey-Kelson mass relations



G. T. Garvey and I. Kelson, Phys. Rev. Lett. **16**, 197 (1966); G. T. Garvey, W. J. Gerace, R. L. Jaffe, I. Talmi, and I. Kelson, Rev. Mod. Phys. **41**, S1 (1969).

Garvey-Kelson mass relations



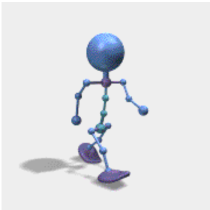
RMSD~150 keV for nuclei with A>=120 based on the AME2012

$$M(N+2,Z)-M(N,Z-2)+M(N+1,Z-2)-M(N+2,Z-1)+M(N,Z-1)-M(N+1,Z)=0.$$

$$M(N+2,Z-2)-M(N,Z)+M(N,Z-1)-M(N+1,Z-2)+M(N+1,Z)-M(N+2,Z-1)=0$$

OUTLINE

- Introduction



- Our recent results based on G-K

It is useful to investigate various extrapolation approaches based on the G-K relations to make reliable predictions further away from known masses.

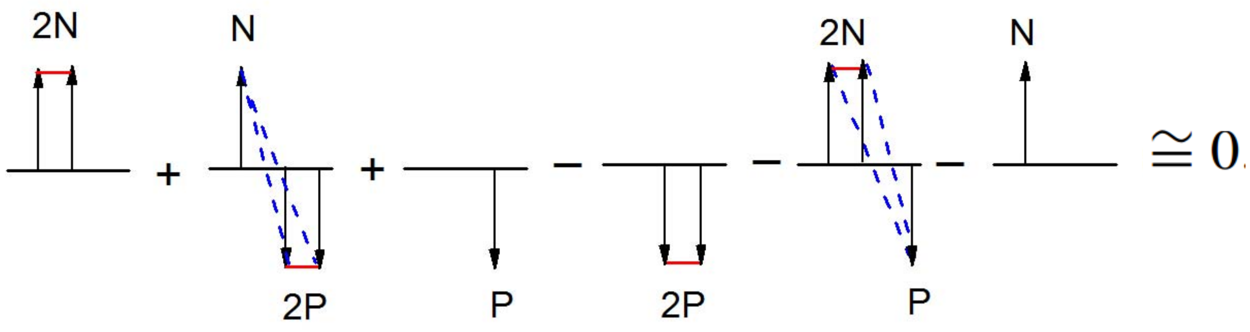
Our recent results based on G-K

- **Generalized G-K mass relations**
- **Mass relations associated with proton-neutron interactions**
- **Local relations of alpha-decay energies**
- **Extraction of the Wigner energy**
- **parameterizations of the symmetry energy coefficients.**
- **Improved Jänecke mass formula**
- **Local relations of separation energies**

(1) Generalized G-K mass relations

Based on an independent-particle shell model, the residual neutron-neutron, proton-proton, and neutron-proton interactions are canceled out.

GKL :

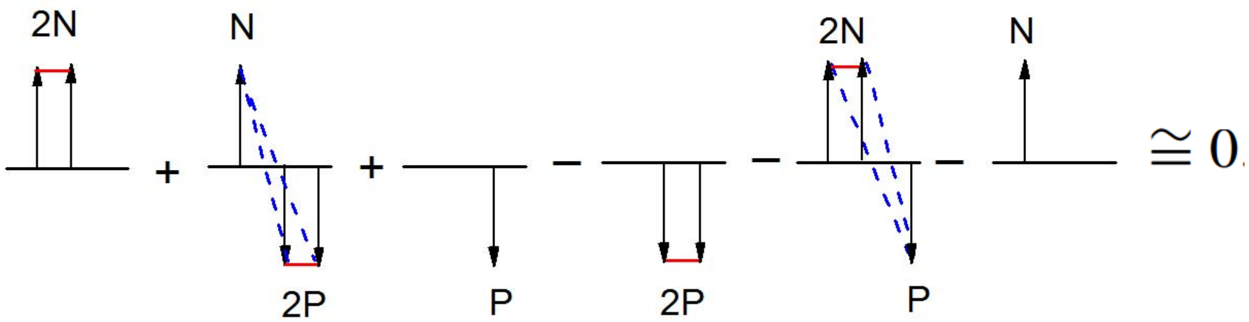


The neutron-neutron and proton-proton interactions are easily canceled out by requiring that each of the N and Z values appears twice with different signs for the six masses in mass equation.

GKL :

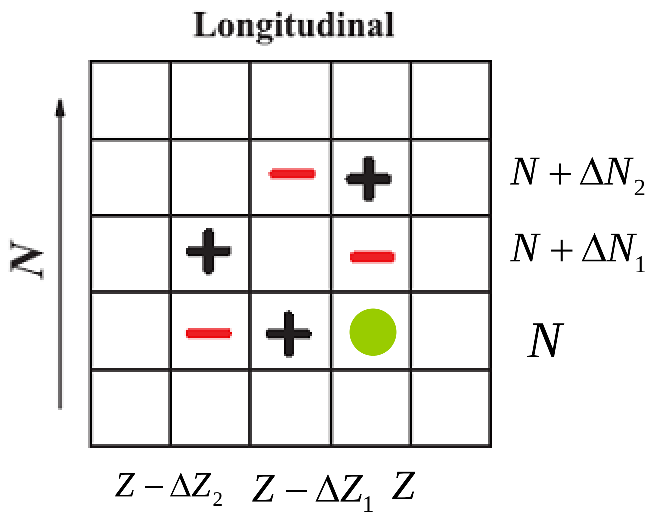
$$M(N+2, Z) - M(N, Z-2) + M(N+1, Z-2) - M(N+2, Z-1) + M(N, Z-1) - M(N+1, Z) = 0.$$

$$M_{+2,0} + M_{+1,-2} + M_{0,-1} - M_{0,-2} - M_{+2,-1} - M_{+1,0} \cong 0,$$



Neutron-proton interactions are canceled out.

GKL :

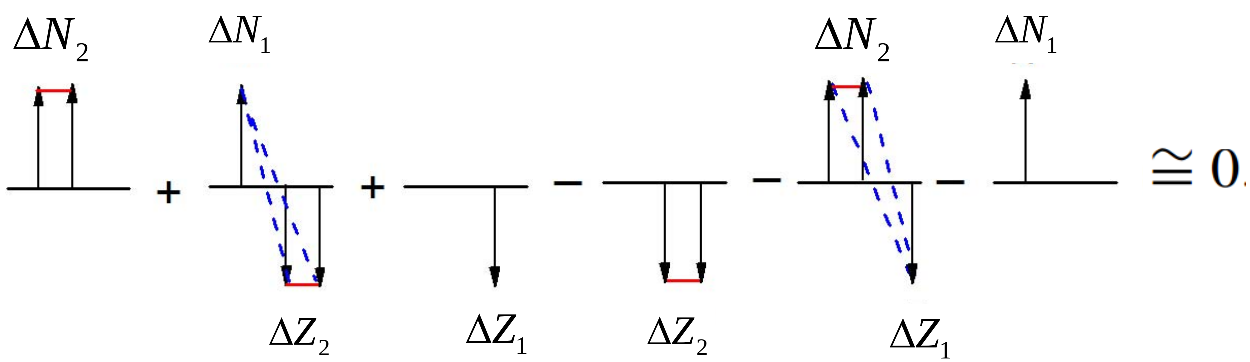


The total neutron-proton interaction of each nucleus is proportional to the product of N and Z.

$$\Delta N_1 \Delta Z_2 = \Delta N_2 \Delta Z_1 = L = 2$$

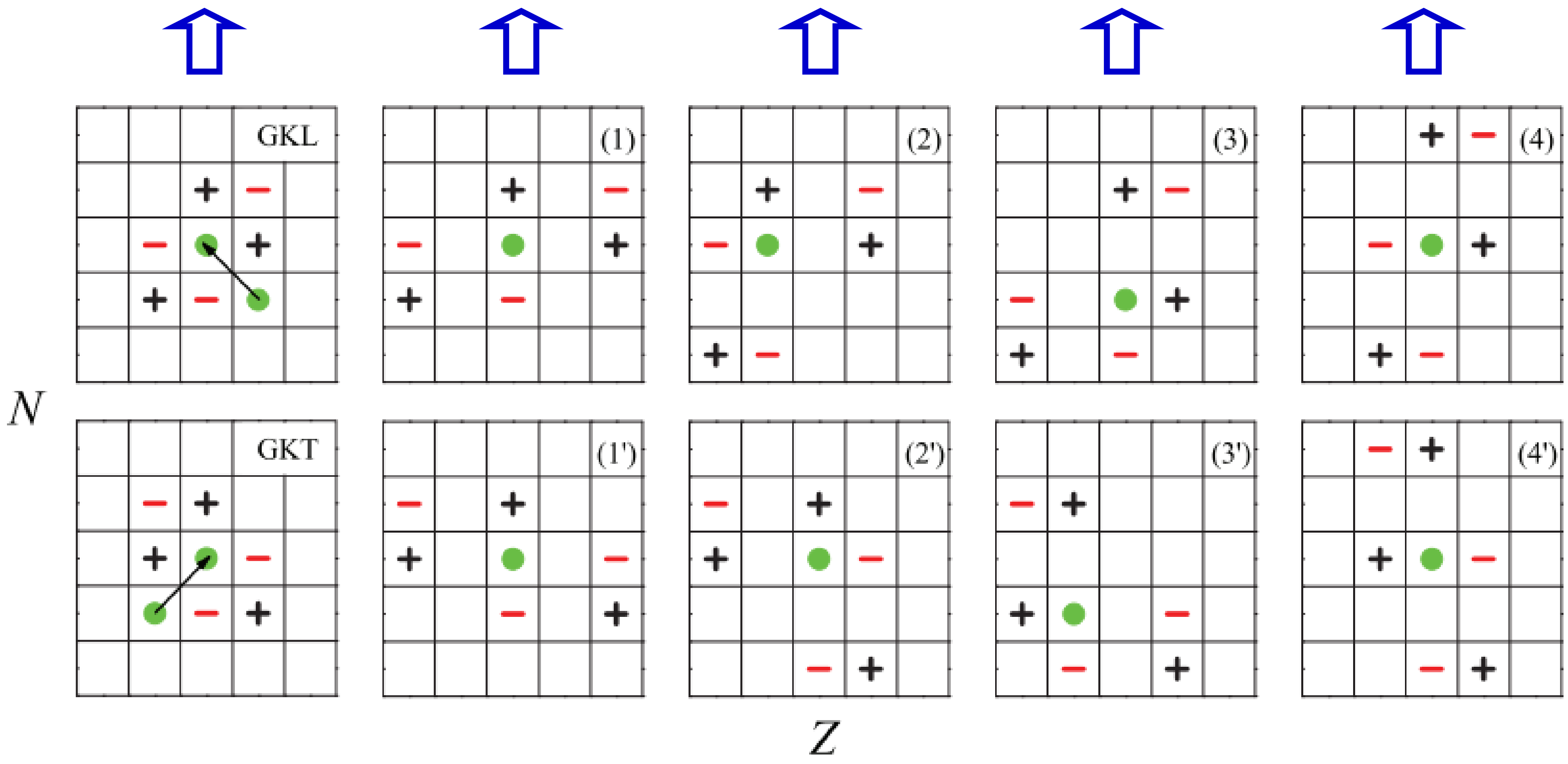
$$\Delta N \le 2; \quad \Delta N_1 = 1, \Delta N_2 = 2$$

$$\Delta Z \le 2; \quad \Delta Z_1 = 1, \Delta Z_2 = 2$$



- For $L=\pm 2$, one obtains 32 local mass relations, among which ten of them are independent. $\Delta N_1 \Delta Z_2 = \Delta N_2 \Delta Z_1 = L$

$\Delta N \le 2$	$\Delta N \le 2$	$\Delta N \le 3$	$\Delta N \le 3$	$\Delta N \le 4$
$\Delta Z \le 2$	$\Delta Z \le 4$	$\Delta Z \le 3$	$\Delta Z \le 3$	$\Delta Z \le 2$



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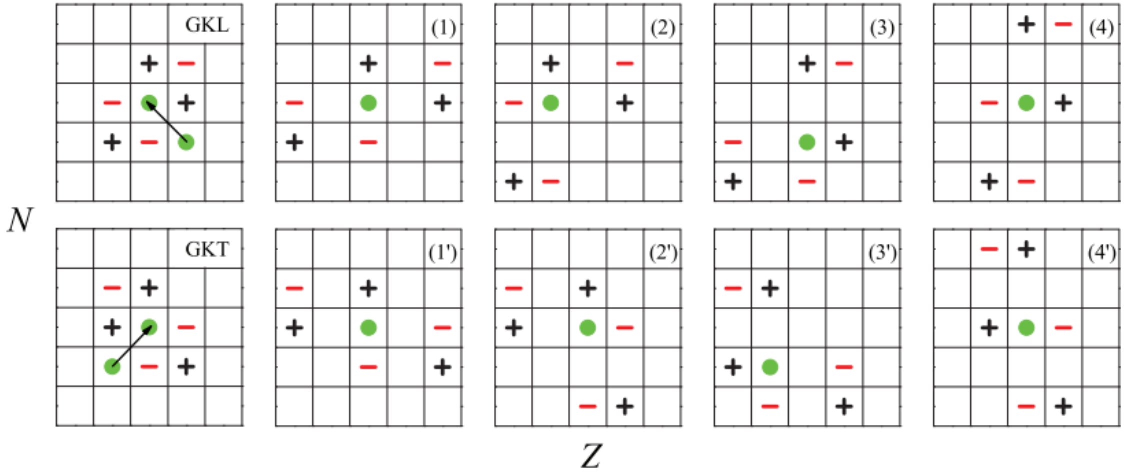
Using these equations, one can enlarge possibilities to predict masses of more nuclei.



$$\text{GKL}: M_{+1,0} + M_{0,-2} + M_{+2,-1} - M_{+1,-2} - M_{0,-1} - M_{+2,0} = D_{\mathrm{L}}(N,Z) \cong 0,$$

$$(1): M_{+1,0} + M_{0,+2} + M_{-1,-2} - M_{+1,+2} - M_{0,-2} - M_{-1,0} = D_{1}(N,Z) \cong 0,$$

$$(2): M_{+1,0} + M_{0,+2} + M_{-2,-1} - M_{+1,+2} - M_{0,-1} - M_{-2,0} = D_{2}(N,Z) \cong 0,$$

$$(3): M_{+2,0} + M_{0,+1} + M_{-1,-2} - M_{+2,+1} - M_{0,-2} - M_{-1,0} = D_{3}(N,Z) \cong 0,$$

$$(4): M_{+2,0} + M_{0,+1} + M_{-2,-1} - M_{+2,+1} - M_{0,-1} - M_{-2,0} = D_{4}(N,Z) \cong 0,$$

$$\text{GKT}: M_{+1,0} + M_{0,+2} + M_{+2,+1} - M_{+1,+2} - M_{0,+1} - M_{+2,0} = D_{\mathrm{T}}(N,Z) \cong 0,$$

$$(1'): M_{+1,0} + M_{0,-2} + M_{-1,+2} - M_{+1,-2} - M_{0,+2} - M_{-1,0} = D_{1'}(N,Z) \cong 0,$$

$$(2'): M_{+1,0} + M_{0,-2} + M_{-2,+1} - M_{+1,-2} - M_{0,+1} - M_{-2,0} = D_{2'}(N,Z) \cong 0,$$

$$(3'): M_{+2,0} + M_{0,-1} + M_{-1,+2} - M_{+2,-1} - M_{0,+2} - M_{-1,0} = D_{3'}(N,Z) \cong 0,$$

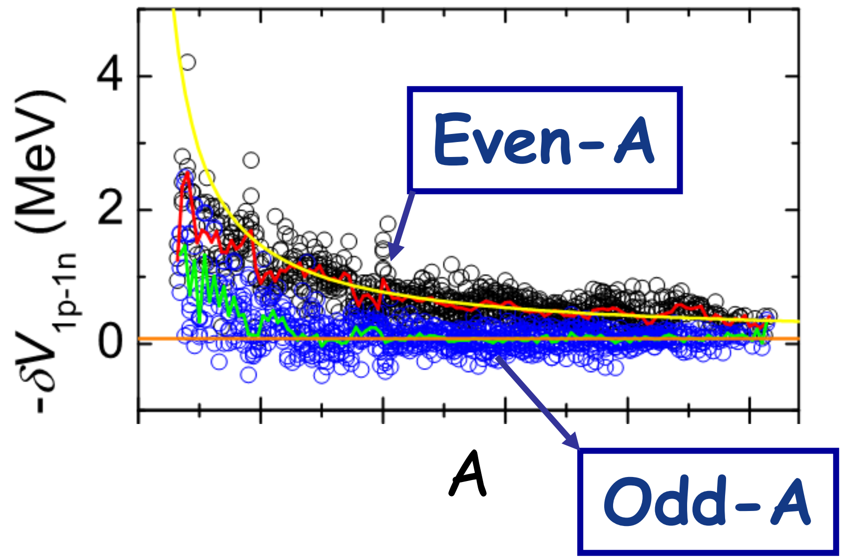
$$(4'): M_{+2,0} + M_{0,-1} + M_{-2,+1} - M_{+2,-1} - M_{0,+1} - M_{-2,0} = D_{4'}(N,Z) \cong 0.$$

(2) Mass relations associated with proton-neutron interactions

The residual proton-neutron interaction

$$\begin{aligned}\delta V_{ip\text{-}jn}(Z,N) &= S_{ip}(Z,N) - S_{ip}(Z,N-j) \\ &= S_{jn}(Z,N) - S_{jn}(Z-i,N) \\ &= [B(Z,N) - B(Z,N-j)] \\ &\quad - [B(Z-i,N) - B(Z-i,N-j)].\end{aligned}$$

Z. C. Gao and Y. S. Chen, Phys. Rev. C **59**, 735 (1999);
Z. C. Gao, Y. S. Chen, and J. Meng, Chin. Phys. Lett. **18**, 1186 (2001).



- **It can be regarded as Garvey-Kelson relations because:**

$$\delta V_{1p\text{-}1n}(e-e) = \delta V_{1p\text{-}1n}(o-o)$$
$$\delta V_{1p\text{-}1n}(e-o) = \delta V_{1p\text{-}1n}(o-e)$$

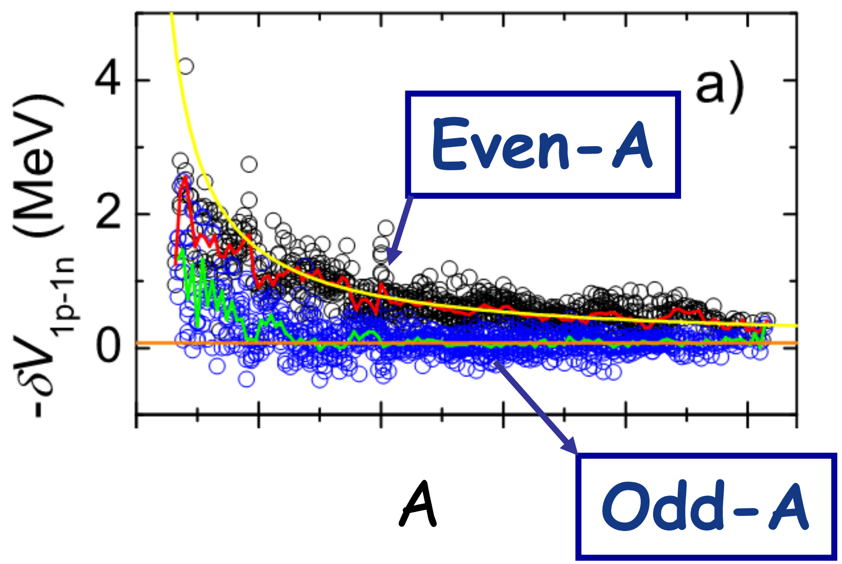
GKL

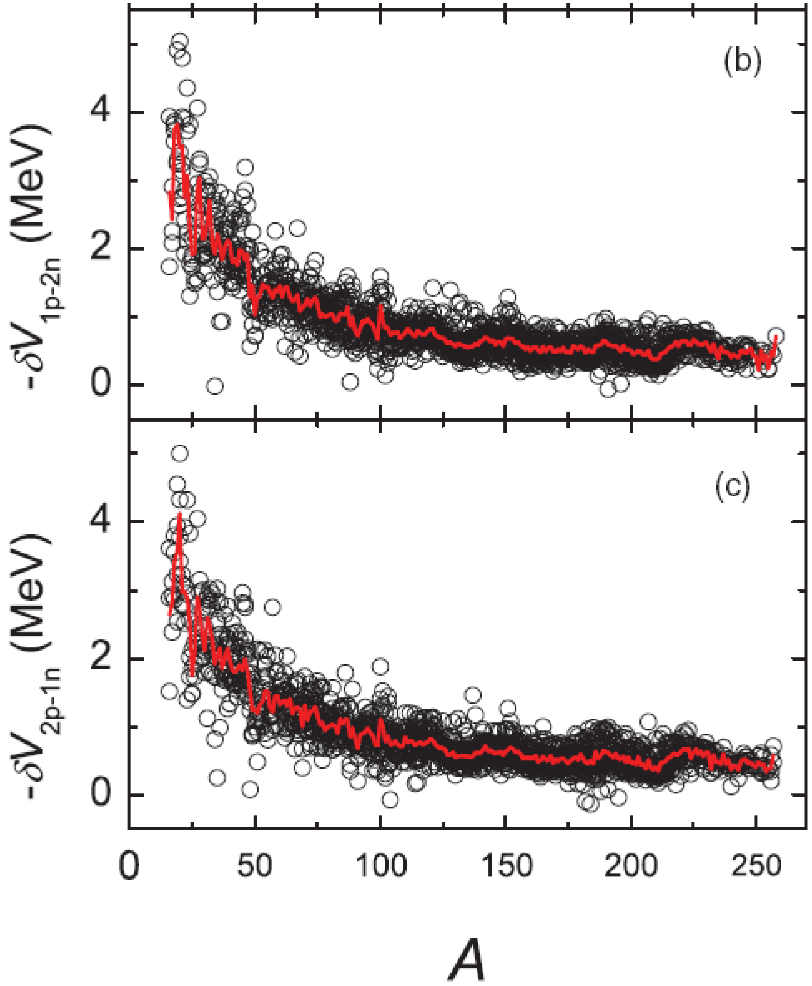
$$\begin{aligned} &M(Z-2,N+2) - M(Z,N) \\ &+M(Z-1,N) - M(Z-2,N+1) \\ &+M(Z,N+1) - M(Z-1,N+2) = 0 \ , \end{aligned}$$

$$\begin{aligned} &M(Z,N+2) - M(Z-2,N) \\ &+M(Z-2,N+1) - M(Z-1,N+2) \\ &+M(Z-1,N) - M(Z,N+1) = 0 \ , \end{aligned}$$

GKT

For $(i, j) = (1, 1), (1, 2), (2, 1)$, the values of $\delta V_{ip\text{-}jn}$ are found to exhibit compact correlations with mass number A and thus are very useful to describe and to predict atomic masses





http://nuclearmasses.org/resources.html

nuclearmasses.org

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[3]Argonne National Laboratory
[4]Japan Atomic Energy Agency

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César Barbero, Jorge G. Hirsch, Alejandro E. Mariano, Nuc. Phys. A, Vol. 874, 2012, Pgs. 81-97

The anatomy of the simplest Duflo-Zuker mass formula
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J. Duflo, A.P. Zuker, Phys. Rev. C5 2 (1995) R23

Theoretical Mass Models: Nucleon Interaction

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Nuclear mass relations based on systematics of proton-neutron interactions
H. Jiang, et al., Phys. Rev. C 82, 054317 (2010)

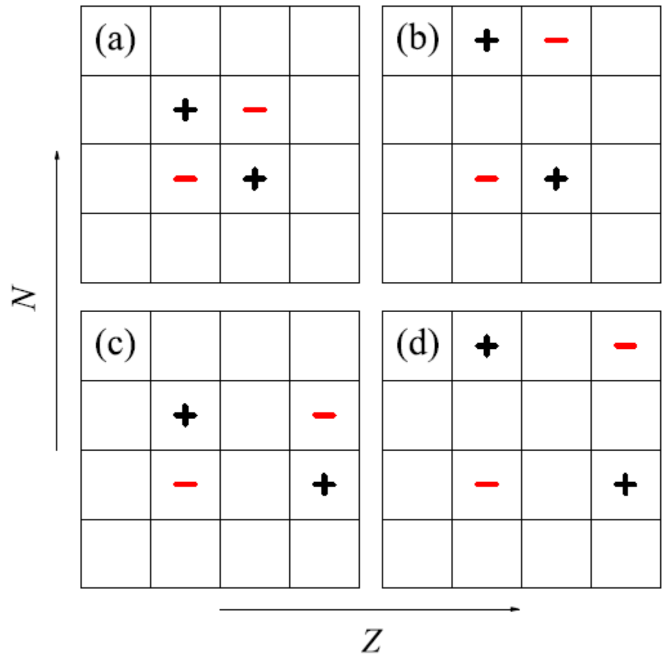
G.J. Fu, Hui Jiang, Y.M. Zhao, S. Pittel, A. Arima, Phys. Rev. C82 (2010) 034304

Partial Databases are in:

http://www.physics.sjtu.edu.cn/~ymzhao/mass-pred-new.txt

(3) Local Relations of alpha-decay Energies

$$Q(N,Z) = -B(N,Z) + B(N-2,Z-2) + B\left({}^4_2\text{He}\right)$$
$$= M(N,Z) - M(N-2,Z-2) - M\left({}^4_2\text{He}\right).$$



$$Q(N+1,Z) + Q(N,Z+1) - Q(N,Z) - Q(N+1,Z+1) \approx 0.$$
$$Q(N+2,Z) + Q(N,Z+1) - Q(N,Z) - Q(N+2,Z+1) \approx 0,$$
$$Q(N+1,Z) + Q(N,Z+2) - Q(N,Z) - Q(N+1,Z+2) \approx 0,$$
$$Q(N+2,Z) + Q(N,Z+2) - Q(N,Z) - Q(N+2,Z+2) \approx 0.$$

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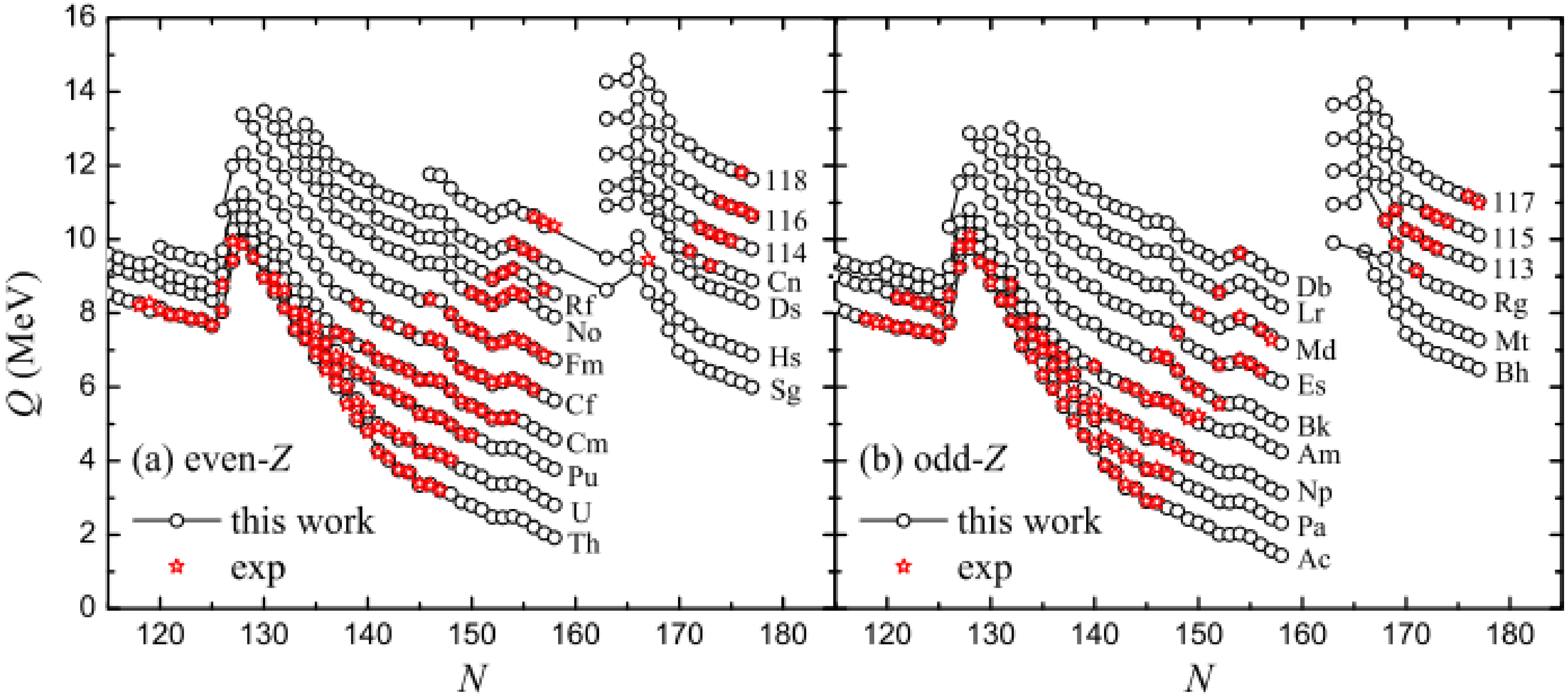
(3) Local Relations of alpha-decay Energies

These relations are found to yield very small derivations from available experimental data.



Z **ranging from 89 to 118 predicted by our formulas.**

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- Summary and prospect

Summary

(1) Simple relations between nuclear masses may be very useful because of their simplicity in making ground-state mass predictions.

(2) The rms deviation of local mass predictions grows rapidly by using the previously predicted masses on each new iteration. However Continuing progresses have been made to achieve more accurate predictions based on local mass relations.

- Mass relations associated with proton-neutron interactions
- Local relations of alpha-decay energies, separation energies
- Generalized G-K mass relations
- Extraction of the Wigner energy
- parameterizations of the symmetry energy coefficients.

Thank you for your attention！