Like- and Unlike-Pairing Correlations in a Deformed Mean Field for Finite Nuclear Systems

- Competition of Deformation and Pairing Correlations in $N = Z$ (Stable or Unstable) Nuclei -

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in collaboration with

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4) Tuebingen University, Germany

5) Bratislava Univ, Slovakia and DUBNA, Moscow, Russia

International Symposium on Simplicity, Symmetry and Beauty of Atomic Nuclei in honor of Professor Akito Arima’s 88 year-old birthday (米寿)

Sep. 25–29, 2018, Shanghai, China
A Recollection on the Dawning of APCTP

JEWAN KIM
EMERITUS PROFESSOR, SEOUL NATIONAL UNIVERSITY

1996

Thanks to Prof. Akito Arima for APCTP & Congratulation on his 88th Birthday!!

Jewan Kim is the professor emeritus in Seoul National University. He has been a research professor at Johns Hopkins University. He served Presidential Science Committee and awarded Science Aw and Technology. He is currently working as Honorary Chairman of Association of Advancement of Scientific I
The First Asia Pacific Physics Conference in Singapore and the Establishment of the Association of Asia Pacific Physical Societies

AKITO ARIMA
PRESIDENT OF THE JAPAN RADIOISOPOTE ASSOCIATION

It is my great pleasure to write my memorandum on the first Asia Pacific Physics Conference in Singapore and the establishment of the Association of Asia Pacific Physical Societies (AAPPS).

I was in Stony Brook, New York for several years between 1971 and 1980. I often discussed in Stony Brook with pore, although China still did not have official diplomatic relations with Singapore. These words convinced me that China would be cooperative towards the Society and receptive to participating in the international conferences.

For a few weeks in the spring of 1981, Professor C. N.

1. Motivation

2. Spin singlet and spin triplet pairing correlations on shape evolution in sd- and pf-shell N=Z nuclei.

3. Effects of the Coulomb and the spin-orbit interaction in a deformed mean field on the residual pairing correlations for N=Z nuclei. The Wigner SU(4) spin–isospin symmetry on the pairing gaps!

4. Competition of deformation and neutron-proton pairing in Gamow-Teller transitions for $^{56,58}$Ni and $^{62,64}$Ni.

5. Summary
**Pairing correlation**

- like-pairing (pp and nn pairing): IV
- unlike-pairing (np pairing): IV & IS

**T=1, S=0**

- Isovector (IV), spin-singlet
- Are there some deuteron-like structures in nuclei?
- For N=Z nuclei, is the np pairing more dominant than the like-pairing?
- In these nuclei, protons and neutrons occupy the same orbital and have the maximal spatial overlap, which makes especially T=0 pairing important.

**T=0, S=1**

- Isoscalar (IS), spin-triplet
- There have been many discussions about the coexistence of IS and IV and their competitions.
- The nuclear structure of N≠Z nuclei, 60< N<70 and 57< Z<64, may also be affected by np correlations. **The np pairing even for N ≠ Z?**

PRL 106, 252502(2011)
M1 spin transition data show the IV quenching for the $N = Z$ $sd$-shell nuclei. $T = 0$ pairing by the tensor force well-known in deuteron structure may become more significant even inside nuclei. PRL 115, 102501(2015)
Nonquenched Isoscalar Spin-M1 Excitations in sd-Shell Nuclei


FIG. 4 (color online). Accumulated sums of the spin-M1 SNMEs for (a) IS and (b) IV transitions up to $E_x = 16$ MeV. The error bars and gray bands indicate the total experimental uncertainties and the partial uncertainties from the spin assignment, respectively. The solid lines and dotted lines are the predictions of shell-model calculations using the USD with bare and effective $g$ factors, respectively.

Fig. 1. Schematic isospin structure of $J^P = 1^+$ states excited from the g.s. of an even-even $N = T = 0$ nucleus with $F = T = 0$. The reactions mainly responsible for each excitation and the type of operator are shown alongside the arrows indicating the transitions. Both isotope analogue relationships among states are shown by broken lines. The Coulomb displacement energies have been removed to show the isospin symmetry of the system clearly.

Fig. 2. A comparison of ($^3$He, t) and ($p$, p$'$) spectra on the $T = 0$, $^{104}$Sn target nucleus. The excitation energies in spectrum (b) are shifted by 9.3 MeV, the amount of the Coulomb displacement energy. The M1 states observed in the ($p$, p$'$) spectrum can have either $T = 1$ or $T = 0$. On the other hand, the ($^3$He, t) reaction can only excite $T = 1$, C3 states that are analogous to the $T = 1$, M1 states. The $E_x$ values in the ($^3$He, t) spectrum are from [22]. The $E_x$ values and the identification of $T = 0$, M1 states in the ($p$, p$'$) spectrum are from [15,23].
In our early papers, the \( np \) pairing was discussed for GT and double-beta decay using spherical QRPA, which did not include the deformation explicitly and the IS \( np \) pairing was taken into account by renormalizing the IV \( np \) pairing. Similar approach has been doing by various DFT for pairing interactions!
M.K. Cheoun \textit{et al.} NPA 561(1993), NPA 564(1993) ... 

But in our recent works, the \textit{effects of deformation and IS np pairing} are taken into account explicitly in the HFB approach and DQRPA approach.

Also some possibilities of \textit{isospin condensation} in nuclei are discussed.
1. Motivation

2. Spin singlet and spin triplet pairing correlations on shape evolution in $sd$- and $pf$-shell $N=Z$ nuclei.

3. Effects of the Coulomb and the spin-orbit interaction in a deformed mean field on the residual pairing correlations for $N=Z$ nuclei.

4. Competition of deformation and neutron-proton pairing in Gamow-Teller transitions for $^{56,58}$Ni.

5. Summary
In a simple shell-filling model, we assume that:
- no smearing, which means that the occupation provability of nucleon, \( v^2 \), is 1 or 0.
- Fermi energy is located on the each outermost shell (black dotted line).
Shell evolution of $^{24}\text{Mg}$ total energy

$$H = H_0 + H_{\text{int}}$$

$$H_0 = T + V_{\text{DWS}} (V_c + V_{\text{SO}} + V_{\text{coul}})$$

$$E_{\text{tot}} = E_{\text{MF}} + E_{\text{pair}} + E_{\text{self}}$$

(a) without np-pairing
(b) with np-pairing

- T=0 contribution makes the bounding more stronger due to its attractive property.
- Enhanced IS \textit{np} pairing correlations may be an indispensable ingredient to understand the prolate deformation.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\beta_2^{E2}$ [34]</th>
<th>$\beta_2^{\text{RMF}}$ [35]</th>
<th>$\beta_2^{\text{FRDM}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>0.605</td>
<td>0.416</td>
<td>0.</td>
</tr>
</tbody>
</table>
Why we consider the Enhanced T=0 pairing correlation for N=Z nuclei

T=0, S=1 (Isoscalar(IS), spin-triplet)

- Isoscalar (deuteron-like or $^3S_1$) pairs T=0, S=1

M1 spin transition data shows the IV quenching for the N = Z nuclei in sd-shell; T = 0 pairing becomes more significant.

enhanced $T_0 = (T=0) \times 1.5$( IV quenching) $\times 2$ ($\uparrow\uparrow+\downarrow\downarrow$)

enhanced $T_0 = 3 \times (T=0)$
Evolution of pairing strength of $^{24}\text{Mg}$

- All results are fitted to reproduce empirical np-pairing gaps. No difference of green and blue results!!
- $g_{np}^*$ becomes smaller in $|\beta_2| > 0.3$. that is, the smaller $g_{np}^*$ we have, but, the larger pairing energy is obtained.
- It indicates that there can be T=0 pairing (Isoscalar) condensation in large deformation.
- There is the coexistence of T=0 and T=1 pairing in $|\beta_2| > 0.3$. 

![Graph showing the evolution of pairing strength of $^{24}\text{Mg}$](image)
Shell evolution of $^{32}$S

- $^{32}$S can be prolate deformed by the strong $T = 0$ pairing correlations.

How about $^{28}$Si which is known as oblate ????
$5/2^+_{1}$ state

$\epsilon_0(n_z, n_{\perp}, m_l) \simeq [(N + \frac{3}{2}) + \delta(\frac{N}{3} - n_z)]$,
### In pf-shell N=Z nuclei

Ha et al. PRC97, 064322(2018)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\beta_2^{E2}$ [9]</th>
<th>$\beta_2^{RMF}$ [10]</th>
<th>$\beta_2^{FRDM}$ [11]</th>
<th>$\Delta_p^{emp}$</th>
<th>$\Delta_n^{emp}$</th>
<th>$\delta_{np}^{emp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{44}$Ti</td>
<td>0.268</td>
<td>0.000</td>
<td>0.011</td>
<td>2.631</td>
<td>2.653</td>
<td>2.068</td>
</tr>
<tr>
<td>$^{48}$Cr</td>
<td>0.368</td>
<td>0.225</td>
<td>0.226</td>
<td>2.128</td>
<td>2.138</td>
<td>1.442</td>
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<tr>
<td>$^{52}$Fe</td>
<td>0.230</td>
<td>0.186</td>
<td>-0.011</td>
<td>1.991</td>
<td>2.007</td>
<td>1.122</td>
</tr>
<tr>
<td>$^{64}$Ge</td>
<td>0.250</td>
<td>0.217</td>
<td>0.207</td>
<td>1.807</td>
<td>2.141</td>
<td>1.435</td>
</tr>
<tr>
<td>$^{68}$Se</td>
<td>-0.250</td>
<td>-0.285</td>
<td>0.233</td>
<td>1.909</td>
<td>2.174</td>
<td>1.522</td>
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<tr>
<td>$^{72}$Kr</td>
<td>-0.350</td>
<td>-0.358</td>
<td>-0.366</td>
<td>2.001</td>
<td>1.985</td>
<td>1.353</td>
</tr>
</tbody>
</table>
pairing gaps & Fermi E evolution in sd- & pf-shell N=Z nuclei

- Empirical pairing gap by five mass formula.
- Theoretical pairing gaps are adjusted to reproduce the empirical pairing gaps. Specifically, np-pairing gaps are almost saturated in pf-shell N=Z nuclei.
- The gap between proton and neutron Fermi E increases as the number of mass increases.
Shell evolution of $^{64}$Ge

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<td>0.233</td>
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$^{72}$Kr: $-0.350$ $-0.358$ $-0.366$

Even the oblate deformation can be explained by the unlike-pairing correlations!
There is also the coexistence of $T=0$ and $T=1$ pairing at large deformation similarly to $sd$-shell $N=Z$ nuclei.

- **Motivations**

- **Formalism**

- **Results**

- **Summary**
In this work, we switch on and off the Coulomb and/or the SO interaction in the deformed WS potential, respectively. Consequently, we may examine the Wigner’s spin-isospin SU(4) symmetry, in which the nuclear Hamiltonian satisfies the following relation

\[ [H, \Sigma_i \tau_i] = [H, \Sigma_i \sigma_i] = [H, \Sigma_i \tau_i \sigma_i] = 0. \]  

Consequently, the SU(4) symmetry is usually broken either by the Coulomb interaction associated with the 1st term or by the SO interaction related to the 2nd term in The
**Pairing gaps of pp, nn, and np for sd-shell N=Z \(^{24}\text{Mg}\)**

- Constant PME (pairing matrix element): the pairing under the Wigner spin-isospin SU(4) symmetry.
- Brueckner G-Matrix PME: state dependent, the realistic description of ground state.

- The charge independence symmetry is approximately conserved for \(^{24}\text{Mg}\).

- The smearing of the Fermi surface decreases by the SO force, which decreases the pairing gap.
in \textit{pf}-shell N=Z nuclei $^{48}\text{Cr}$

- The SO force increases the smearing at the Fermi surface, which increases the pairing gap.
- The charge independence symmetry is approximately conserved for $^{48}\text{Cr}$.
- The SO force increases the smearing at the Fermi surface, which increases the pairing gap.
The Coulomb effects appear explicitly by increasing the pairing gap with G-Mat PME.

The SU(4) symmetry is more or less violated by the SO and the Coulomb force on the pairing gaps. But it is still a good symmetry even on the pairing (see green triangles).
Reordering of SPSE in $^{128}$Gd by the Coulomb force

- With Coulomb force

- Without Coulomb force

- The occupation probability: $0h_{11/2} + 1d_{5/2}$ (with CF) > $0h_{11/2} + 2s_{1/2}$ (w/o CF)

- The large smearing by the CF makes a large pairing gap.
Ratio of isovector and isoscalar np-pairing

- IS condensation by the enhanced T=0 np pairing may happen in deformed $^{24}$Mg and $^{48}$Cr.
- There is a rapid phase transition from IV to IS component in the np pairing. But it may happen slower in heavy nuclei, which may mean the coexistence in some deformation region.
- For heavy nuclei such as $^{108}$Xe the phase transition may happen more easily even with the normal T=0.
1. Motivation
2. Spin singlet and spin triplet pairing correlations on shape evolution in sd- and pf-shell N=Z nuclei.
3. Effects of the Coulomb and the spin-orbit interaction in a deformed mean field on the residual pairing correlations for N=Z nuclei.

4. Competition of deformation and neutron-proton pairing in Gamow-Teller transitions for $^{56,58}$Ni and $^{62,64}$Ni.

5. Summary
● Gamow-Teller strength for $^{56}$Ni

- In particular, $^{56}$Ni is thought to be almost spherical because of its double magic numbers.
- If we take $\alpha$-cluster model for $^{56}$Ni, the ground state may be slightly deformed. PRC 84, 024302(2011)
- The $np$ pairing effects turn out to be able to properly explain the GT strength although the deformation is also another important property. The high-lying GT peak in the two peaks stems from the repulsive $np$ pairing through the reduction of Fermi energies of protons and neutrons.
The shift of the GT strength distributions by the enhanced T=0 np pairing is mainly attributed to the IS coupling condensation. Even with the small deformation, the second peak appears by the T=0 pairing.
**Gamow-Teller strength and IAR for $^{58}$Ni (N=Z+2)**

- The $np$ pairing makes the IAR (isobaric analogue resonance) concentrated around 12 MeV, which is consistent with the results in PRC 69(2004) at $\beta_2=0.2$.

- The deformation effect turned out to be more important rather than the $np$ pairing correlations since the $np$ pairing effects become the smaller with the increase of $N-Z$ number. Some spurious states peculiar to QRPA lead to small distribution of IAR state.
Summary

1. We find a coexistence of two types of superconductivities (T=0 and T=1) at the $|\beta_2| > 0.3$ region in $^{24}\text{Mg}$.

3. The IS condensation by the enhanced $T=0$ pairing may happen not only in $sd$-shell, but also in $pf$-shell nuclei.

4. The IS condensation part plays a vital role to explain the GT strength distribution of $^{56,58,62,64}\text{Ni}$ nucleus, with the deformation and the unlike-pairing correlations.

5. The Coulomb force and the SO force are shown to change the smearing by change of ordering of SPS. Remember the splitting by the SO as well as the deformation.

6. The state-dependent Brueckner G-PME takes into account shell structure effects on the residual interaction and enables us to do realistic description of ground states of the $N=Z$ nuclei.

7. For heavy $N=Z$ nuclei, the transition may happen more easily even with the normal $T=0$ pairing with a phase transition.


**References of our recent papers**

1. Spin singlet and spin triplet pairing correlations on shape evolution in $sd$-shell $N=Z$ nuclei. Ha, MKC et al. PRC97,024320(2018)

2. Neutron-proton pairing correlations and deformation for $N = Z$ nuclei in $pf$-shell by the deformed BCS and HFB approach.
   Ha, MKC et al. PRC97, 064322(2018)

3. Competition of deformation and neutron-proton pairing in Gamow-Teller transitions for $^{56,58}\text{Ni}$. Ha, MKC et al. accepted to PRC

4. Effects of the Coulomb and the spin-orbit interaction in a deformed mean field on the residual pairing correlations for $N=Z$ nuclei.
   Ha, MKC et al. submitted to PRC.

5. **Isoscalar condensation in $N = Z$ nuclei.**

6. ...
Thanks for your attention !!

Long and Happy Life for Prof. Akito Arima !!
Back-up files
How to include the deformation?

Deformed Woods-Saxon (WS) potential
(cylindrical WS, Damgaard et al 1969)

\[ V(\ell) = -\frac{V_0}{1 + \exp\left(\ell / a\right)}, \quad V_{so} = -\lambda(h/2mc)^2 \text{grad} V(\ell)(\vec{\sigma} \times \vec{p}) \]

\[ \ell(u, v; \beta_2, \beta_4) = \text{CS}(u, v) / |\nabla_{u,v} S(u, v)|, \quad z = \text{Cu}, \quad \rho = \text{Cv} \]

\[ \beta_2 : \text{quadrupole deformation parameter} \]

\[ \beta_4 : \text{hexadecapole deformation parameter} \]

- We can determine these two parameters by taking values giving the minimum ground state energy.

- To exploit G-matrix elements, which is calculated on the spherical basis, deformed bases are **expanded in terms of the spherical bases**.

\[ |a\Omega_\alpha \rangle = \sum_a B^\alpha_a |a\Omega_\alpha \rangle, \]

Deformed SPS \quad Sph. HO w. f.
In sd-shell N=Z nuclei, \( Q_{\text{exp}} \) of \(^{28}\text{Si} \) is different from \(^{24}\text{Mg} \) and \(^{32}\text{S} \).

<table>
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<tr>
<th>Nucleus</th>
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<th>( \beta_2^{RMF} ) [11]</th>
<th>( \beta_2^{FRDM} ) [12]</th>
<th>( Q_{\text{exp.}} ) [14, 15]</th>
<th>( \Delta_p^{\text{emp}} )</th>
<th>( \Delta_n^{\text{emp}} )</th>
<th>( \delta_{np}^{\text{emp}} )</th>
</tr>
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<td>0.605</td>
<td>0.416</td>
<td>0.0</td>
<td>-0.29 \sim -0.07</td>
<td>3.123</td>
<td>3.193</td>
<td>1.844</td>
</tr>
<tr>
<td>(^{28}\text{Si} ) (prolate)</td>
<td>0.407</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>2.841(^a)</td>
<td>2.917(^a)</td>
<td>1.384(^a)</td>
</tr>
<tr>
<td>(^{28}\text{Si} ) (oblate)</td>
<td>x</td>
<td>-0.374</td>
<td>-0.363</td>
<td>0.16 \sim 0.18</td>
<td>2.841(^a)</td>
<td>2.917(^a)</td>
<td>1.384(^a)</td>
</tr>
<tr>
<td>(^{32}\text{S} )</td>
<td>0.312</td>
<td>0.186</td>
<td>0.221</td>
<td>-0.12 \sim -0.18</td>
<td>2.141</td>
<td>2.207</td>
<td>1.047</td>
</tr>
</tbody>
</table>

\[
\beta_2 = \frac{4\pi}{3ZR_0^2} \left[ \frac{B(E2 \uparrow)}{\theta^2} \right]^{1/2} \quad (R_0 = 1.2A^{1/3})
\]

In the rotational model, \( Q_{J\pi} = \frac{3K^2-J(J+1)}{(J+1)(2J+3)}Q_0 \).

For \( 2^+ \), \( Q_{2^+} = -2/7 \) \( Q_0 \).

\( Q_{2^+} \) : experimental quadrupole moment

\( Q_0 \) : intrinsic quadrupole moment

\( ^{28}\text{Si} \) is not heavy. Where does it come from?
Pairing correlation

**BCS**

\[ J=0, T=1 \]
\[ j \quad m \quad j \quad -m \]

**deformed BCS**

\[ \Omega \quad -\Omega \]
\[ (J=0, 1, 2, 3 \cdots) \]
\[ T=0 \quad J=1, 3, 5, \cdots \]
\[ T=1 \quad J=0, 2, 4, \cdots \]

Since the deformed SPS are expanded in terms of the spherical SP bases, the different total angular momenta of the SP basis states would be mixed.


**Pairing correlation**

### deformed HFB

\[ \Delta_{p\bar{p}\alpha} = \Delta_{\alpha p\bar{\alpha} p} = - \sum_{J,c} g_{pp} F_{\alpha\dot{a}\alpha\dot{a}}^{J0} F_{\gamma\dot{c}\gamma c}^{J0} G(aacc, J, T = 1)(u_{1pc}^* v_{1pc} + u_{2pc}^* v_{2pc}) \]

\[ \Delta_{p\bar{n}\alpha} = \Delta_{\alpha p\bar{n} n} = - \sum_{J,c} g_{np} F_{\alpha\dot{a}\alpha\dot{a}}^{J0} F_{\gamma\dot{c}\gamma c}^{J0} [G(aacc, J, T = 1) Re(u_{1nc}^* v_{1pc} + u_{2nc}^* v_{2pc}) \]

\[ + iG(aacc, J, T = 0) Im(u_{1nc}^* v_{1pc} + u_{2nc}^* v_{2pc})] \]

### deformed BCS

\[ \Delta_{p\bar{p}\alpha} = \Delta_{\alpha p\bar{\alpha} p} = - \sum_{J,c,d} g_{pp} F_{\alpha\dot{a}\alpha\dot{a}}^{J0} F_{\gamma\dot{c}\gamma c}^{J0} G(aacd, J, T = 1)(u_{1pc}^* v_{1pd} + u_{2pc}^* v_{2pd}) \]
Self energy in BCS

\[ H_0 = \sum_b^A 2\left[ v_b^2 \left( \eta_b + \frac{1}{2} \mu_b \right) - \frac{1}{2} u_b v_b \Delta_b \right] \]

\[ \downarrow \quad \downarrow \quad \downarrow \]

\[ E_{\text{mean}} \quad E_{\text{self}} \quad E_{\text{pair}} \]

BCS eq.

\[ \eta_b \equiv \varepsilon_b - \lambda - \mu_b \]

\[ \mu_b = -\frac{1}{2} \sum_{a,J} v_a^2 \hat{J}^2 \langle ab : J | V | ab : J \rangle \quad : \text{self energy} \]

\[ \Delta_b = -\sum_a u_a v_a \langle aa;0 | V | bb : 0 \rangle \quad : \text{pairing gap} \]

The self energy term was usually neglected in BCS eq. because it results from particle-hole correlations beyond the BCS and affects a renormalization of the single particle energy.

- **Motivations**
- **Formalism**
- **Results**
- **Summary**
Shell evolution & the simplest shell model of $^{28}$Si
Shell evolution & the simplest shell model of $^{32}$S
Parameter set of Deformed Woods-Saxon

Set of parameter values defined by the program according to the input value of the ICHOIC variable. The symbols P (N) refer to the protons (neutrons). The $\lambda$ values in the case of the Chepurnov parametrisation are defined by $\lambda = 23.8 \times (1 + 2 \times (N - Z)/A)$. Blomqvist-Wahlborn stands for Blomqvist and Wahlborn. The values of $r_0$ and $a$ are in fermi, $V_0$ in MeV, $\kappa$ and $\lambda$ dimensionless.

<table>
<thead>
<tr>
<th>Parametrisation</th>
<th>$\lambda$ (P)</th>
<th>$\lambda$ (N)</th>
<th>$r_{0-so}$ (P)</th>
<th>$r_{0-so}$ (N)</th>
<th>$r_0$ (P)</th>
<th>$r_0$ (N)</th>
<th>$\kappa$</th>
<th>$V_0$</th>
<th>$a$</th>
<th>ICHOIC</th>
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<tr>
<td>Blomqv.-Wahlb.</td>
<td>32.0</td>
<td>32.0</td>
<td>1.270</td>
<td>1.270</td>
<td>1.270</td>
<td>1.270</td>
<td>0.67</td>
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<td>Rost</td>
<td>17.8</td>
<td>31.5</td>
<td>0.932</td>
<td>1.280</td>
<td>1.275</td>
<td>1.347</td>
<td>0.86</td>
<td>49.6</td>
<td>0.70</td>
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<td>Chepurnov</td>
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<td>$A$-dependent</td>
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<td>1.347</td>
<td>0.86</td>
<td>49.6</td>
<td>0.70</td>
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<tr>
<td>“universal”</td>
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<td>35.0</td>
<td>1.20</td>
<td>1.310</td>
<td>1.275</td>
<td>1.347</td>
<td>0.86</td>
<td>49.6</td>
<td>0.70</td>
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<td>“input”</td>
<td>parameters read from input</td>
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<tr>
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<td>deformation-dependent</td>
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<tr>
<td>INCREA = 1</td>
<td>only for $\beta_2 &gt; 0.325$</td>
<td></td>
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<td>0–5</td>
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</table>
In gd-shell $N=Z$ nuclei

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\beta_2^{RMF}$ [10]</th>
<th>$\beta_2^{FRDM}$ [11]</th>
<th>$\beta_2^{KTUY}$ [10]</th>
<th>$\Delta_p^{emp}$</th>
<th>$\Delta_n^{emp}$</th>
<th>$\delta_{np}^{emp}$</th>
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</thead>
<tbody>
<tr>
<td>$^{104}$Te</td>
<td>--</td>
<td>-0.011</td>
<td>0.039</td>
<td>1.520</td>
<td>1.548</td>
<td>0.665</td>
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<tr>
<td>$^{116}$Ce</td>
<td>0.285</td>
<td>0.282</td>
<td>0.145</td>
<td>1.452</td>
<td>1.530</td>
<td>0.697</td>
</tr>
<tr>
<td>$^{128}$Gd</td>
<td>0.350</td>
<td>0.341</td>
<td>0.194</td>
<td>1.415</td>
<td>1.393</td>
<td>0.592</td>
</tr>
</tbody>
</table>
Used parameters in this work.

* $N_{\text{max}} = 10$ (spherical basis)
* $N_{\text{max}} = 5$ (deformed basis)
* WS input parameters: universal param.
* pairing gap: five-term mass formula
* $g_{pp}(g_{ph}) = 0.99(1.15)$ particle – particle (particle – hole) int. strength
- ground state $E$ of $^{104}$Te
deformed Hartree Fock Bogoliubov (DHFB) transformation,

\[
\begin{pmatrix}
    a_1^\dagger \\
    a_2^\dagger \\
    a_\bar{1} \\
    a_\bar{2}
\end{pmatrix}_{\alpha} =
\begin{pmatrix}
    u_{1p} & u_{1n} & v_{1p} & v_{1n} \\
    u_{2p} & u_{2n} & v_{2p} & v_{2n} \\
    -v_{1p} & -v_{1n} & u_{1p} & u_{1n} \\
    -v_{2p} & -v_{2n} & u_{2p} & u_{2n}
\end{pmatrix}_{\alpha}
\begin{pmatrix}
    c_p^\dagger \\
    c_n^\dagger \\
    c_{\bar{p}} \\
    c_{\bar{n}}
\end{pmatrix}_{\alpha}
\]
- ground state $E$ of $^{116}$Ce
Motivations

Summary

Formalism

Results

- ground state $E$ of $^{128}\text{Gd}$

![Graphs showing energy levels for different cases of pairing and without pairing.](image)
Two-body interaction

Realistic two body interaction inside nuclei was taken by Brueckner g-matrix, which is a solution of the Bethe-Salpeter Eq., derived from the Bonn-CD potential for nucleon-nucleon interaction in free space.

\[
g(\omega)_{ab,cd} = V_{ab,cd} + V_{ab,cd} \frac{Q_p}{\omega - H_0} g(\omega)_{ab,cd}
\]

\(a,b,c,d\) : single particle states from the Woods-Saxon potential.
\(V_{ab,cd}\) : phenomenological nucleon-nucleon potential in free space.