Like- and Unlike-Pairing Correlations in a Deformed Mean Field for Finite Nuclear Systems

- Competition of Deformation and Pairing Correlations in N = Z (Stable or Unstable) Nuclei -

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in collaboration with

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4) Tuebingen University, Germany
5) Bratislava Univ, Slovakia and DUBNA, Moscow, Russia

International Symposium on Simplicity, Symmetry and Beauty of Atomic Nuclei in honor of Professor Akito Arima's 88 year-old birthday (米寿)
Sep. 25-29, 2018, Shanghai, China

OCTOBER 2016 VOL. 26 NO. 5

ASSOCIATION OF ASIA PACIFIC PHYSICAL SOCIETIES
AAPPS
Volume 26 | Number 5 | OCTOBER 2016 Bulletin

A Recollection on the Dawning of APCTP

JEWAN KIM
EMERITUS PROFESSOR, SEOUL NATIONAL UNIVERSITY

Thanks to Prof. Akito Arima for APCTP & Congratulation on his 88th Birth day !!

1996


Professor Arima visiting S.N.U.

Jewan Kim is the professor emeritus in Seoul National University. He has been a research professor at Univer
professor at Johns Hopkins University. He served Presidential Science Committee and awarded Science Awa
and Technology. He is currently working as Honorary Chairman of Association of Advancement of Scientific C

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The First Asia Pacific Physics Conference in Singapore and the Establishment of the Association of Asia Pacific Physical Societies

AKITO ARIMA
PRESIDENT OF THE JAPAN RADIOISOTOPE ASSOCIATION

Thanks to Prof. Akito Arima for AAPPS & Congratulation on his 88th Birth day !!

It is my great pleasure to write my memorandum on the first Asia Pacific Physics Conference in Singapore and the establishment of the Association of Asia Pacific Physical Societies (AAPPS).

I was in Stony Brook, New York for several years between 1971 and 1980. I often discussed in Stony Brook with

pore, although China still did not have official diplomatic relations with Singapore. These words convinced me that China would be cooperative towards the Society and receptive to participating in the international conferences.

For a few weeks in the spring of 1981, Professor C. N.

Akito Arima was born in Osaka, Japan in 1930. He is President of the Japan Radioisotope Association. He was President of the University of Tokyo (1989-1993), President of the Institute of Physical and Chemical Research (RIKEN) (1993–1998), Minister of Education, Science, Sports and Culture (1998-1999), member of the Japanese House of Councilors (1998-2004).
His research field is theoretical nuclear physics.
Humboldt Award (1987), Wetherill Medal, The Franklin Institute (1990), Bonner Prize and The Japan Academy Prize (1993), Order of Culture (2010).

Contents

1. **Motivation**

2. **Spin singlet and spin triplet pairing correlations on shape evolution in *sd*- and *pf*-shell N=Z nuclei.**

3. **Effects of the Coulomb and the spin-orbit interaction in a deformed mean field on the residual pairing correlations for N=Z nuclei.**

   The Wigner SU(4) spin–isospin symmetry on the pairing gaps!

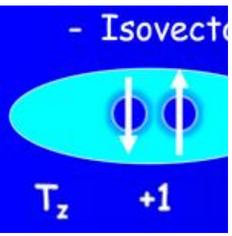
4. **Competition of deformation and neutron-proton pairing in Gamow-Teller transitions for $^{56,58}Ni$ and $^{62,64}Ni$.**

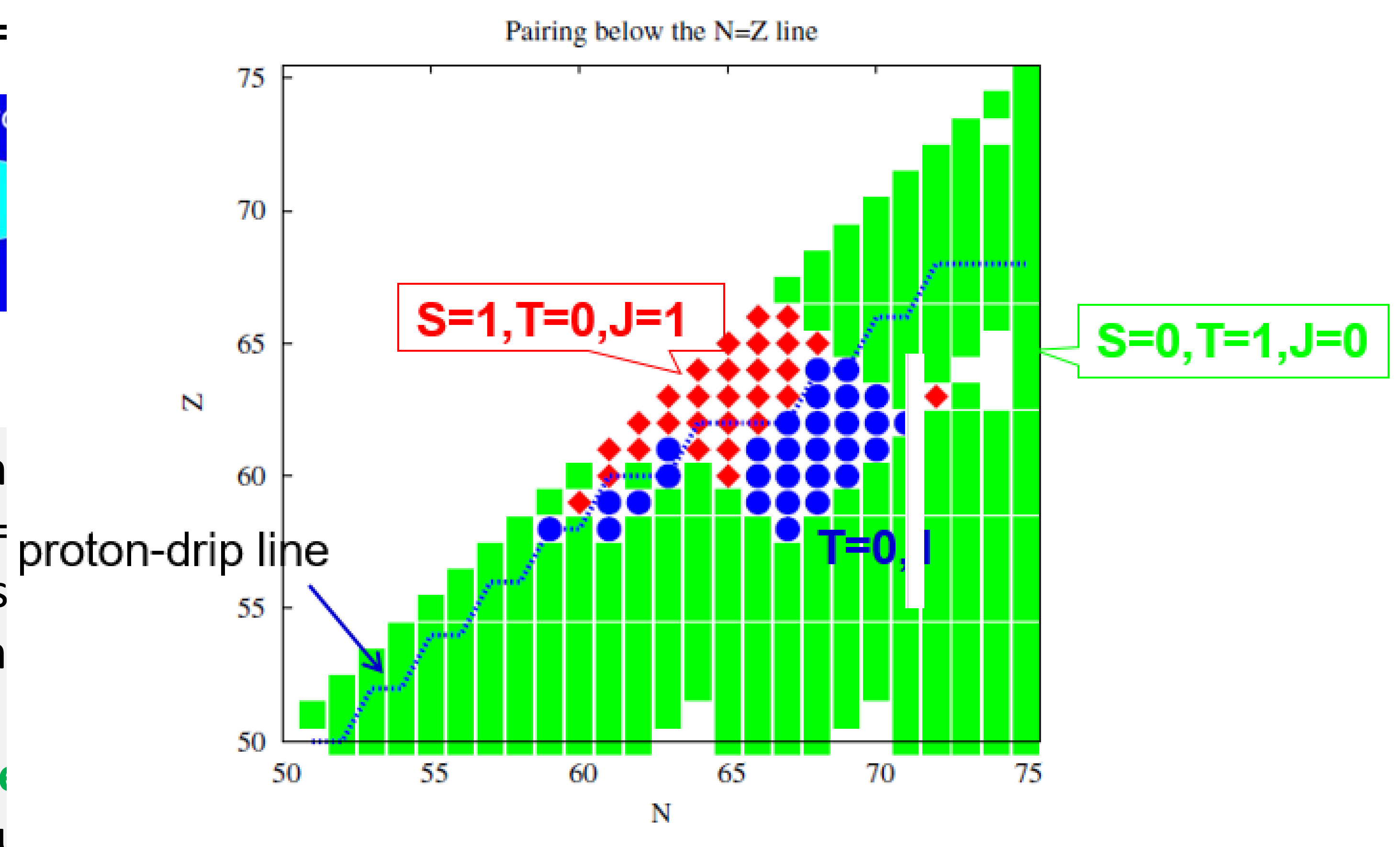
5. **Summary**

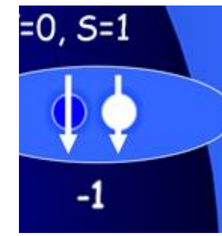
❖ Pairing correlation

- like-pairing (*pp* and *nn* pairing): IV
- unlike-pairing (*np* pairing)   : IV & IS

❖ T=1, S= … riplet)







- ➢ Are th
- ➢ For N= … In thes … e maxim
- ➢ There … **their compe**
- ➢ The nu … affected by np correlations. **The np pairing even for N ≠ Z ??** PRL 106, 252502(2011)

- M1 spin transition data show the IV quenching for the N = Z *sd*-shell nuclei. ;T = 0 pairing by the tensor force well-known in deuteron structure may become more significant even inside nuclei. PRL 115, 102501(2015)

Nonquenched Isoscalar Spin-$M1$ Excitations in sd-Shell Nuclei

H. Matsubara,[1,†] A. Tamii,[1] H. Nakada,[2] T. Adachi,[1] J. Carter,[3] M. Dozono,[5,‡] H. Fujita,[1] K. Fujita,[1,§] Y. Fujita,[1] K. Hatanaka,[1] W. Horiuchi,[6] M. Itoh,[7] T. Kawabata,[4,∥] S. Kuroita,[5] Y. Maeda,[9] P. Navrátil,[10] P. von Neumann-Cosel,[11] R. Neveling,[12] H. Okamura,[1,*] L. Popescu,[13,¶] I. Poltoratska,[11] A. Richter,[11] B. Rubio,[14] H. Sakaguchi,[1] S. Sakaguchi,[4,§] Y. Sakemi,[7] Y. Sasamoto,[4] Y. Shimbara,[15,**] Y. Shimizu,[4,††] F. D. Smit,[12] K. Suda,[1,††] Y. Tameshige,[1,‡‡] H. Tokieda,[4] Y. Yamada,[5] M. Yosoi,[1] and J. Zenihiro[8,††]

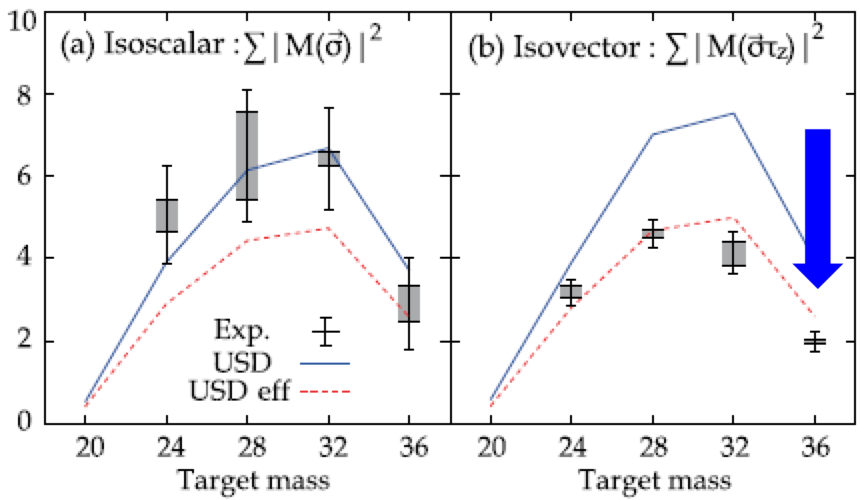


FIG. 4 (color online). Accumulated sums of the spin-$M1$ SNMEs for (a) IS and (b) IV transitions up to $E_x = 16$ MeV. The error bars and gray bands indicate the total experimental uncertainties and the partial uncertainties from the spin assignment, respectively. The solid lines and dotted lines are the predictions of shell-model calculations using the USD with bare and effective g factors, respectively.

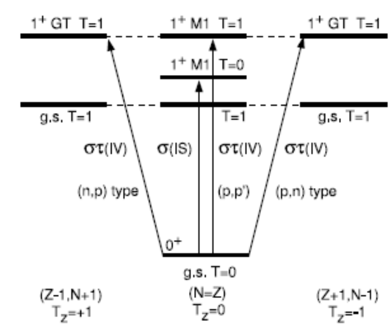


Fig. 1. Schematic isospin structure of $J^\pi = 1^+$ states excited from the g.s of an even–even $J^\pi = 0^+$ nucleus with $T = T_0 = 0$ $(N = Z)$. The reactions mainly responsible for each excitation and the types of operator are shown alongside the arrows indicating the transitions. Isobaric analogue relationships among states are shown by broken lines. The Coulomb displacement energies have been removed to show the isospin symmetry of the system clearly.

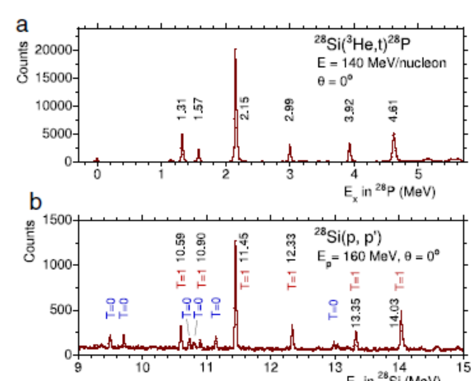


Fig. 2. A comparison of (^{3}He, t) and (p, p') spectra on the $T = 0$, ^{28}Si target nucleus. The excitation energies in spectrum (b) are shifted by 9.3 MeV, the amount of the Coulomb displacement energy. The $M1$ states observed in the (p, p') spectrum can have either $T = 1$ or $T = 0$. On the other hand, the (^{3}He, t) reaction can only excite $T = 1$, GT states that are analogous to the $T = 1$, $M1$ states. The E_x values in the (^{3}He, t) spectrum are from [22]. The E_x values and the identification of $T = 0$, $M1$ states in the (p, p') spectrum are from [15,22].

- In our early papers, the *np* pairing was discussed for GT and double-beta decay using spherical QRPA, which did not include the deformation explicitly and the IS *np* pairing was taken into account by renormalizing the IV *np* pairing. **Similar approach has been doing by various DFT for pairing interactions !**
  M.K. Cheoun *et al.* NPA 561(1993), NPA 564(1993) …

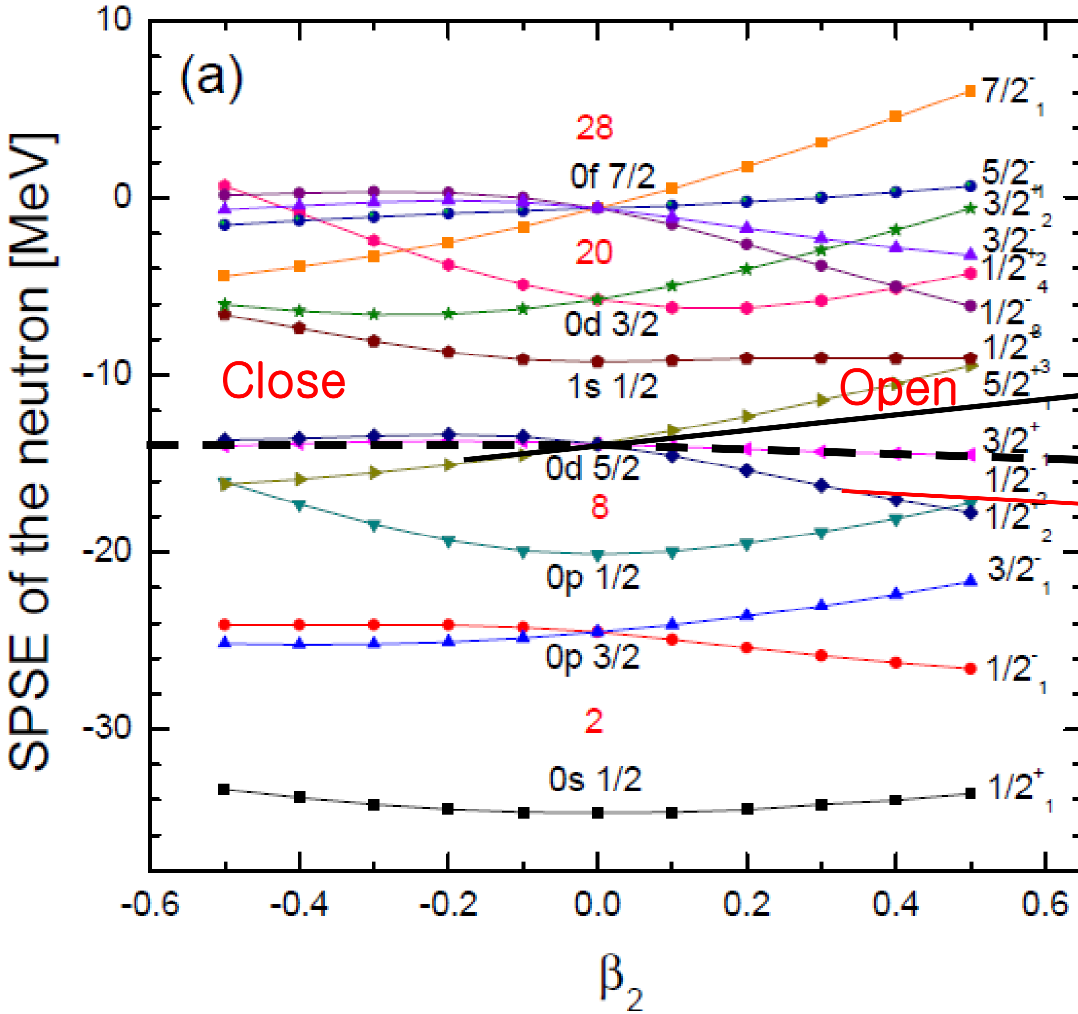
- But in our recent works, the effects of deformation and IS *np* pairing are taken into account explicitly in the HFB approach and DQRPA approach.

- Also some possibilities of **isospin condensation** in nuclei are discussed.

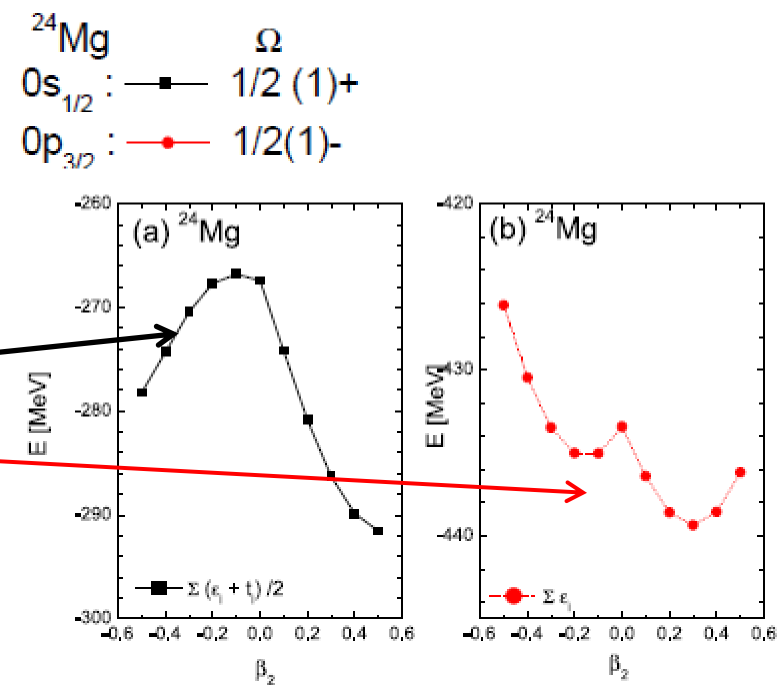
Contents

Ha *et al*. PRC97,024320(2018)

❖ Single particle state E(SPSE) by DWS of ^{24}Mg



❖ A simple shell-filling model



$$E_{GSE} = \sum_{i}^{A/2} \frac{1}{2}\left[\left(\epsilon_i^p + t_i^p\right) + \left(\epsilon_i^n + t_i^n\right)\right] \quad E_{TSPE} = \sum_{i=1}^{Z/2(N/2)} \left(\epsilon_i^{\,p} + \epsilon_i^{\,n}\right)$$

Strutinsky correction is in progress !

➢ In a simple shell-filling model, we assume that

- no smearing, which means that the occupation provability of nucleon, v^2, is 1 or 0.
- Fermi energy is located on the each outermost shell (black dotted line).

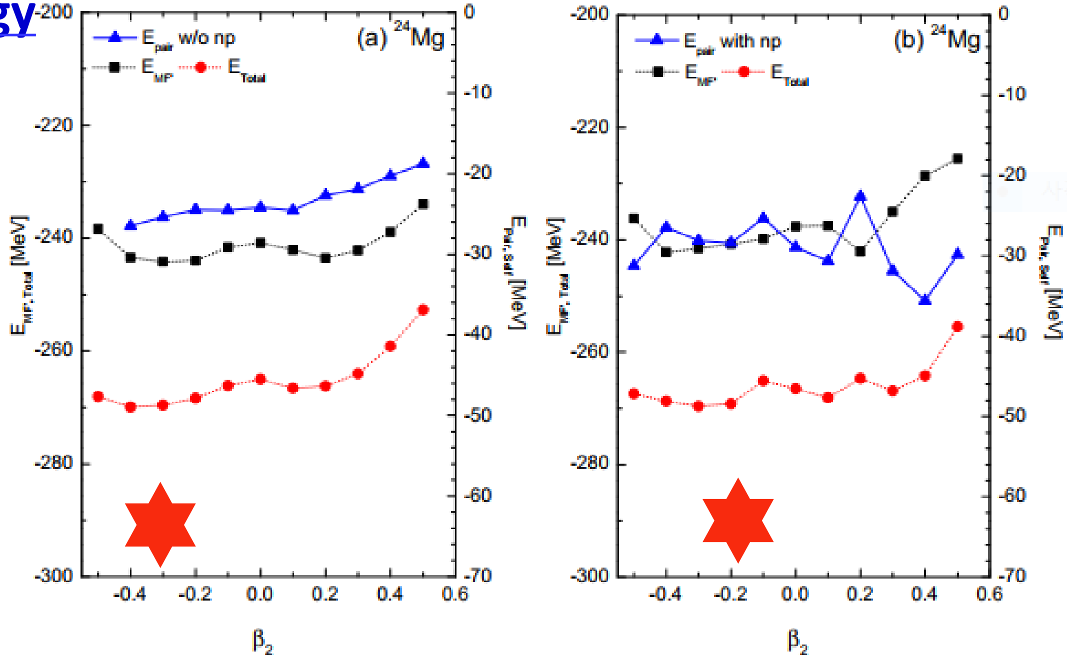
❖ Shell evolution of ^{24}Mg total energy

$$H = H_0 + H_{int}$$

$$H_0 = T + V_{DWS}\ (V_c + V_{SO} + V_{coul})$$

$$E_{tot} = E_{MF'} + E_{pair} + E_{self}$$

(a) without np-pairing
(b) with np-pairing

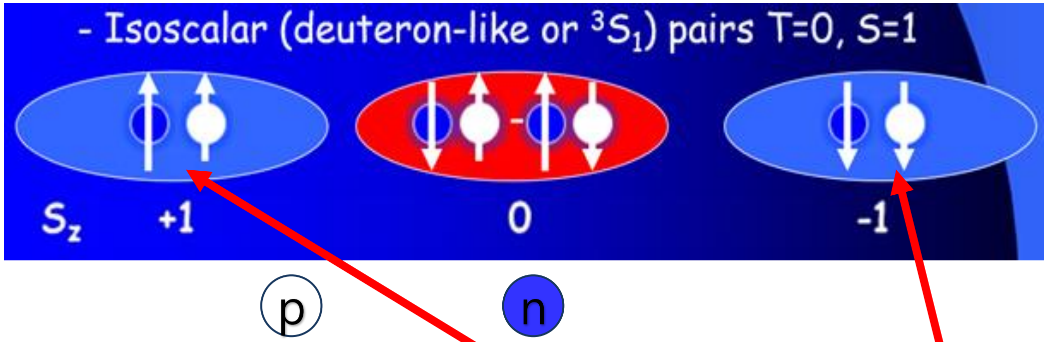


- T=0 contribution makes the bounding more stronger due to its attractive property.
- Enhanced IS *np* pairing correlations may be an indispensable ingredient to understand the prolate deformation.

Nucleus	β_2^{E2} [34]	β_2^{RMF} [35]	β_2^{FRDM}
^{24}Mg	0.605	0.416	0.

❖ Why we consider the <u>Enhanced T=0 pairing correlation for N=Z nuclei</u>

T=0, S=1 (Isoscalar(IS), spin- triplet)



- M1 spin transition data shows the IV quenching for the N = Z nuclei in *sd*-shell ; T = 0 pairing becomes more significant.
  enhanced T_0 = (T=0) × 1.5(IV quenching) × 2 (↑↑+↓↓)

Sagawa *et al.*

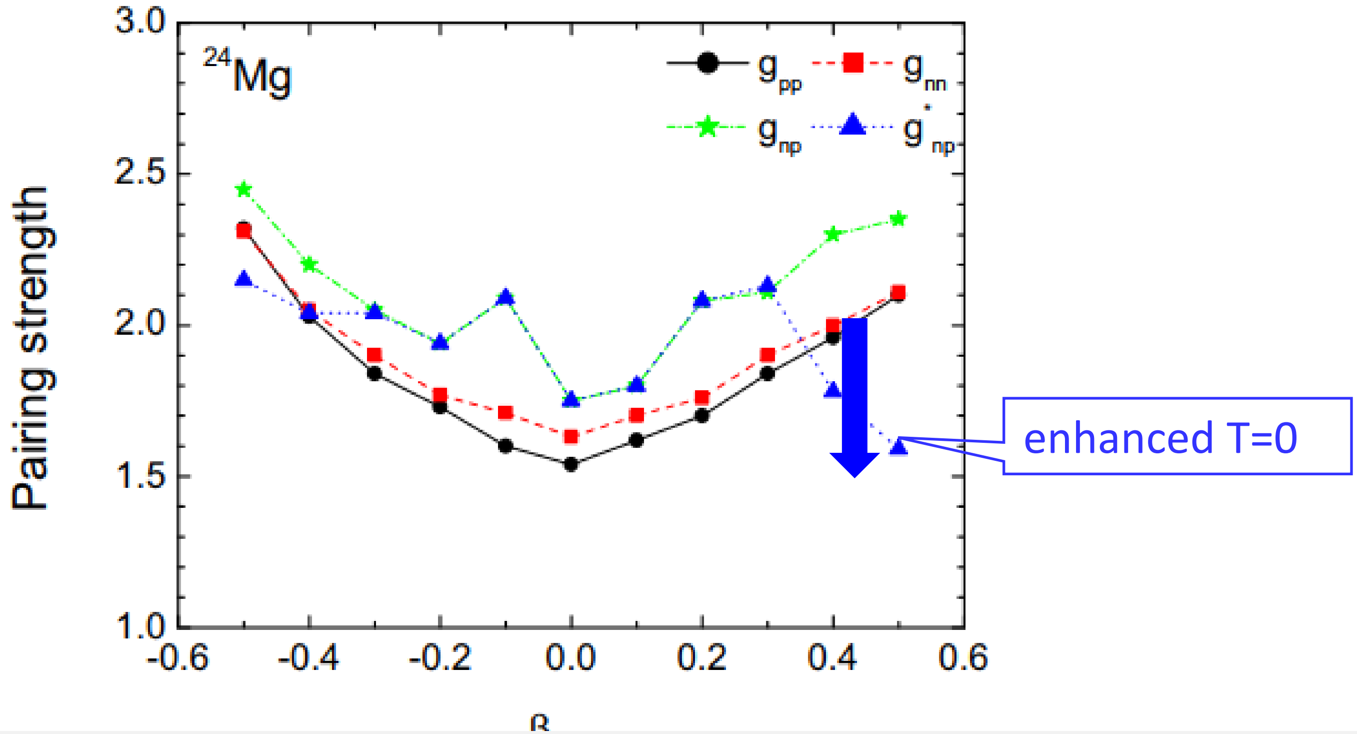
<u>enhanced T_0 = 3 × (T=0)</u>

$$\langle \alpha n \alpha p, T=0|V_{\text{pair}}|\beta n \beta p, T=0\rangle = \langle \alpha n \alpha p, T=0|V_{\text{pair}}|\bar{\beta} n \bar{\beta} p, T=0\rangle, \quad (22)$$

then Im $\Delta^{T=0}_{\alpha n \alpha p} = 0$ and Re $\Delta^{T=0}_{\alpha n \alpha p}$ = Im $\Delta^{T=0}_{\alpha n \bar{\alpha} p}$ by Eqs. (5) to (7) in Ref. [3]. It leads to

$$\Delta^{2\,(T=0)}_{np} = 2|\Delta^{T=0}_{\alpha p \bar{\alpha} n}|^2 + 2|\Delta^{T=0}_{\alpha p \alpha n}|^2 = 4|\Delta^{T=0}_{\alpha p \bar{\alpha} n}|^2, \quad (23)$$

❖ Evolution of pairing strength of ^{24}Mg



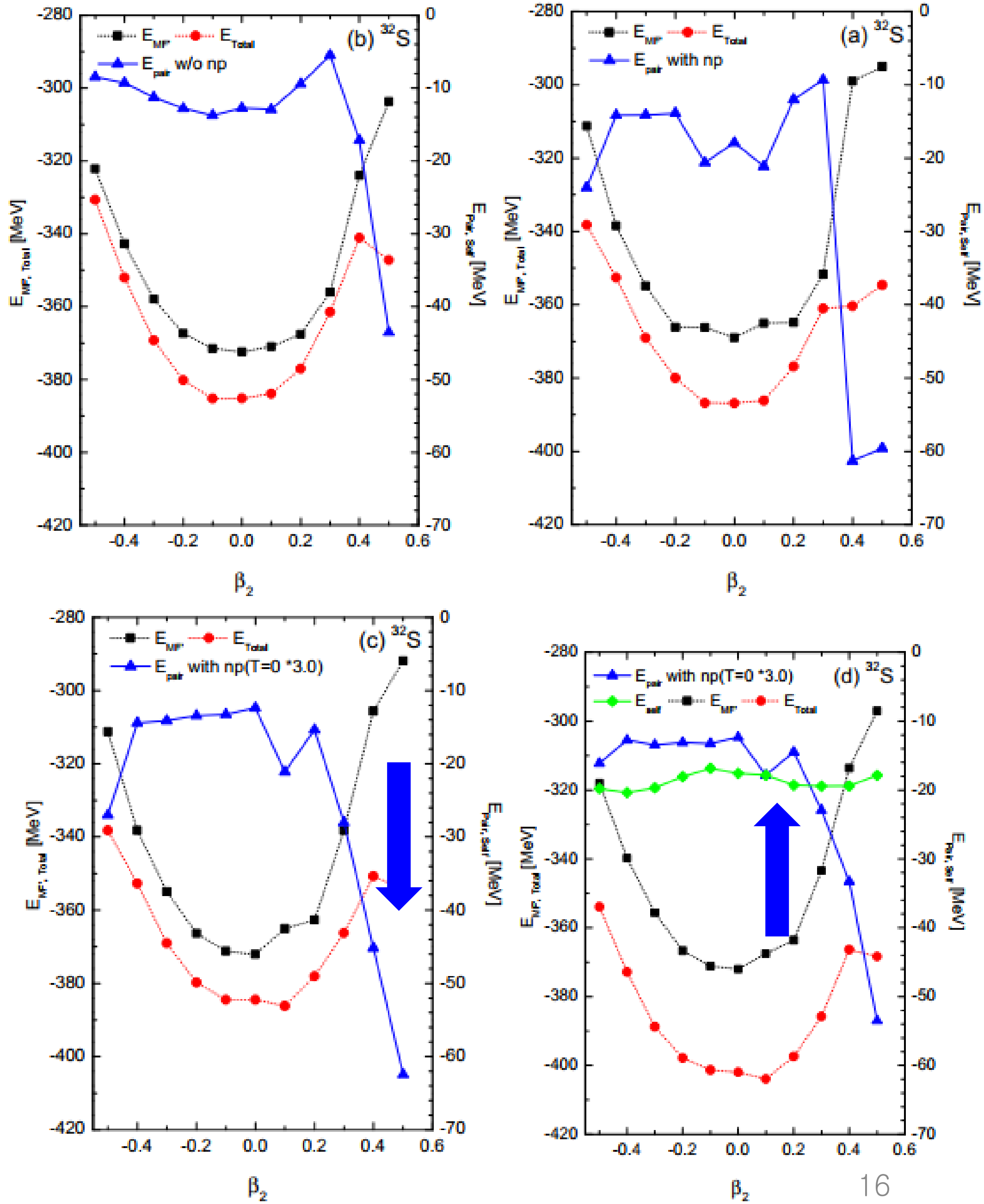
- All results are fitted to reproduce empirical np-pairing gaps. No difference of green and blue results !!
- g_{np}* becomes smaller in $|\beta_2| > 0.3$. , that is, **the smaller g_np^* we have , but, the larger pairing energy is obtained.**
- It indicates that there can be T=0 pairing **(Isoscalar)** condensation in large deformation.
- There is the coexistence of T=0 and T=1 pairing in $|\beta_2| > 0.3$.

❖ Shell evolution of ^{32}S

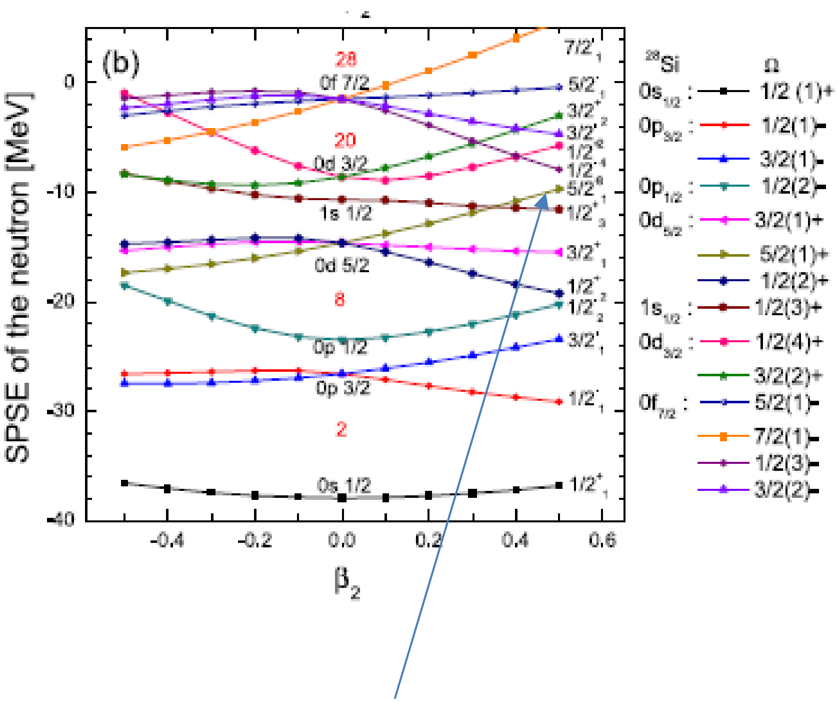
Nucleus	β_2^{E2} [10]	β_2^{RMF} [11]	β_2^{FRDM} [12]
^{24}Mg	0.605	0.416	0.
^{28}Si (prolate)	0.407	x	x
^{28}Si (oblate)	x	− 0.374	− 0.363
^{32}S	0.312	0.186	0.221

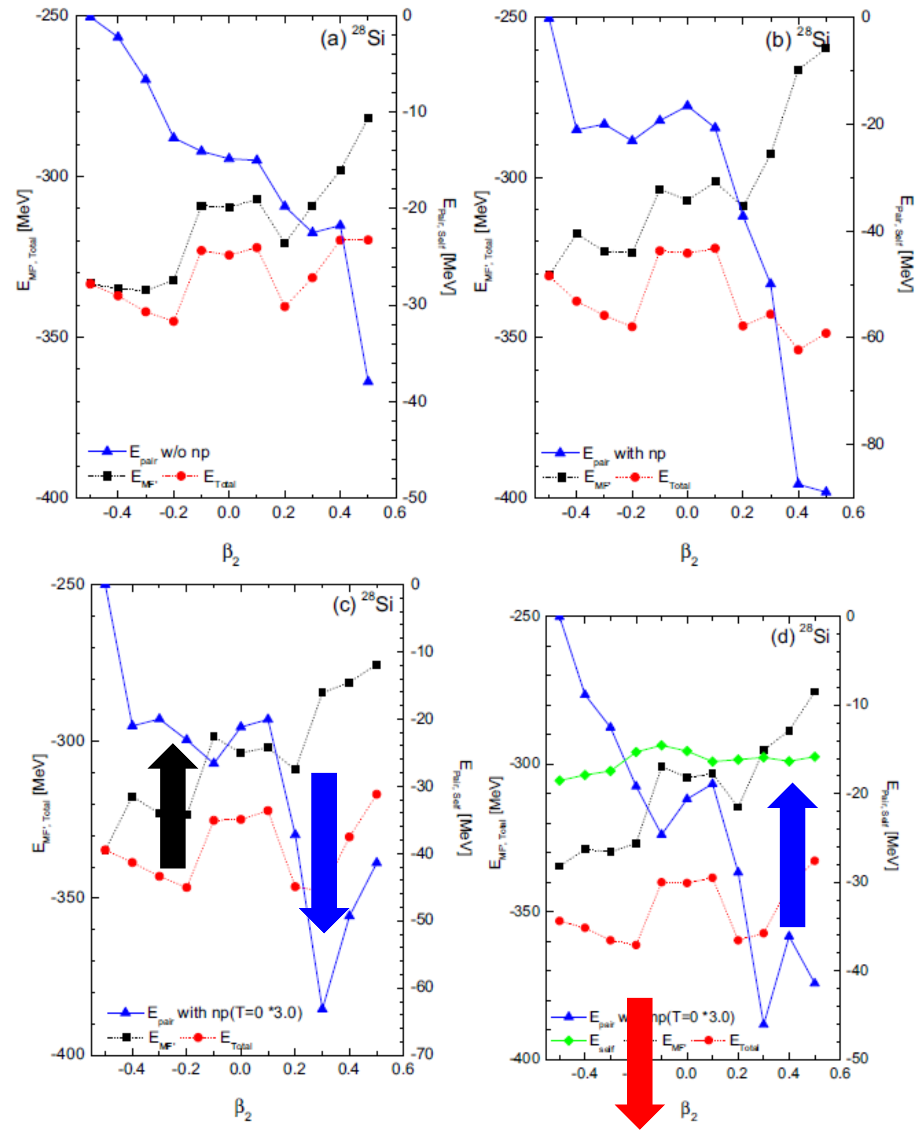
➢ ^{32}S can be prolate deformed by the strong T = 0 pairing correlations.

How about ^{28}Si
which is known as oblate ????



SPIN SINGLET AND SPIN TRIPLET PAIRING … PHYSICAL REVIEW C 97, 024320





$5/2^+_1$ state

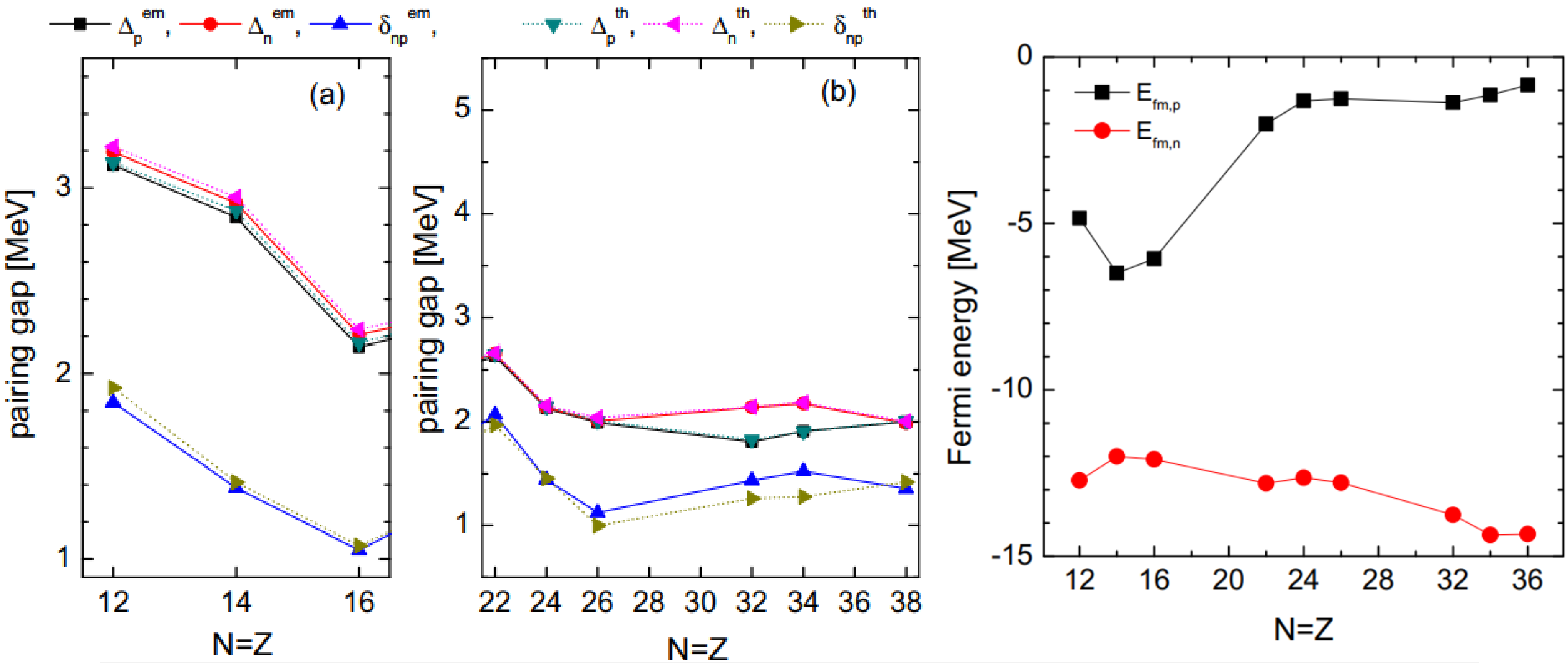
$$\epsilon_0(n_z, n_\perp, m_l) \simeq [(N + \frac{3}{2}) + \delta(\frac{N}{3} - n_z)] \ ,$$

❖In pf-shell N=Z nuclei

Ha *et al*. PRC97, 064322(2018)

Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]	Δ_p^{emp}	Δ_n^{emp}	δ_{np}^{emp}
^{44}Ti	0.268	0.000	0.011	2.631	2.653	2.068
^{48}Cr	0.368	0.225	0.226	2.128	2.138	1.442
^{52}Fe	0.230	0.186	–0.011	1.991	2.007	1.122
^{64}Ge	0.250	0.217	0.207	1.807	2.141	1.435
^{68}Se	–0.250	–0.285	0.233	1.909	2.174	1.522
^{72}Kr	–0.350	–0.358	–0.366	2.001	1.985	1.353

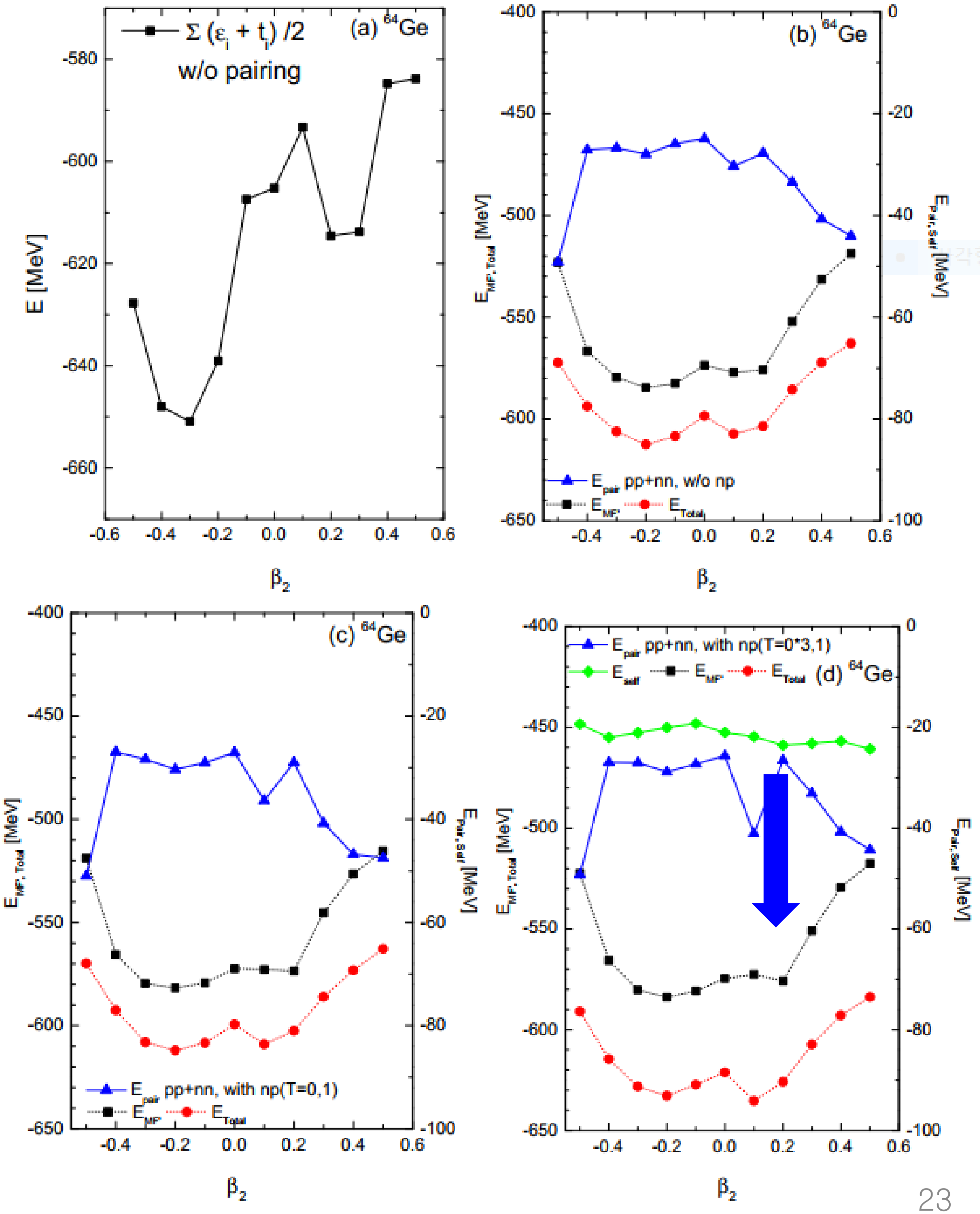
❖ Pairing gaps & Fermi E evolution in sd- & pf-shell N=Z nuclei



- Empirical pairing gap by five mass formula.
- Theoretical pairing gaps are adjusted to reproduce the empirical pairing gaps. Specifically, np-pairing gaps are almost saturated in pf-shell N=Z nuclei.
- The gap between proton and neutron Fermi E increases as the number of mass increases.

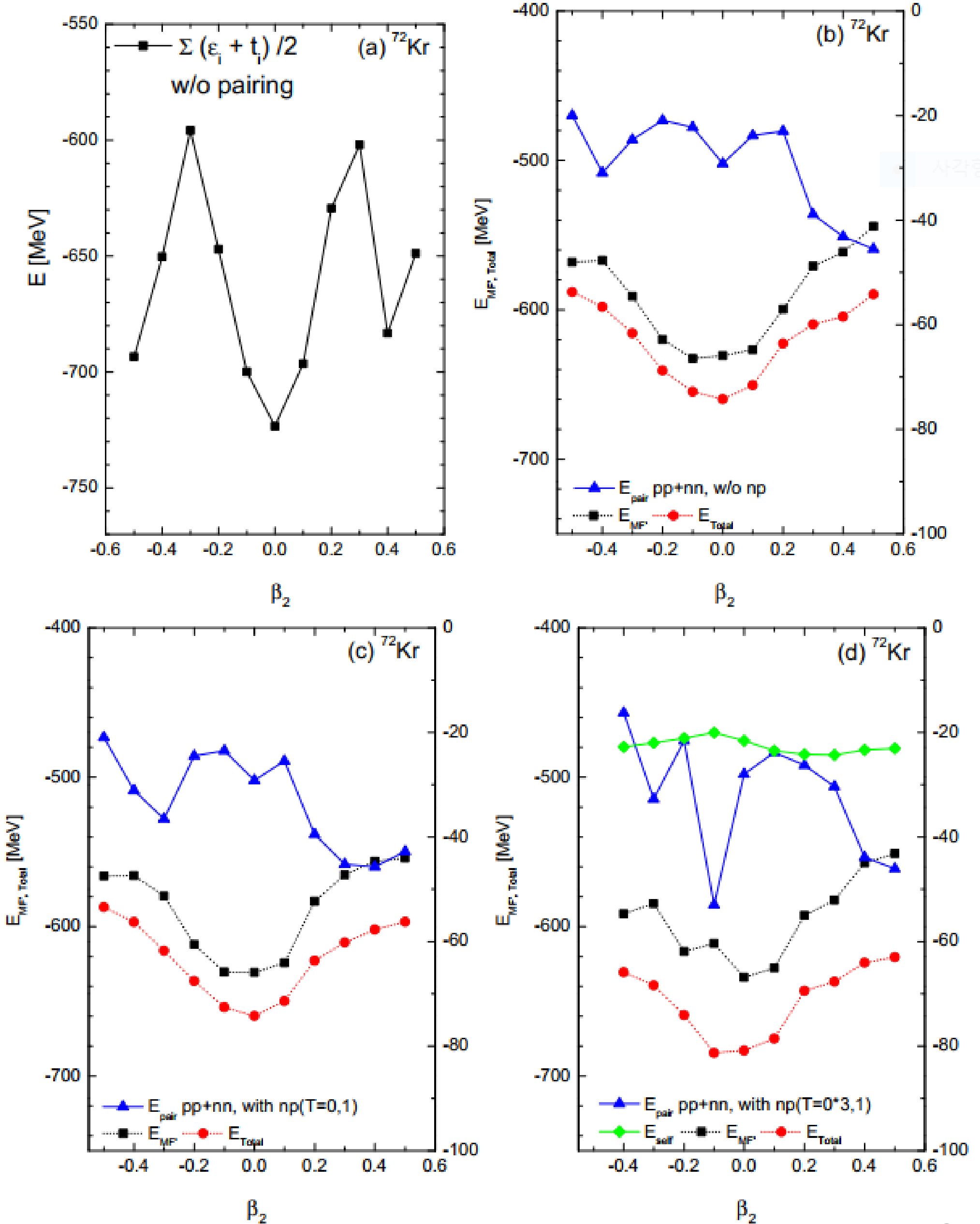
❖ Shell evolution of ^{64}Ge

Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	−0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	−0.250	−0.285	0.233
^{72}Kr	−0.350	−0.358	−0.366



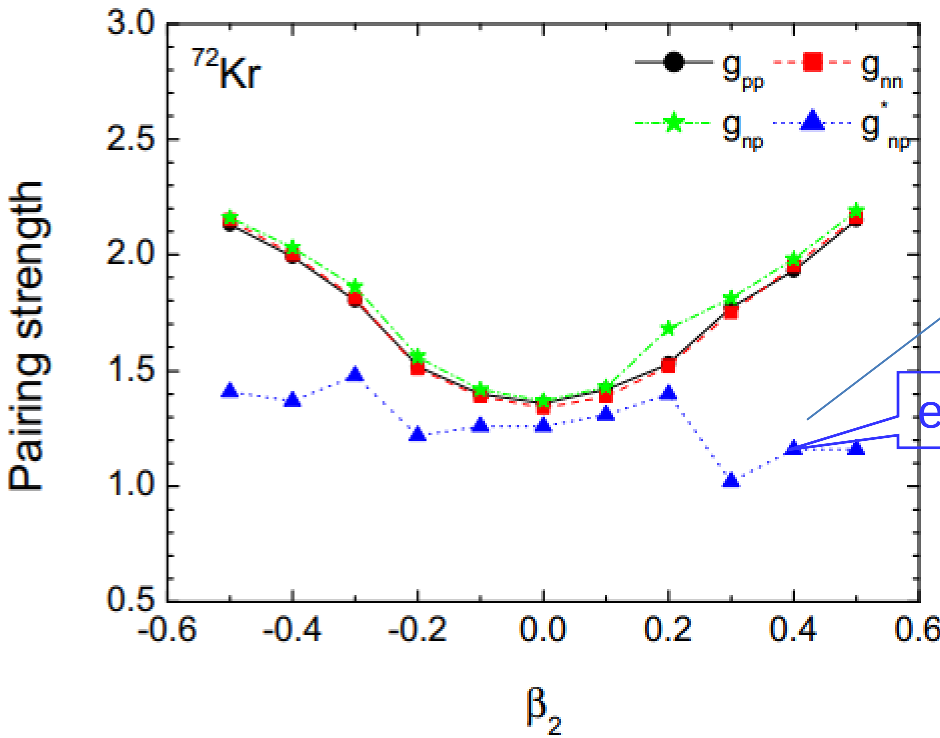
❖ Shell evolution of ^{72}Kr

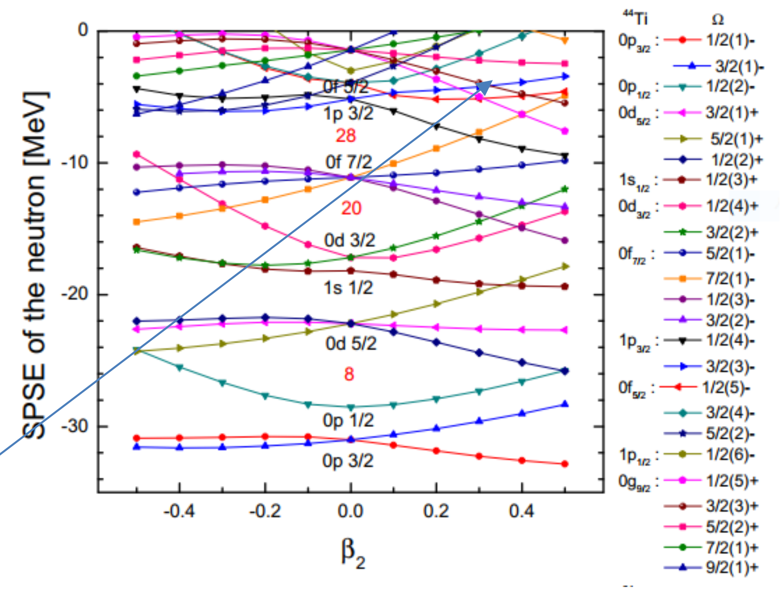
Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	−0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	−0.250	−0.285	0.233
^{72}Kr	−0.350	−0.358	−0.366



Even the oblate deformation
can be explained
by the unlike-pairing
correlations !

❖ Evolution of pairing strength of ^{72}Kr





enhanced T=0

➢ There is also the coexistence of T=0 and T=1 pairing at large deformation similarly to *sd*-shell N=Z nuclei.

Contents

The Wigner SU(4) spin-isospin symmetry on the pairing gaps!

In this work, we switch on and off the Coulomb and/or the SO interaction in the deformed WS potential, respectively. Consequently, we may examine the Wigner's spin-isospin SU(4) symmetry, in which the nuclear Hamiltonian satisfies the following relation

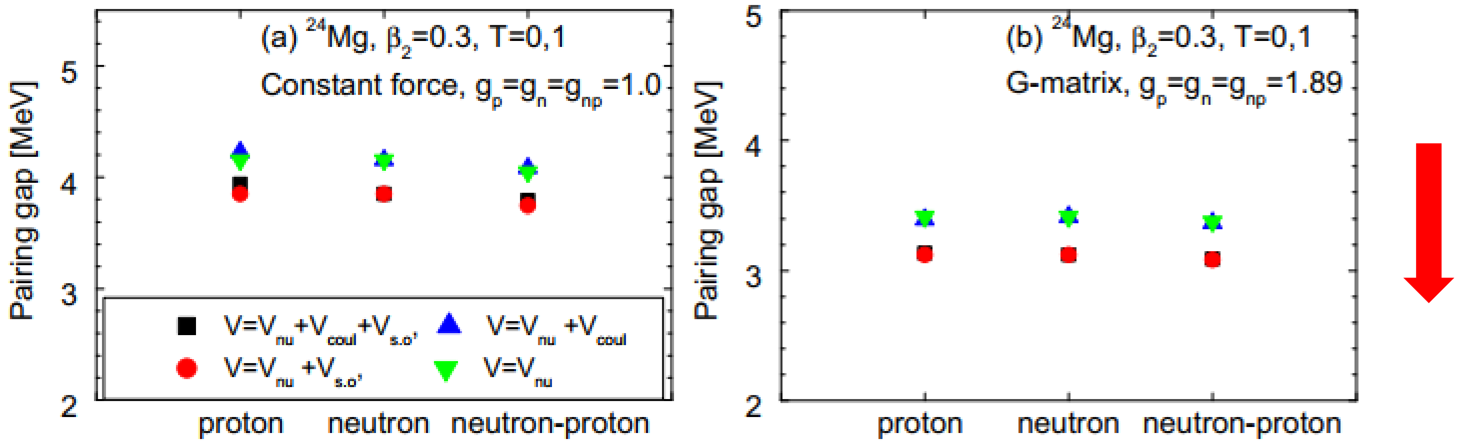
$$[H, \Sigma_i \tau_i] = [H, \Sigma_i \sigma_i] = [H, \Sigma_i \tau_i \sigma_i] = 0. \quad (7)$$

Consequently, the SU(4) symmetry is usually broken either by the Coulomb interaction associated with the 1st term or by the SO interaction related to the 2nd term in The

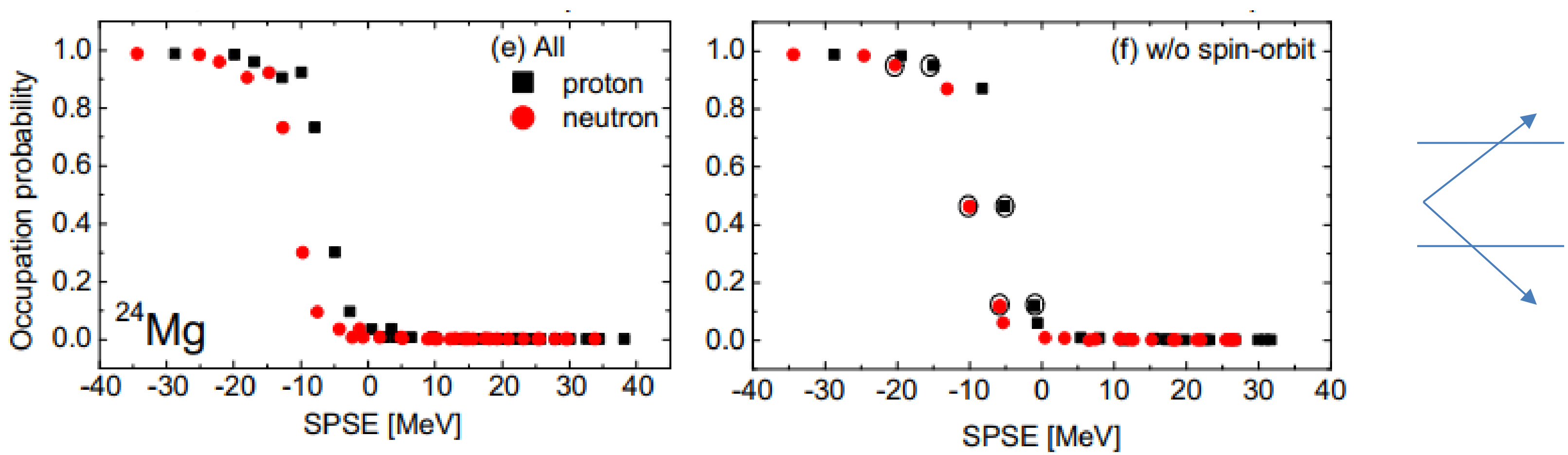
❖ Pairing gaps of pp, nn, and np for sd-shell N=Z ^{24}Mg

Ha *et al.* submitted to PRC

- Constant PME(pairing matrix element): the pairing under the Wigner spin-isospin SU(4) symmetry.
- Brueckner G-Matrix PME : state dependent, the realistic description of ground state.

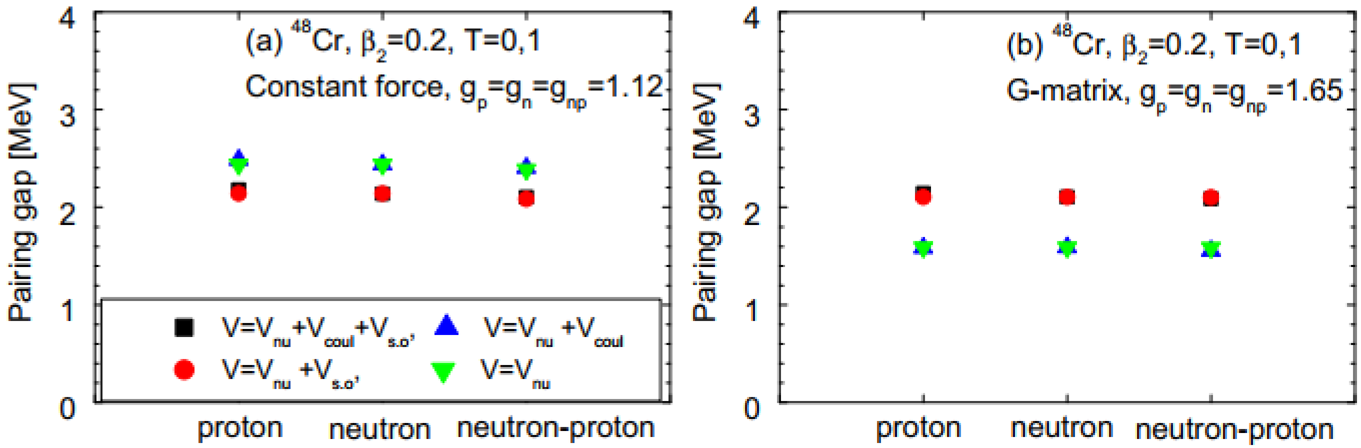


➢ The charge independence symmetry is approximately conserved for ^{24}Mg.

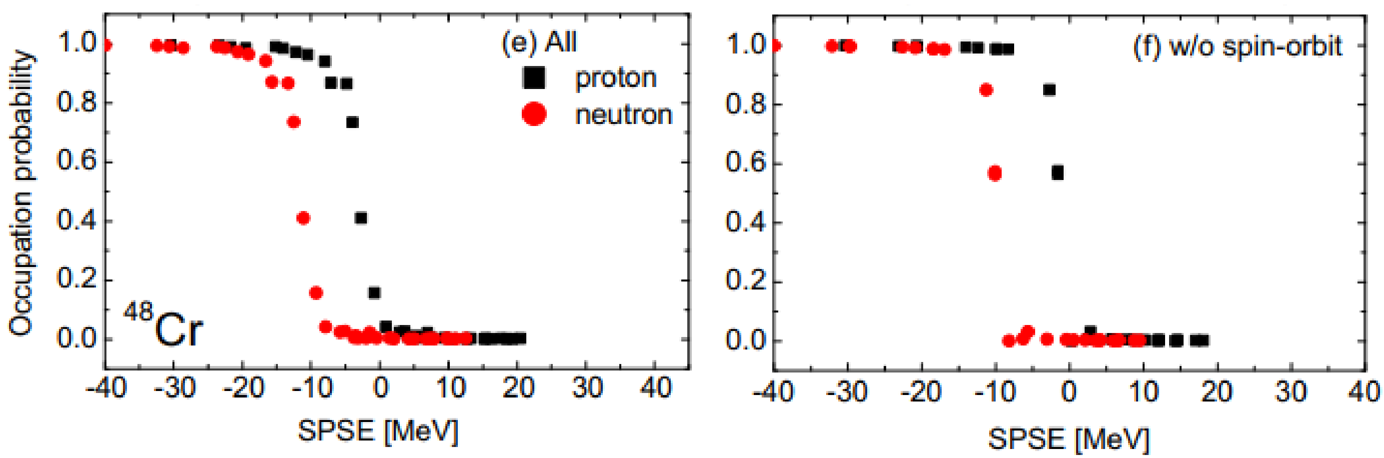


➢ The smearing of the Fermi surface decreases by the SO force, which decreases the pairing

❖ in *pf*-shell N=Z nuclei ^{48}Cr

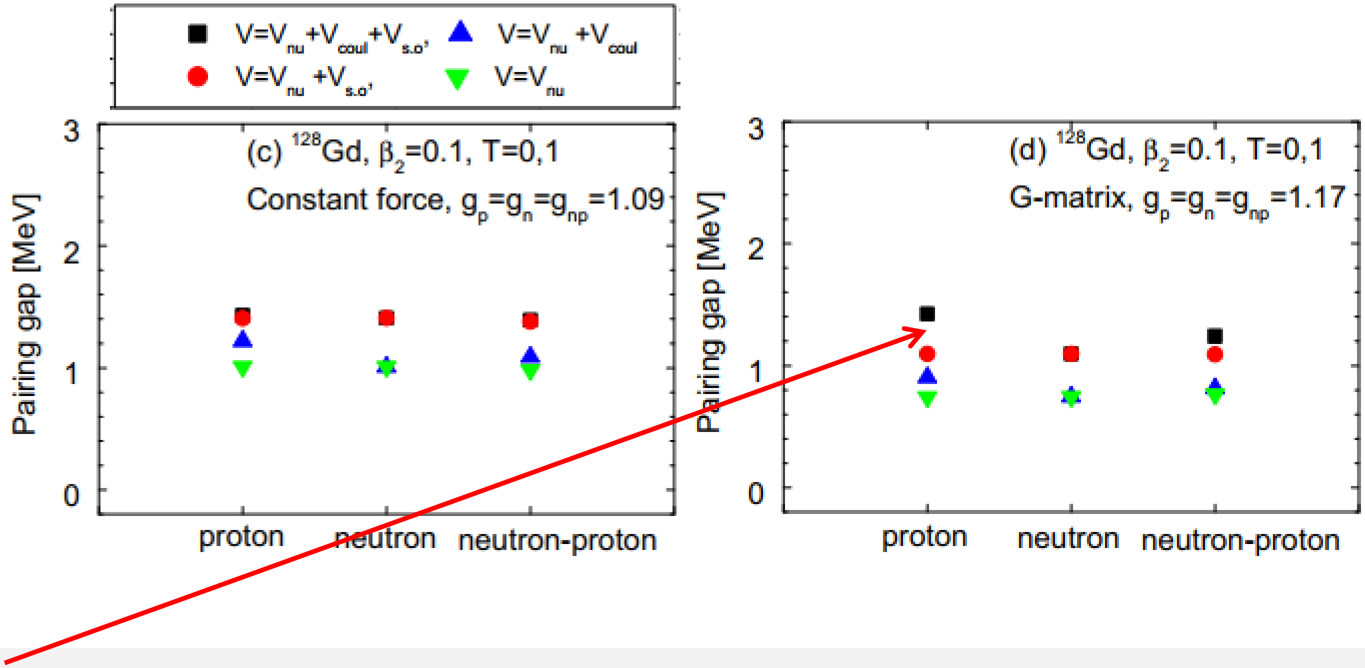


➢ The charge independence symmetry is approximately conserved for ^{48}Cr.



➢ The SO force increases the smearing at the Fermi surface, which increases the pairing gap.

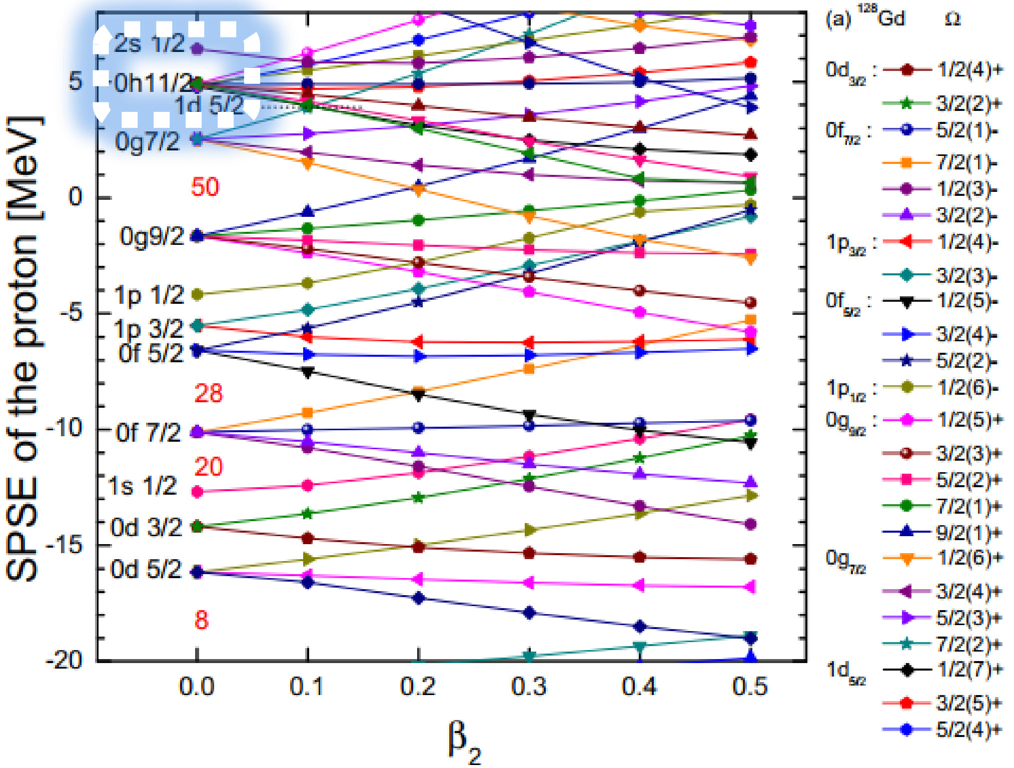
❖ in *sdgh*-shell N=Z nuclei ^{128}Gd



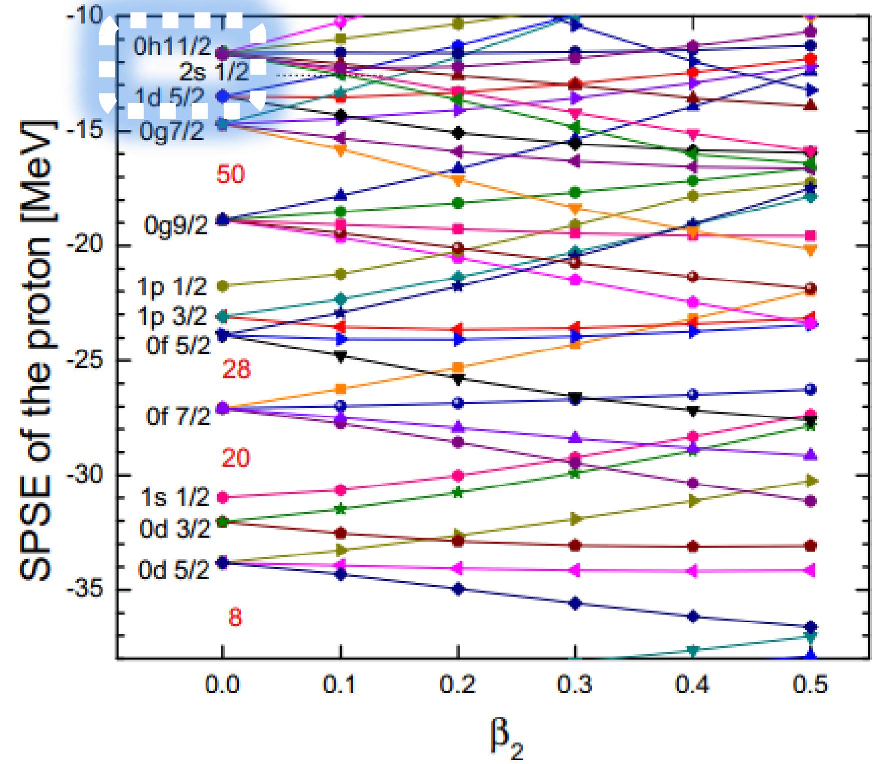
- The Coulomb effects appear explicitly by increasing the pairing gap with G-Mat PME.
- The SU(4) symmetry is more or less violated by the SO and the Coloumb force on the pairing gaps. But it is still a good symmetry even on the pairing (see green triangles).

❖ Reordering of SPSE in ^{128}Gd by the Coulomb force

- **With Coulomb force**



- **Without Coulomb force**

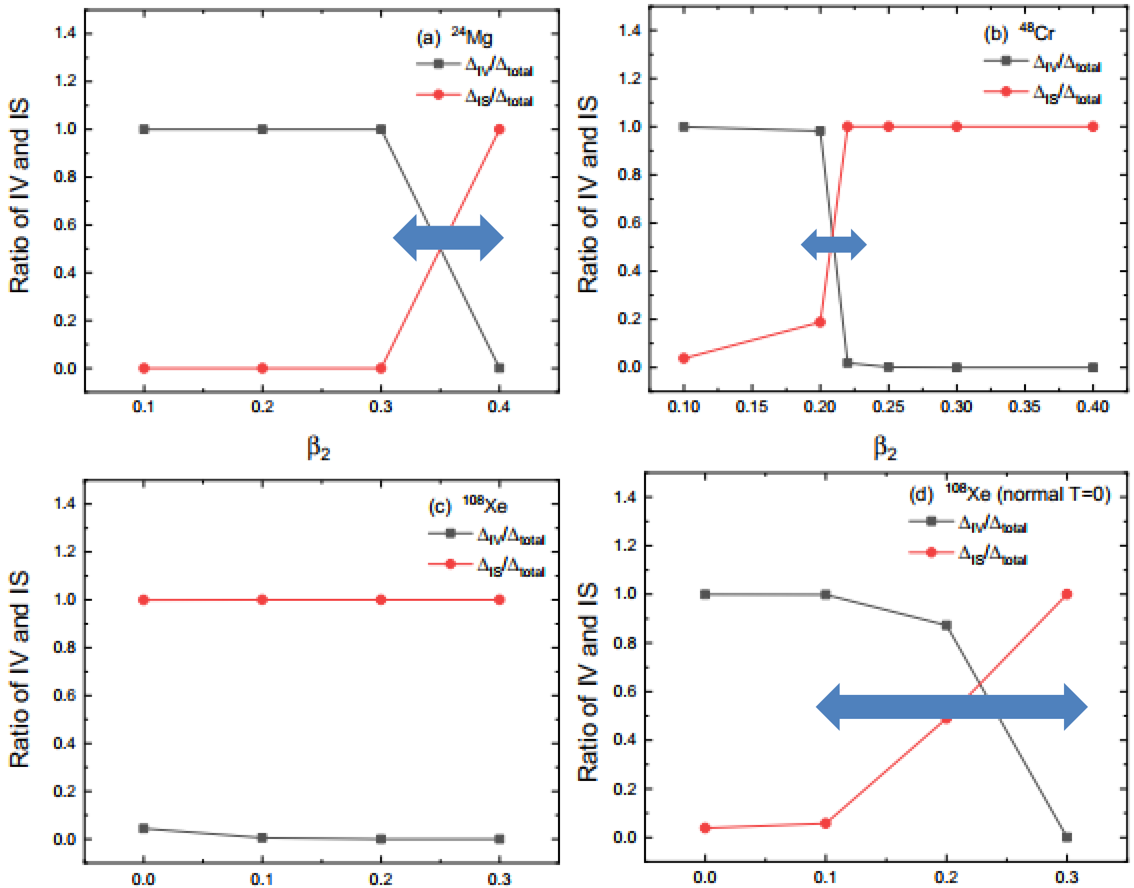


➢ The occupation probability : $0h_{11/2} + 1d_{5/2}$ (with CF) > $0h_{11/2} + 2s_{1/2}$ (w/o CF)

➢ The large smearing by the CF makes a large pairing gap.

❖ Ratio of isovector and isoscalar *np*-pairing

a)~c): enhanced T=0, d) normal T=0

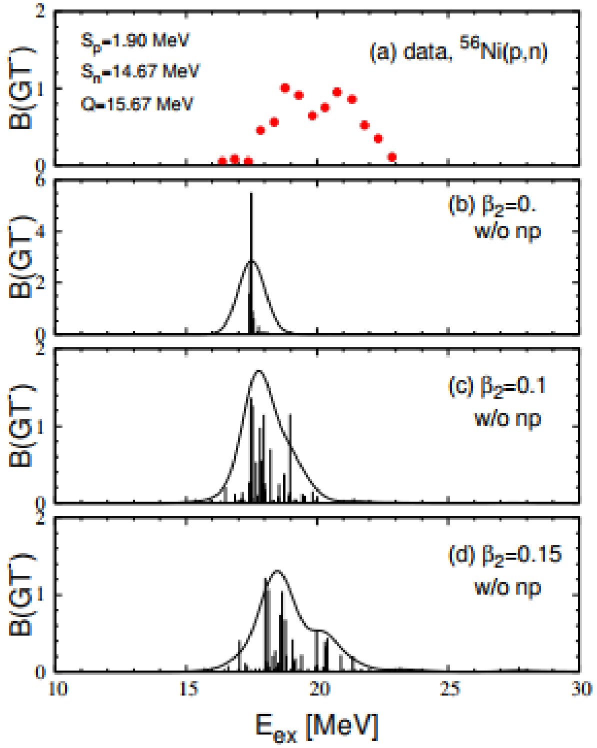


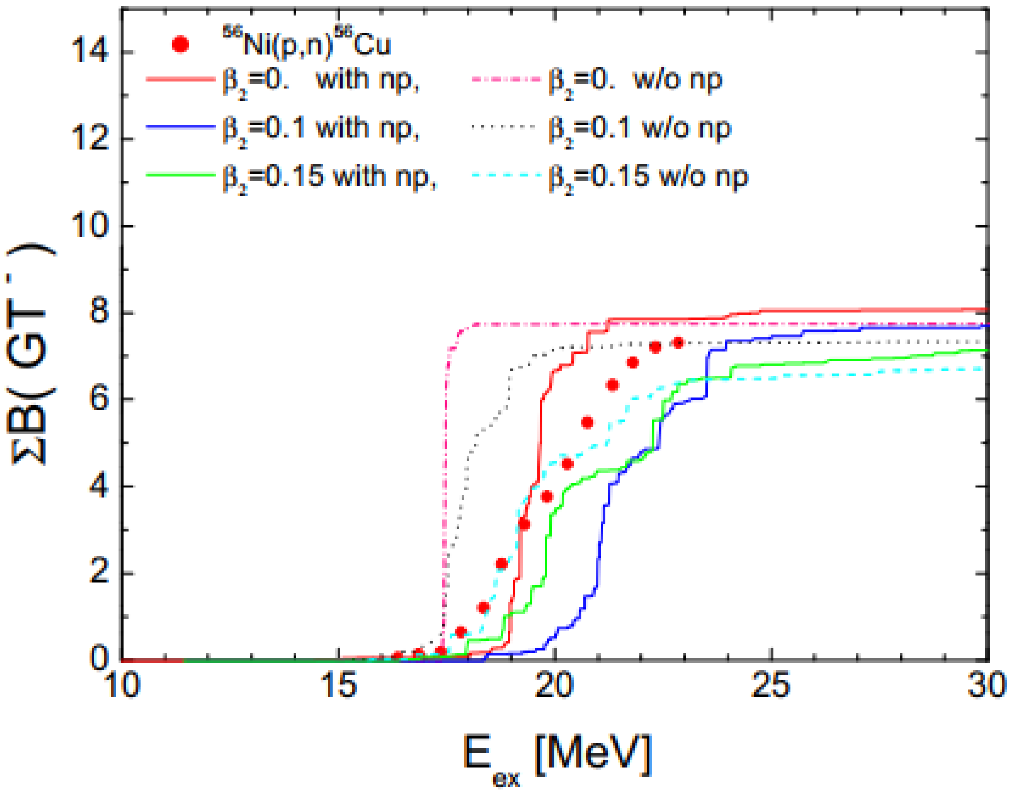
- IS condensation by the enhanced T=0 *np* pairing may happen in deformed ^{24}Mg and ^{48}Cr.
- There is a rapid phase transition from IV to IS component in the *np* pairing. **But it may happen slower in heavy nuclei, which may mean the coexistence in some deformation region.**
- For heavy nuclei such as ^{108}Xe the phase transition may happen more easily even with the normal T=0.

Contents

❖ Gamow-Teller strength for ^{56}Ni

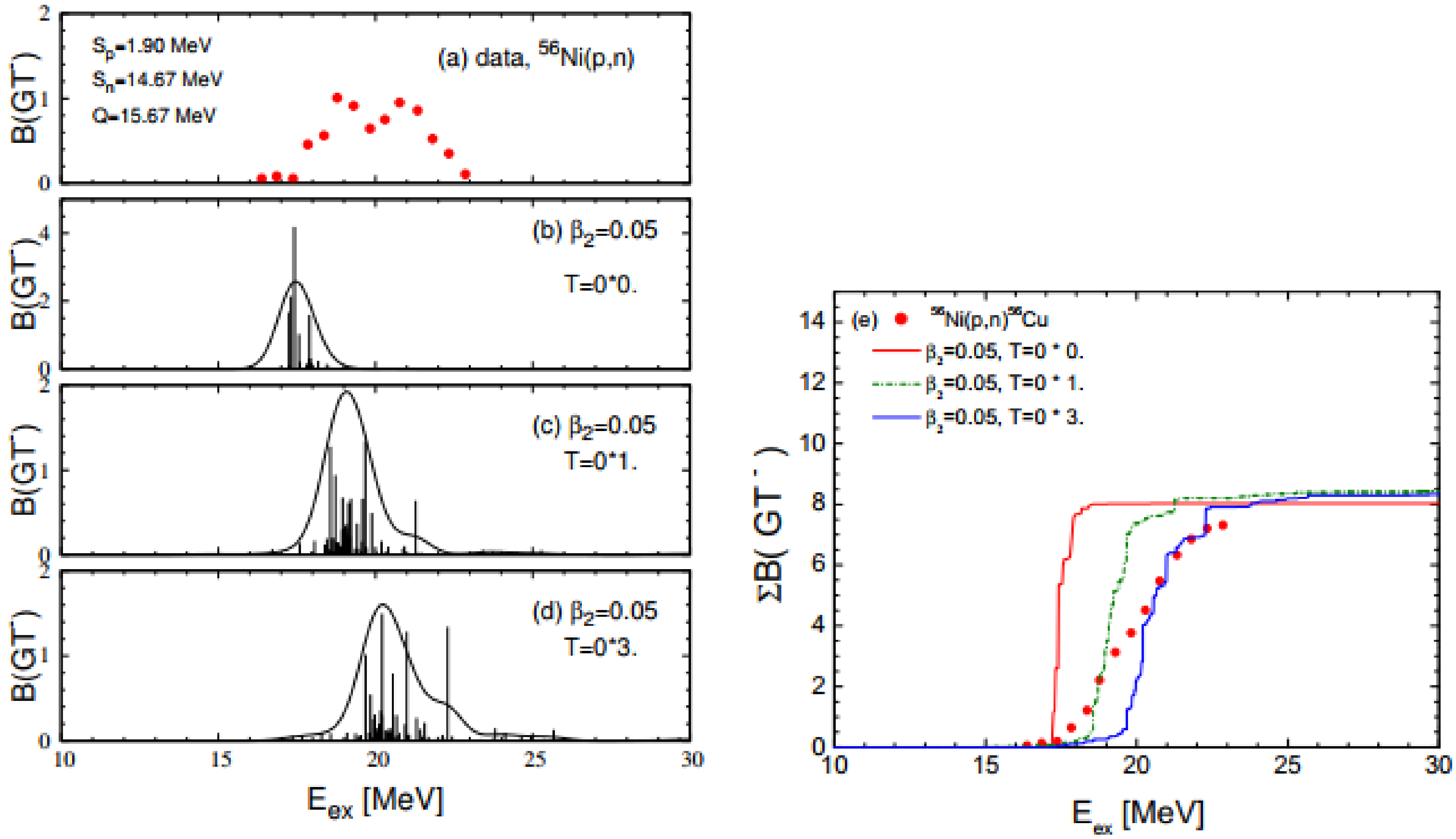
Ha *et al.* accepted to PRC





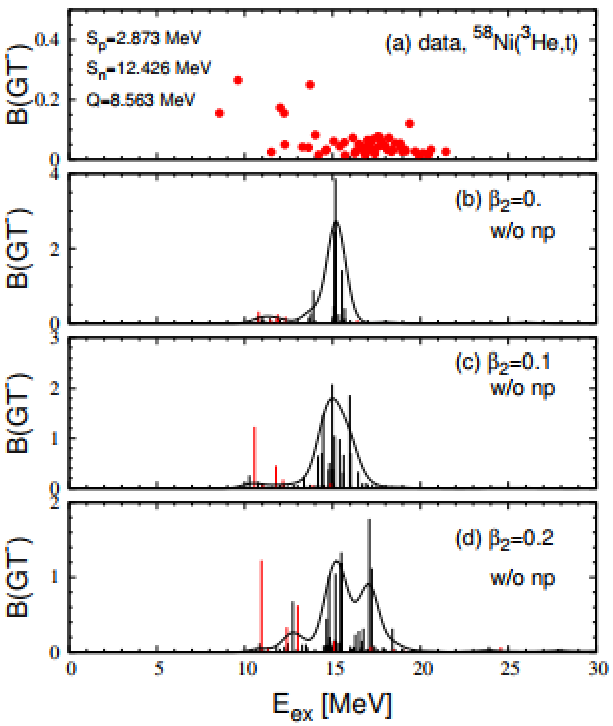
- In particular, ^{56}Ni is thought to be almost spherical because of its double magic numbers.
- If we take α-cluster model for ^{56}Ni, the ground state may be slightly deformed. PRC 84, 024302(2011)
- The *np* pairing effects turn out to be able to properly explain the GT strength although the deformation is also another important property. **The high-lying GT peak in the two peaks stems from the repulsive np pairing through the reduction of Fermi energies of protons and neutrons.**

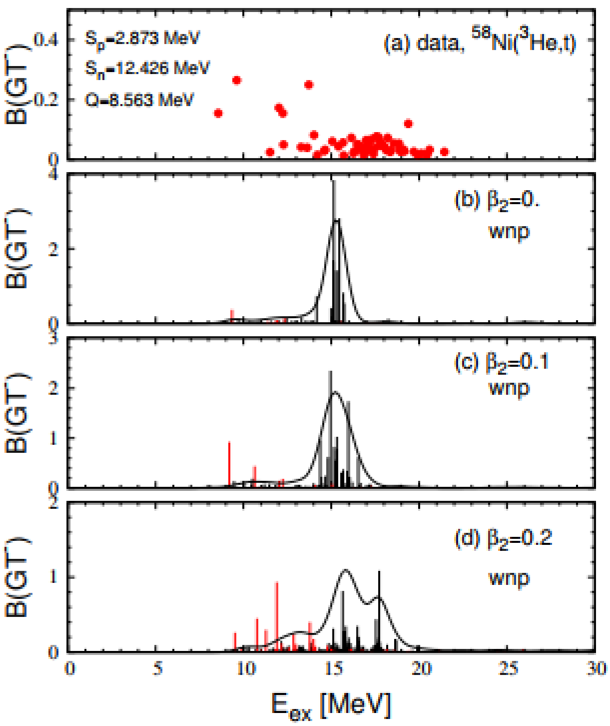
❖ IS *np* pairing effects on B(GT)



➢ The shift of the GT strength distributions by the enhanced T=0 *np* pairing is mainly attributed to the IS coupling condensation. Even with the small deformation, the second peak appears by the T=0 pairing.

❖ Gamow-Teller strength and IAR for ^{58}Ni (N=Z+2)





- The *np* pairing makes the IAR(isobaric analogue resonance) concentrated around 12 MeV, which is consistent with the results in PRC 69(2004) at β_2= 0.2.
- The deformation effect turned out to be more important rather than the *np* pairing correlations since the *np* pairing effects become the smaller with the increase of *N* - *Z* number. Some spurious states peculiar to QRPA lead to small distribution of IAR state.

Summary

1. We find a coexistence of two types of superconductivities (T=0 and T=1) at the $|\beta_2| > 0.3$ region in ^{24}Mg .
3. The IS condensation by the enhanced $T = 0$ pairing may happen not only in *sd*-shell, but also in *pf*-shell nuclei.
4. The IS condensation part plays a vital role to explain the GT strength distribution of ^{56,58,62,64}Ni nucleus, **with the deformation and the unlike-pairing correlations.**

5. The Coulomb force and the SO force are shown to change the smearing by change of ordering of SPS. Remember the splitting by the SO as well as the deformation.

6. The state-dependent Brueckner G-PME takes into account shell structure effects on the residual interaction and enables us to do realistic description of ground states of the $N = Z$ nuclei.

7. For heavy N=Z nuclei, the transition may happen more easily even with the normal T=0 pairing with a phase transition.

❖ References of our **recent papers**

1. Spin singlet and spin triplet pairing correlations on shape evolution in *sd*-shell N=Z nuclei. Ha, MKC *et al*. PRC97,024320(2018)
2. Neutron-proton pairing correlations and deformation for N = Z nuclei in *pf*-shell by the deformed BCS and HFB approach.
   Ha, MKC *et al*. PRC97, 064322(2018)
3. Competition of deformation and neutron-proton pairing in Gamow-Teller transitions for ^{56,58}Ni. Ha, MKC *et al*. accepted to PRC
4. Effects of the Coulomb and the spin-orbit interaction in a deformed mean field on the residual pairing correlations for N=Z nuclei.
   Ha, MKC *et al*. submitted to PRC.
5. **Isoscalar condensation in N = Z nuclei.**
   Ha,MKC et al. to be published Acta Physica Polonica B (2018).
6. …

Thanks for your attention !!

Long and Happy Life for Prof. Akito Arima !!

Back-up files

❖ How to include the deformation?

Deformed Woods-Saxon(WS) potential

(cylindrical WS, Damgaard *et al* 1969)

$$V(\ell) = \frac{-V_0}{1+\exp\left(\ell / a\right)}, \qquad V_{so} = -\lambda(\hbar / 2mc)^2 \operatorname{grad} V(\ell)(\vec{\sigma} \times \vec{p})$$

$$\ell(u, v; \beta_2, \beta_4) = CS(u, v) / \left|\nabla_{u,v} S(u, v)\right|, \qquad z = Cu, \ \rho = Cv$$

distance function — surface function

β_2 : quadrupole deformation parameter
β_4 : hexadecapole deformation parameter

- We can determine these two parameters by taking values giving the minimum ground state energy.

- To exploit G-matrix elements, which is calculated on the spherical basis, deformed bases are **expanded in terms of the spherical bases.**

$$|\alpha\Omega_\alpha> = \sum_a B_a^\alpha \, |a\Omega_\alpha>,$$

Deformed SPS — Sph. HO w. f.

❖ In sd-shell N=Z nuclei , Q_{exp} of ^{28}Si is different from ^{24}Mg and ^{32}S

Nucleus	β_2^{E2} [10]	β_2^{RMF} [11]	β_2^{FRDM} [12]	$Q_{exp.}$ [14, 15]	Δ_p^{emp}	Δ_n^{emp}	δ_{np}^{emp}
^{24}Mg	0.605	0.416	0.	− 0.29 ∼ − 0.07	3.123	3.193	1.844
^{28}Si (prolate)	0.407	x	x	x	2.841[a]	2.917 [a]	1.384[a]
^{28}Si (oblate)	x	− 0.374	− 0.363	0.16 ∼ 0.18	2.841[a]	2.917[a]	1.384[a]
^{32}S	0.312	0.186	0.221	− 0.12 ∼ − 0.18	2.141	2.207	1.047

$$\beta_2 = \frac{4\pi}{3ZR_0^2}\left[\frac{B(E2\uparrow)}{e^2}\right]^{1/2} \qquad (R_0 = 1.2A^{1/3})$$

in the rotational model, $Q_{J^\pi} = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_0.$

for 2^+, $Q_{2+} = -2/7\, Q_0$

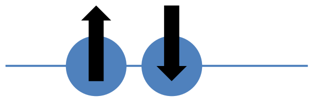
Q_{2+} : **experimental quadrupole moment**

Q_0 : **intrinsic quadrupole moment**

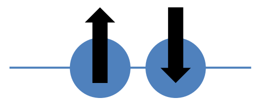
➢ ^{28}Si is not heavy. Where does it come from ?

❖ Pairing correlation

BCS



deformed BCS



(J=0,1, 2, 3 •••)

T=0 J=1, 3, 5, •••

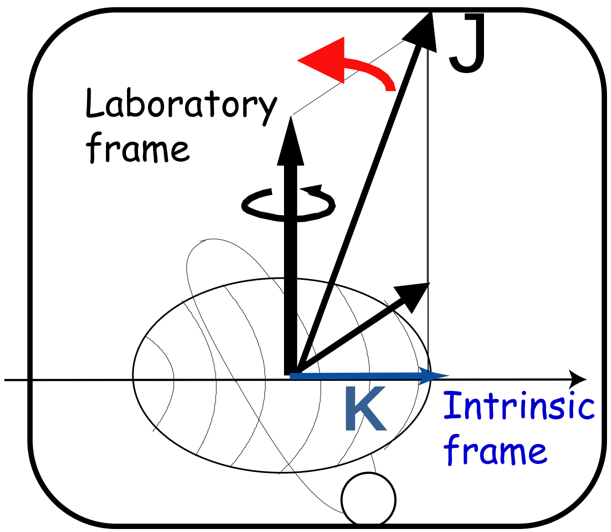
T=1 J=0, 2, 4, •••

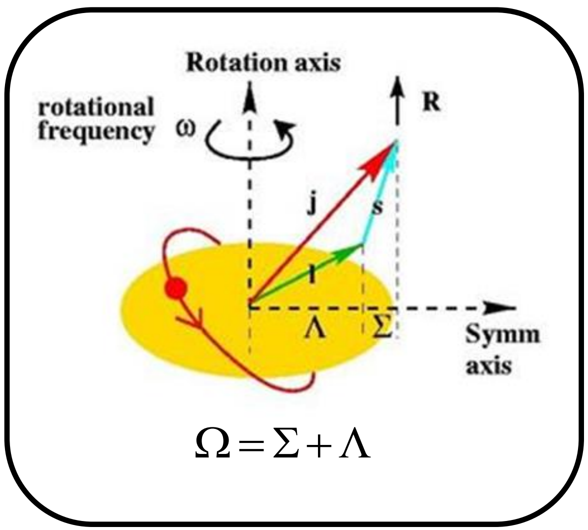
$\Omega = \frac{1}{2}$

$j \geq \Omega$

j	Ω
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{2}$	$\frac{1}{2}$
$\frac{5}{2}$	$\frac{1}{2}$
$\frac{7}{2}$	$\frac{1}{2}$
…	

(Coupled system)

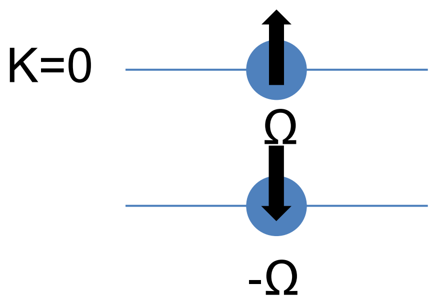




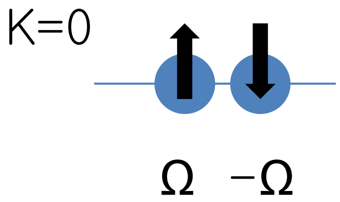
➢ Since the deformed SPS are expanded in terms of the spherical SP bases the different total angular momenta of the SP basis states would be mixed.

❖ Pairing correlation

deformed HFB



deformed BCS



BCS

$$\Delta_{p\bar{p}_\alpha} = \Delta_{\alpha p \bar{\alpha} p} = -\sum_{J,c} g_{\rm pp} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\gamma} c} G(\underline{aacc}, J, T=1)(u^*_{1p_c} v_{1p_c} + u^*_{2p_c} v_{2p_c})$$

$$\Delta_{p\bar{n}_\alpha} = \Delta_{\alpha p \bar{\alpha} n} = -\sum_{J,c} g_{\rm np} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\gamma} c} [G(aacc, J, T=1) Re(u^*_{1n_c} v_{1p_c} + u^*_{2n_c} v_{2p_c})$$
$$+ \; iG(aacc, J, T=0) Im(u^*_{1n_c} v_{1p_c} + u^*_{2n_c} v_{2p_c})] \;,$$

HFB

$$\Delta_{p\bar{p}_\alpha} = \Delta_{\alpha p \bar{\alpha} p} = -\sum_{J,c,d} g_{\rm pp} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\delta} c} G(\overline{aacd}, J, T=1)(u^*_{1p_c} v_{1p_d} + u^*_{2p_c} v_{2p_d})$$

❖ Self energy in BCS

$$H_0 = \sum_b^A 2\left[v_b^2\left(\eta_b + \tfrac{1}{2}\mu_b\right) - \tfrac{1}{2}u_b v_b \Delta_b\right]$$

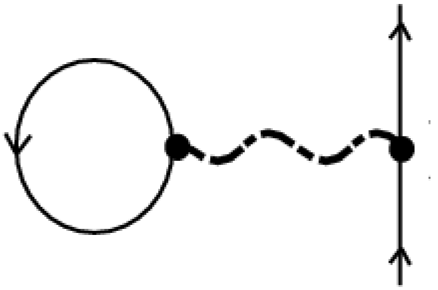
E_{mean} E_{self} E_{pair}

BCS eq.

$$\eta_b \equiv \varepsilon_b - \lambda - \mu_b$$

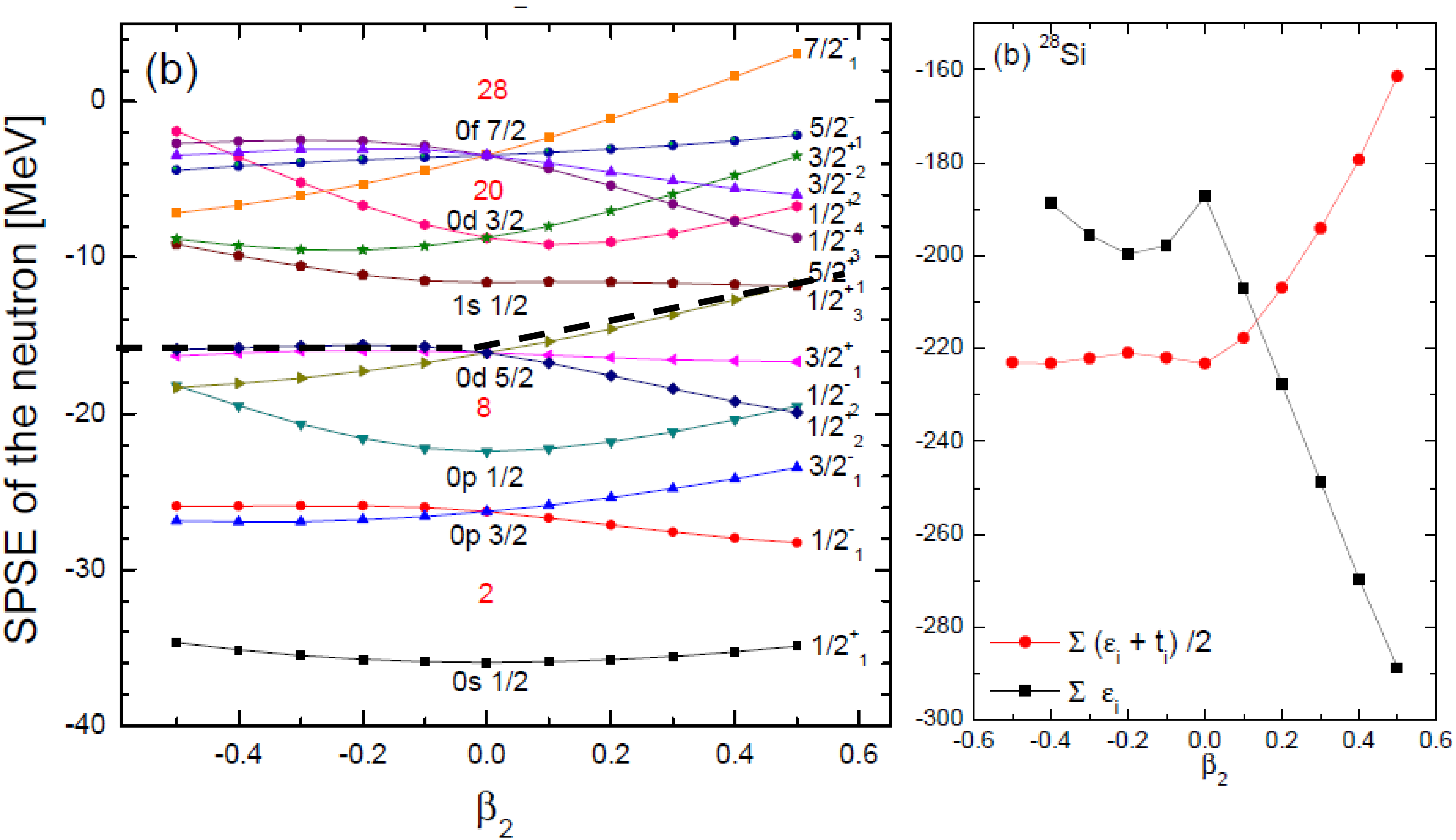
$$\mu_b = -\tfrac{1}{2}\sum_{a,J} v_a^2 \hat{J}^2 \langle ab:J|V|ab:J\rangle \quad \text{: self energy}$$

$$\Delta_b = -\sum_a u_a v_a \langle aa;0|V|bb:0\rangle \quad \text{: pairing gap}$$

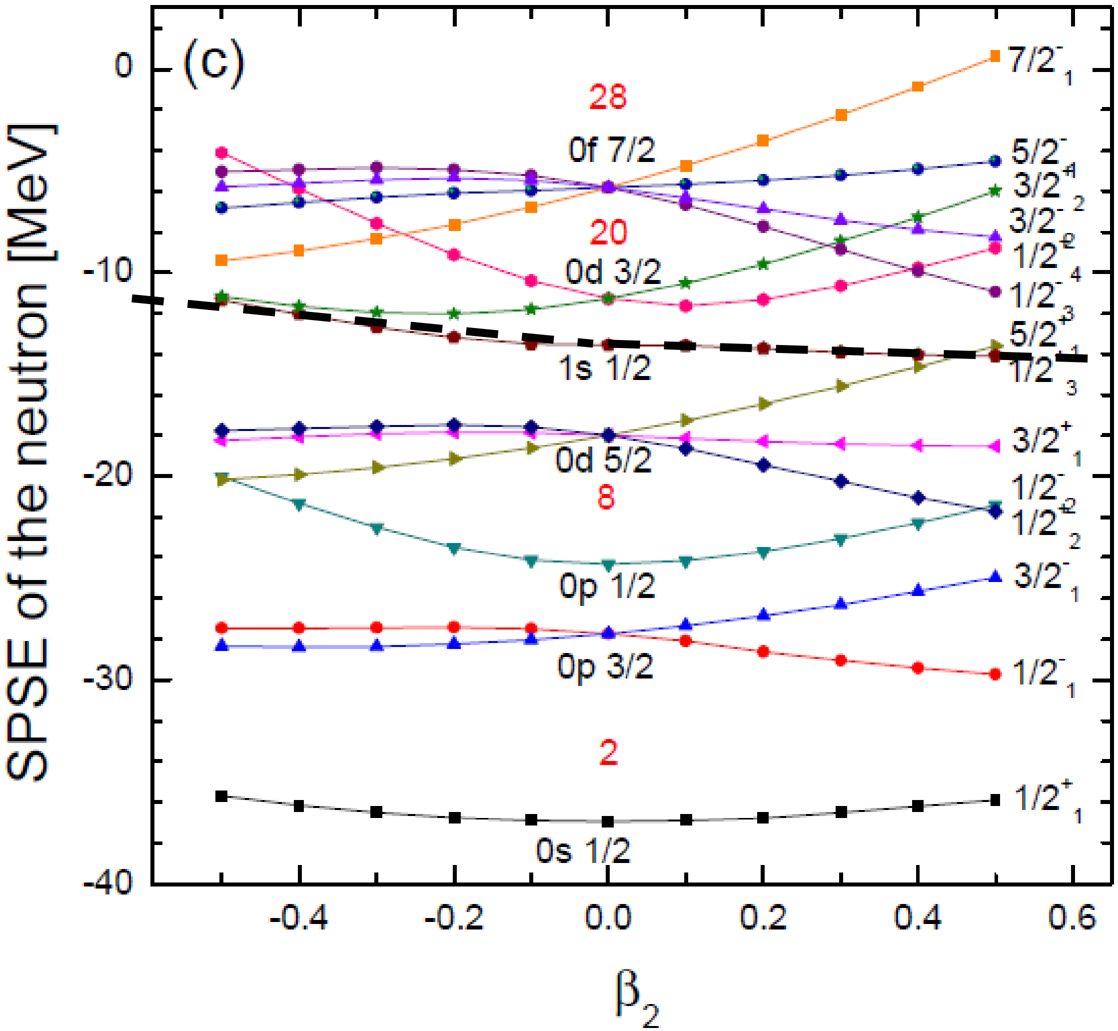


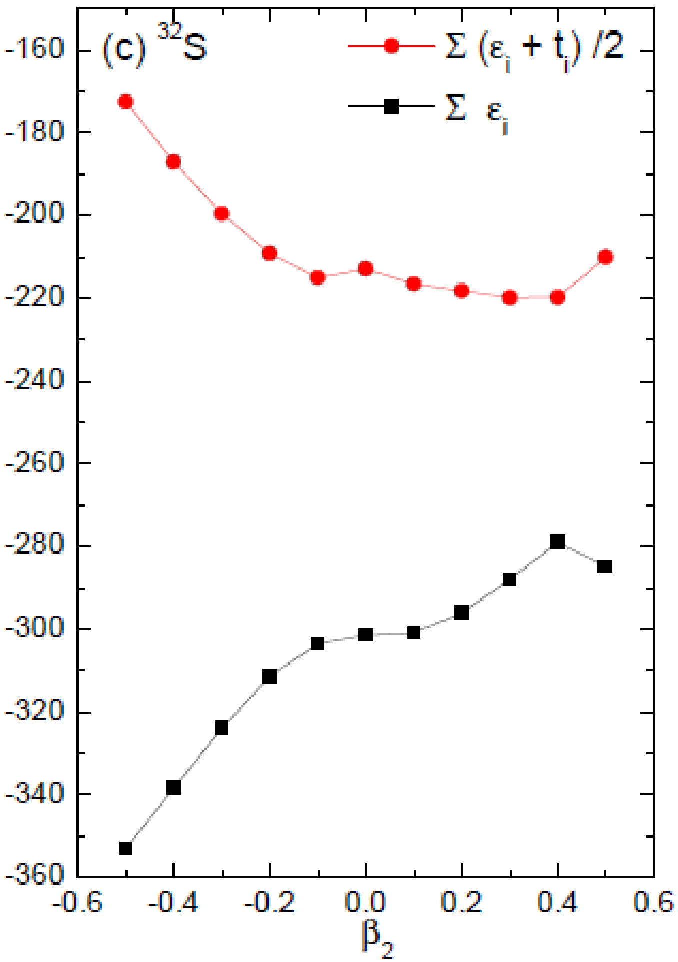
➢ The self energy term was usually neglected in BCS eq. because it results from particle-hole correlations beyond the BCS and affects a renormalization of the single particle energy.

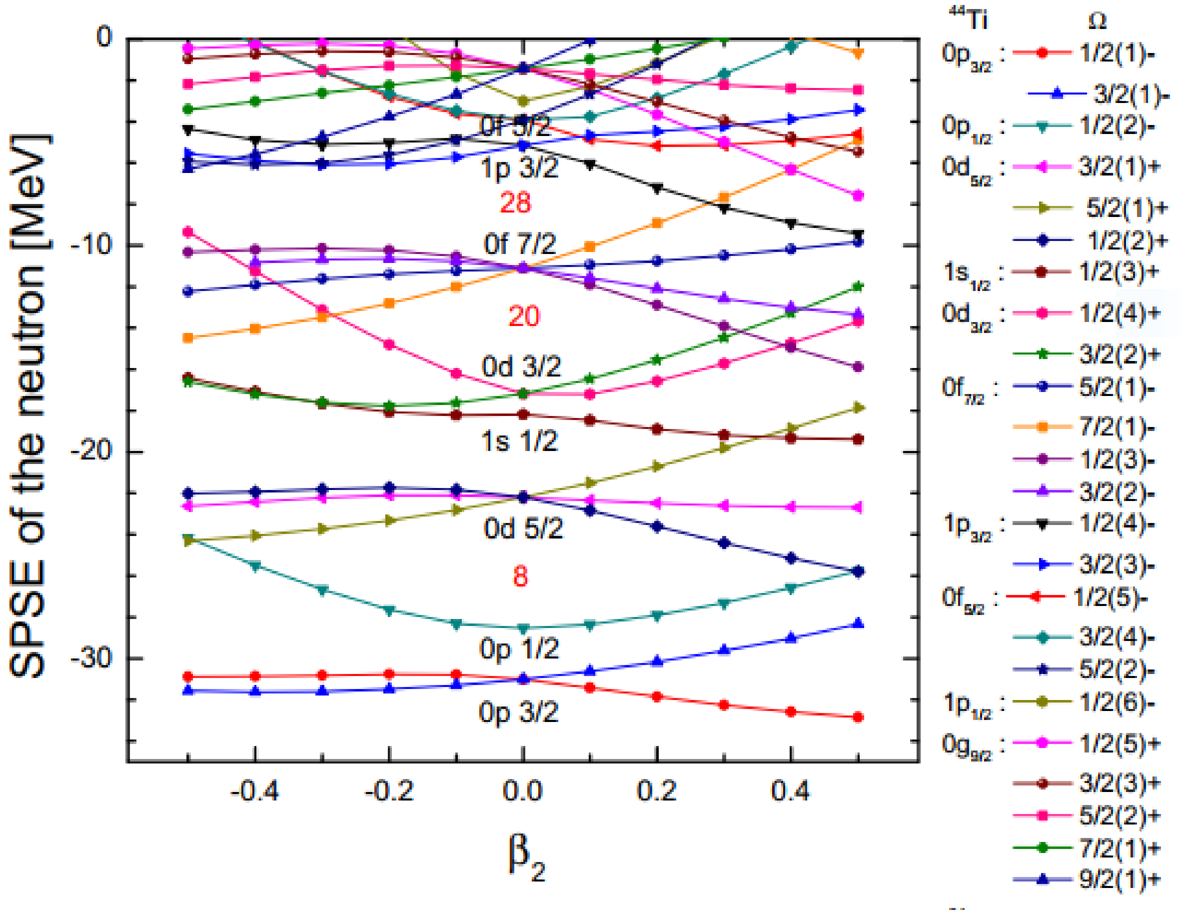
❖ Shell evolution & the simplest shell model of ^{28}Si



❖ Shell evolution & the simplest shell model of ^{32}S







SPSE of the neutron [MeV]
0
-10
-20
-30
-0.4
-0.2
0.0
0.2
0.4
β_2
0f 5/2
1p 3/2
28
0f 7/2
20
0d 3/2
1s 1/2
0d 5/2
8
0p 1/2
0p 3/2
^{44}Ti
Ω
$0p_{3/2}$: 1/2(1)-
3/2(1)-
$0p_{1/2}$: 1/2(2)-
$0d_{5/2}$: 3/2(1)+
5/2(1)+
1/2(2)+
$1s_{1/2}$: 1/2(3)+
$0d_{3/2}$: 1/2(4)+
3/2(2)+
$0f_{7/2}$: 5/2(1)-
7/2(1)-
1/2(3)-
3/2(2)-
$1p_{3/2}$: 1/2(4)-
3/2(3)-
$0f_{5/2}$: 1/2(5)-
3/2(4)-
5/2(2)-
$1p_{1/2}$: 1/2(6)-
$0g_{9/2}$: 1/2(5)+
3/2(3)+
5/2(2)+
7/2(1)+
9/2(1)+

backup file

❖ Parameter set of Deformed Woods-Saxon

Table 1

Set of parameter values defined by the program according to the input value of the ICHOIC variable. The symbols P (N) refer to the protons (neutrons). The λ values in the case of the Chepurnov parametrisation are defined by $\lambda = 23.8\ (1+2*(N-Z)/A)$. Blomqv.–Wahlb. stands for Blomqvist and Wahlborn. The values of r_0 and a are in fermi, V_0 in MeV, κ and λ dimensionless

Parametrisation	λ (P)	λ (N)	r_{0-so} (P)	r_{0-so} (N)	r_0 (P)	r_0 (N)	κ	V_0	a	ICHOIC
Blomqv.–Wahlb.	32.0	32.0	1.270	1.270	1.270	1.270	0.67	51.0	0.67	0
Rost	17.8	31.5	0.932	1.280	1.275	1.347	0.86	49.6	0.70	1
Chepurnov	calc.		1.240	1.240	1.240	1.240	0.63	53.3	0.63	2
"optimal"	A-dependent				1.275	1.347	0.86	49.6	0.70	3
"universal"	36.0	35.0	1.20	1.310	1.275	1.347	0.86	49.6	0.70	4
"input"				parameters read from input						5
def.-dependent INCREA = 1	deformation-dependent (only for $\beta_2 > 0.325$)				depend on ICHOIC					0–5

❖ In gd-shell N=Z nuclei

Nucleus	β_2^{RMF} [10]	β_2^{FRDM} [11]	β_2^{KTUY} [10]	Δ_p^{emp}	Δ_n^{emp}	δ_{np}^{emp}
^{104}Te	–	–0.011	0.039	1.520	1.548	0.665
^{116}Ce	0.285	0.282	0.145	1.452	1.530	0.697
^{128}Gd	0.350	0.341	0.194	1.415	1.393	0.592

❖ Used parameters in this work.

$* N_{max} = 10 (spherical\ basis)$

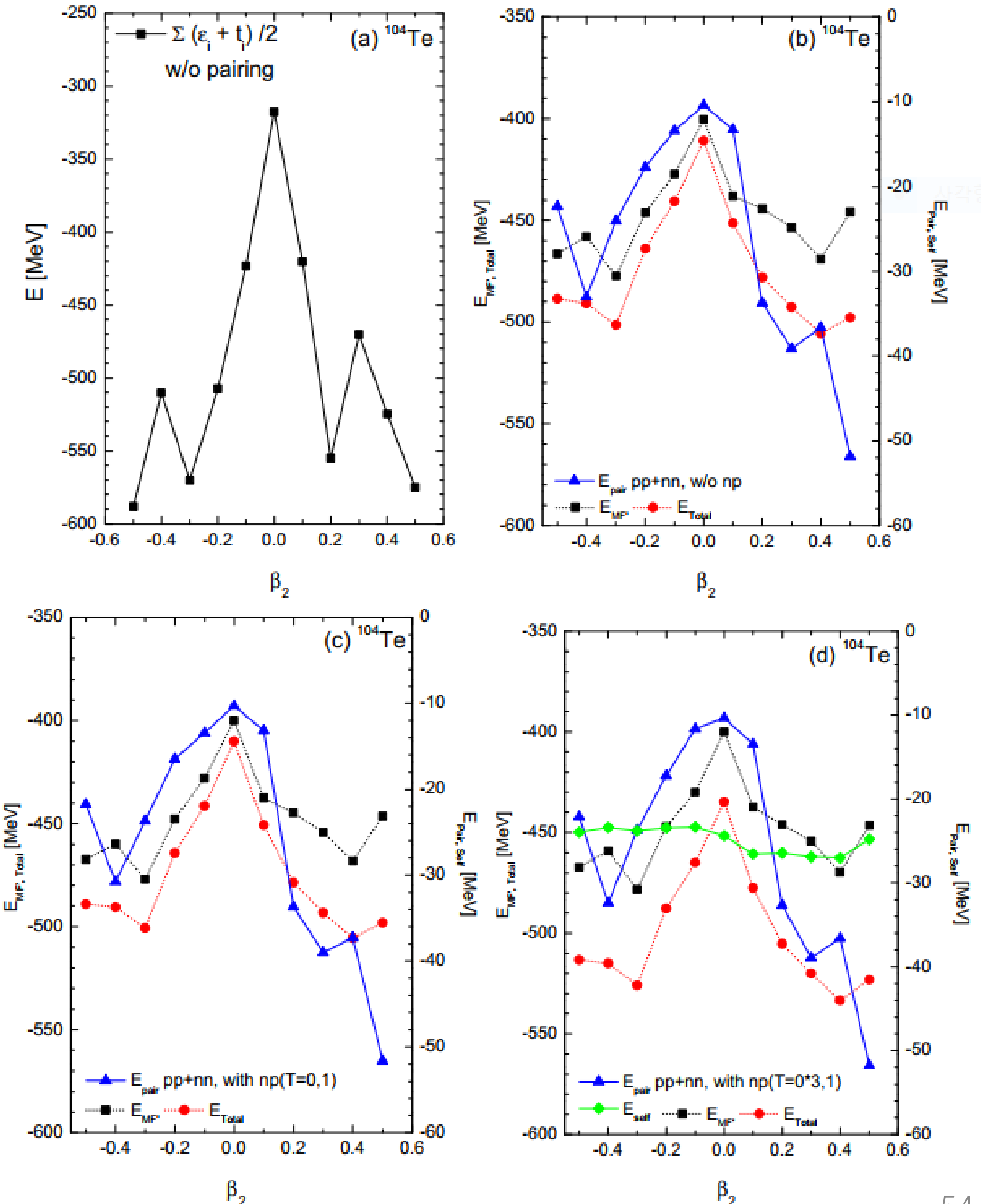
$* N_{max} = 5 (deformed\ basis)$

$* WS\ input\ \ parameters : universal\ param.$

$* pairing\ gap : five - term\ mass\ formula$

$* g_{pp}(g_{ph}) = 0.99(1.15)\ particle - particle(particle - hole)\ int.\ strength$

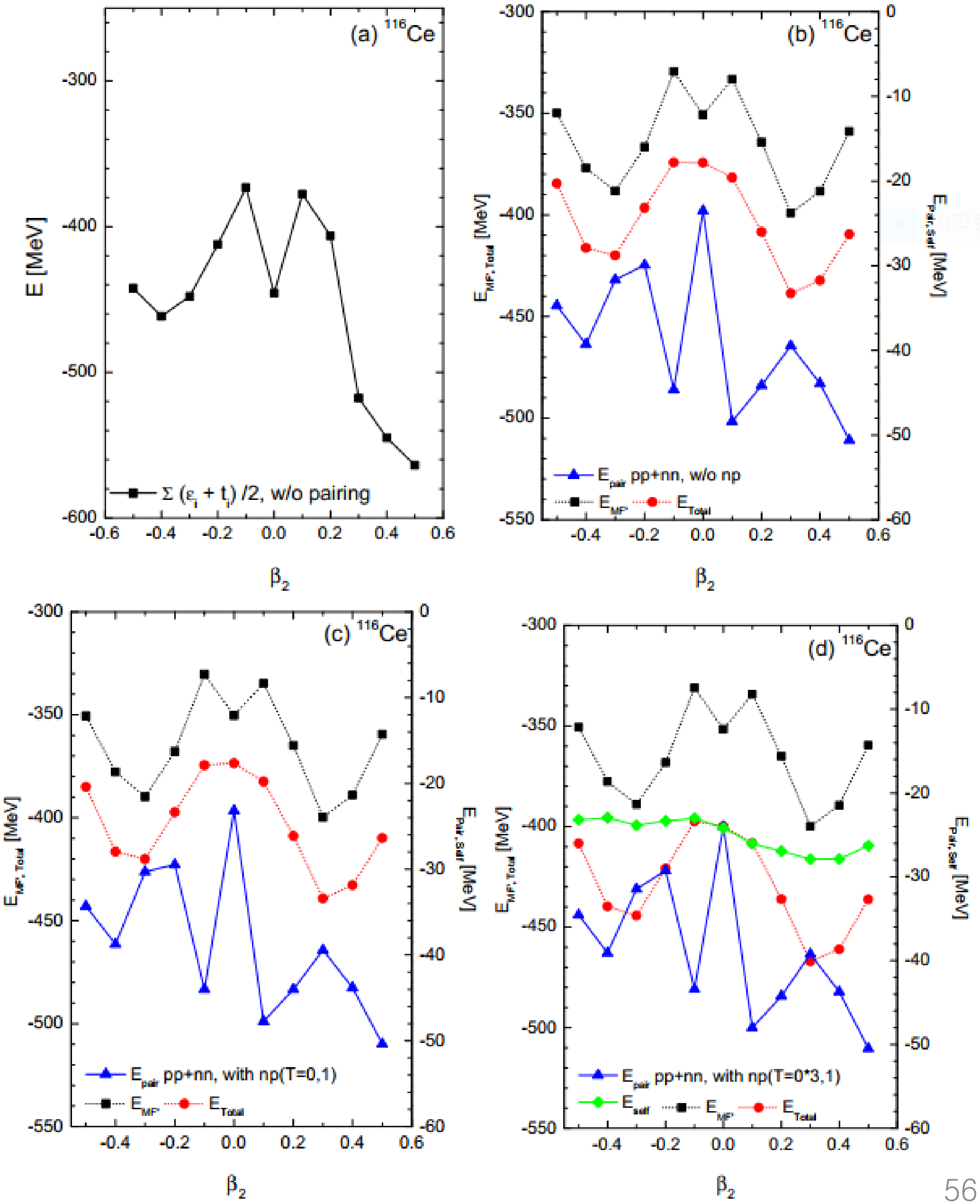
❖ ground state E of 104Te



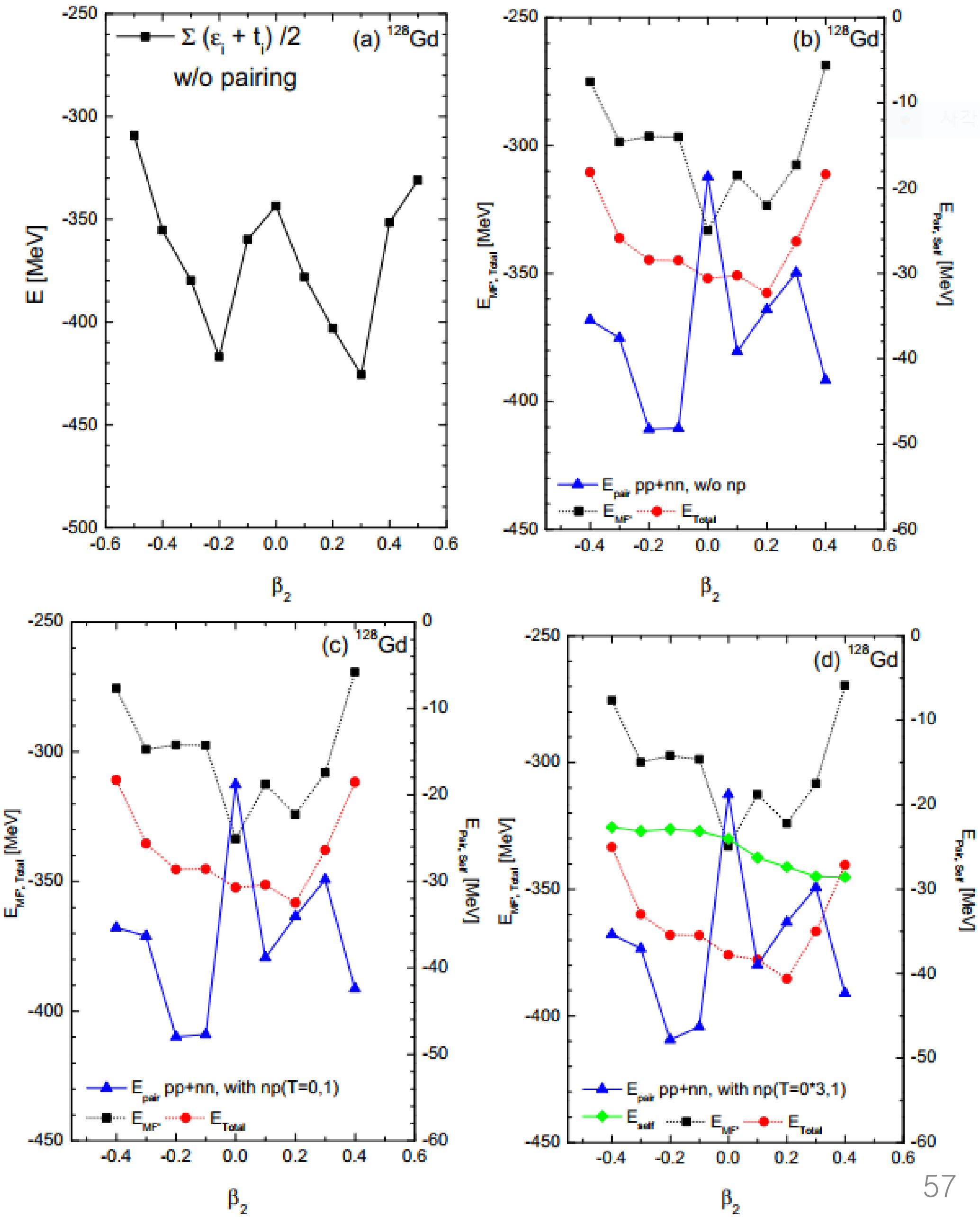
deformed Hartree Fock Bogoliubov (DHFB) transformation,

$$\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_{\bar{1}} \\ a_{\bar{2}} \end{pmatrix}_\alpha = \begin{pmatrix} u_{1p} & u_{1n} & v_{1p} & v_{1n} \\ u_{2p} & u_{2n} & v_{2p} & v_{2n} \\ -v_{1p} & -v_{1n} & u_{1p} & u_{1n} \\ -v_{2p} & -v_{2n} & u_{2p} & u_{2n} \end{pmatrix}_\alpha \begin{pmatrix} c_p^\dagger \\ c_n^\dagger \\ c_{\bar{p}} \\ c_{\bar{n}} \end{pmatrix}_\alpha$$

❖ ground state E of 116Ce



❖ ground state E of 128Gd



❖ Two-body interaction

Realistic two body interaction inside nuclei was taken by Brueckner g-matrix, which is a solution of the Bethe-Salpeter Eq., derived from the Bonn-CD potential for nucleon-nucleon interaction in free space.

$$g(\omega)_{ab,cd} = V_{ab,cd} + V_{ab,cd}\frac{Q_p}{\omega - H_0} g(\omega)_{ab,cd}$$

a,b,c,d : single particle states from the Woods-Saxon potential.
$V_{ab,cd}$: phenomenological nucleon-nucleon potential in free space.