



# **Pair-truncation for particle-hole configuration spaces**

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**International Symposium of Nuclear Science:**

**Simplicity, Symmetry and Beauty**

**In honor of the Rice Age of Professor Akito Arima**

**Shanghai, September 26<sup>th</sup>-28<sup>th</sup>, 2018**

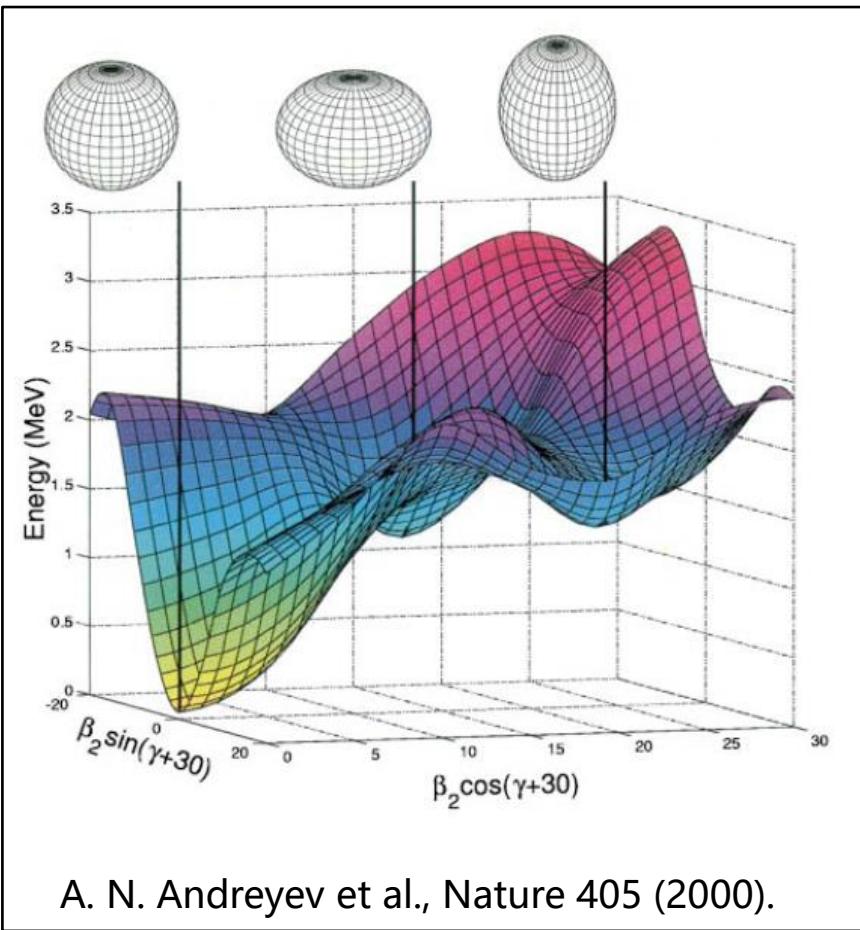


HAPPY HAPPY Rice-Age Birthday  
to  
Arima Sensei !

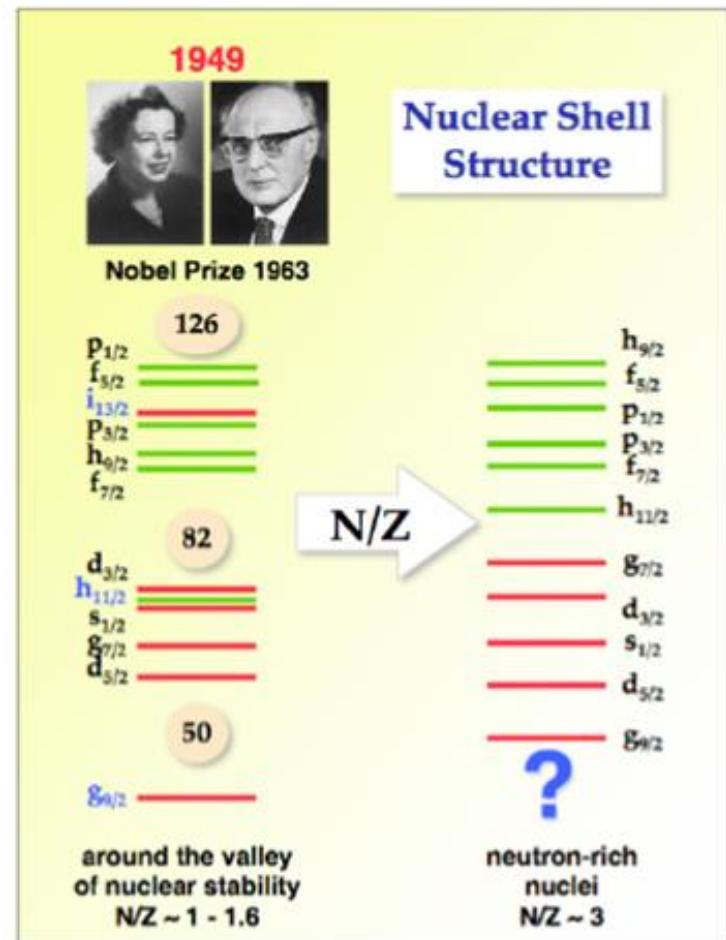


# WHY particle-hole configurations

## Shape coexistence

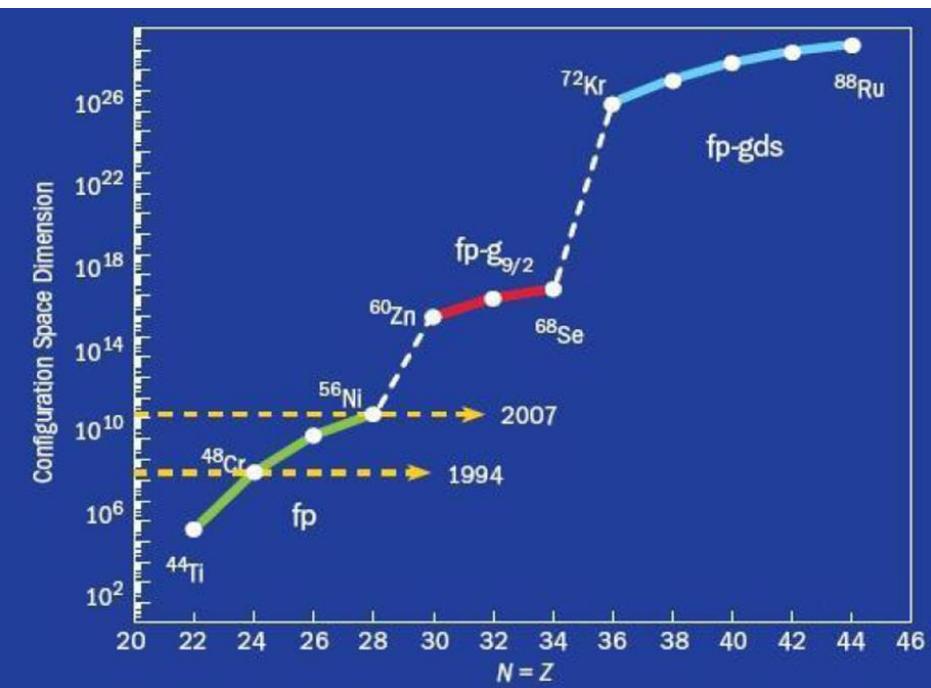


## Shell evolution

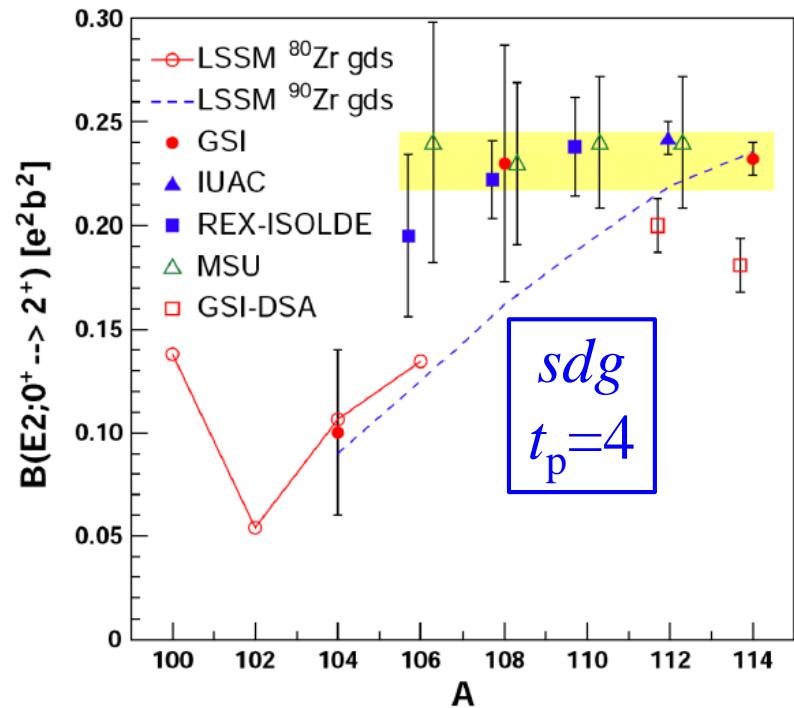


# WHY truncation

## Shell-Model dimensions

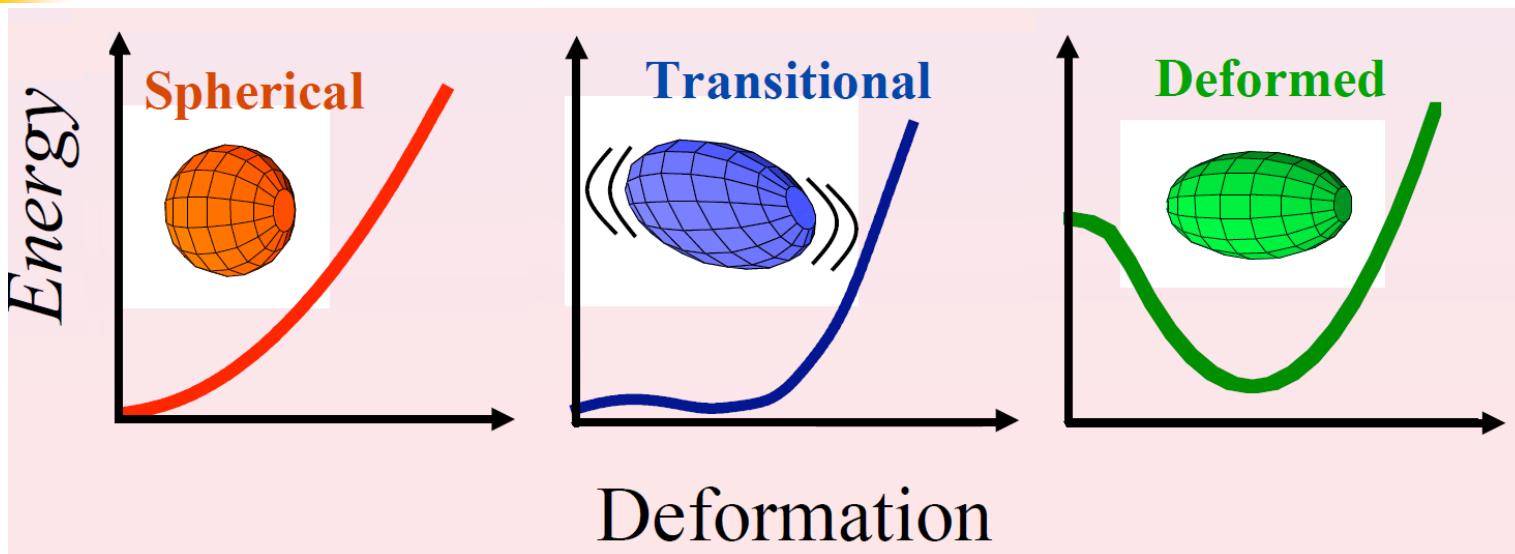


## LSSM cal. for Sn isotopes



Guastalla et al.  
Phys. Rev. Lett. 110, 172501 (2013)

# WHY truncation



To describe Shape Coexistence:

HUGE space when using **spherical s.p. basis**

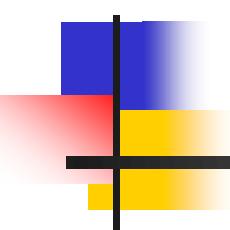
Deformed nuclei: Deformed DFT + angular momentum projection 😊

Spherical nuclei: Spherical s.p. basis

+ truncation !

Transitional nuclei: Spherical s.p. basis





# HOW to truncate

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"A crucial step is to truncate the shell-model basis into a subspace that allows us to treat **low-lying quadrupole excitations and intruder excitations moving across closed shells (or subshells)**." ---- Heyde and Wood, Rev. Mod. Phys. 83, 1467 (2011)

# Interacting Boson Model

VOLUME 35, NUMBER 16

PHYSICAL REVIEW LETTERS

20 OCTOBER 1975

## Collective Nuclear States as Representations of a SU(6) Group\*

A. Arima

*Department of Physics, University of Tokyo, Tokyo, Japan*

and

F. Iachello

*Kernfysisch Versneller Instituut, University of Groningen, Groningen, The Netherlands†, and  
Argonne National Laboratory, Argonne, Illinois 60439*

(Received 11 August 1975)

We propose a description of collective quadrupole states in even-even nuclei in terms of representations of a boson SU(6) group. We show that within this model both the vibrational and the rotational limit can be recovered.

- P. D. Duval, and B. R. Barrett, Phys. Lett. B 100, 223 (1981).  
P. D. Duval, and B. R. Barrett, Nucl. Phys. A 376, 213 (1982).  
K. Heyde, C. De Coster, J. Jolie, and J. L. Wood, Phys. Rev. C 46, 541 (1992).
- .....

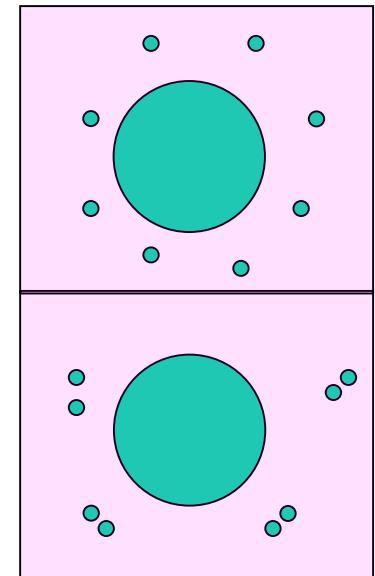
# Nucleon-pair approximation

Shell-model basis (m-scheme):

$$|\phi\rangle = a_{j_1 m_1}^+ a_{j_2 m_2}^+ \cdots a_{j_n m_n}^+ |0\rangle$$

nucleon pair:

$$A^{(r)+} = \sum_{j_1 j_2} y(j_1 j_2 r) A^{(r)+}(j_1 j_2), \quad A^{(r)+}(j_1 j_2) = (a_{j_1}^+ \times a_{j_2}^+)^{(r)}$$



pair basis (J-scheme):

$$|\varphi\rangle = \left( \cdots \left( \left( A^{(r_1)+} \times A^{(r_2)+} \right)^{(J_2)} \times A^{(r_3)+} \right)^{(J_3)} \cdots \times A^{(r_N)+} \right)^{(J_N)}_{M_N} |0\rangle$$

J. Q. Chen, Nucl. Phys. A 626, 686 (1997).

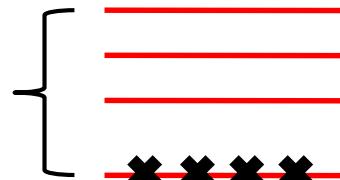
Y. M. Zhao and A. Arima, Physics Reports 545, 1 (2014).

## Nucleon-pair approximation with particle-hole excitations

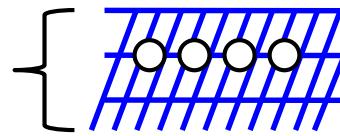
Y. Y. Cheng,<sup>1,2</sup> Y. M. Zhao,<sup>2,3,\*</sup> and A. Arima<sup>4,2</sup>

### “Mixed representation”

not  
change



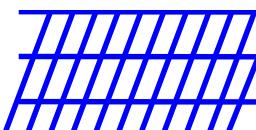
ph  
conjugate  
trans.



**particle-particle**  
**pair**



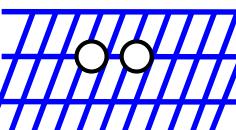
$$\begin{pmatrix} A_{n \times n} & 0 \\ 0 & 0 \end{pmatrix}$$



**hole-hole**  
**pair**



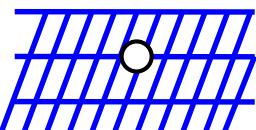
$$\begin{pmatrix} 0 & 0 \\ 0 & B_{m \times m} \end{pmatrix}$$



**particle-hole**  
**pair**



$$\begin{pmatrix} 0 & C_{n \times m} \\ 0 & 0 \end{pmatrix}$$



Two types of excitations across closed shell  
can be flexibly included.

# Calculation of $^{100}\text{Sn}$

## 1. Space

holes : 28-50 major shell , particles : 50-82 major shell  
up to 4-particle-4-hole

## 2. Shell-model Hamiltonian

$$H = H_{\text{SM}} + \beta \left( H_{\text{CM}} - \frac{3}{2} \hbar \omega \right), \quad H_{\text{CM}} = \frac{\tilde{P}^2}{2mA} + \frac{1}{2} mA \omega^2 \tilde{R}^2.$$

$$H_{\text{SM}} = \sum_j \varepsilon_j N_j + \sum_{j_1 \leq j_2} \sum_{j_3 \leq j_4} \sum_{JM} \sum_{TM_T} \frac{V_{JT}(j_1 j_2 j_3 j_4)}{\sqrt{(1 + \delta_{j_1 j_2})(1 + \delta_{j_3 j_4})}} A_{MM_T}^{JT\dagger}(j_1 j_2) A_{MM_T}^{JT}(j_3 j_4)$$

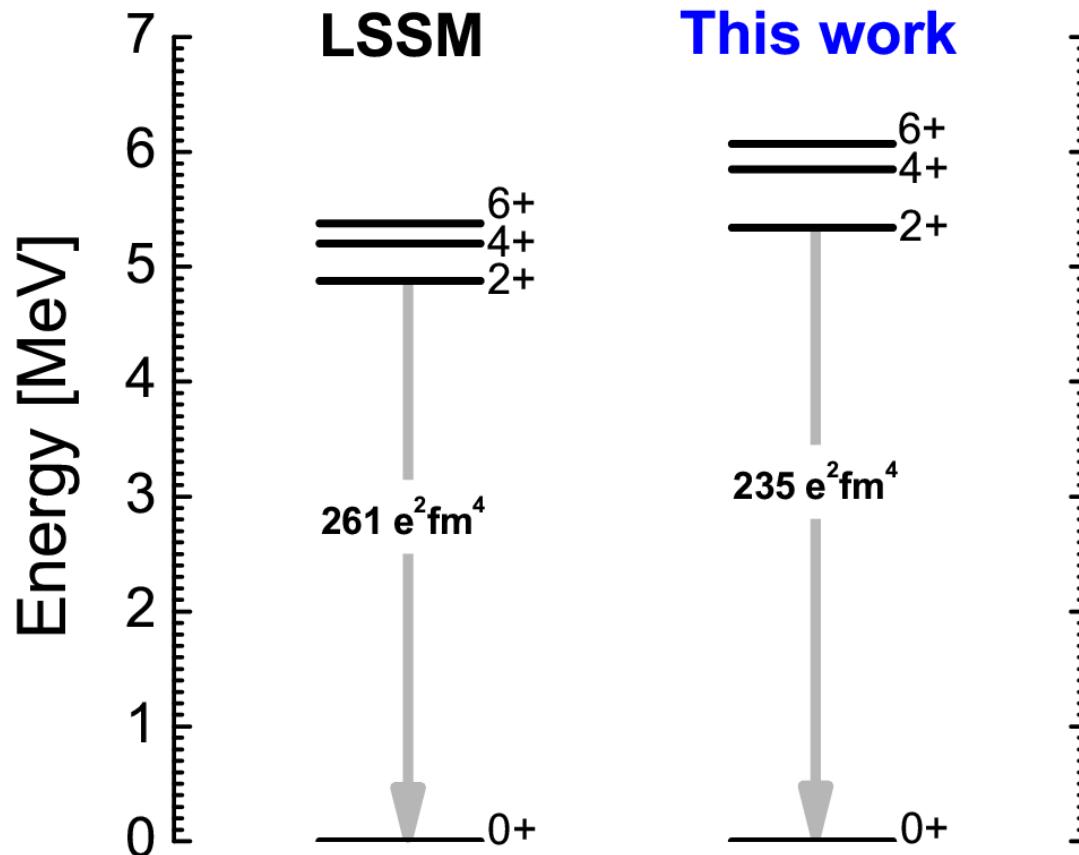
TBME : CD-Bonn + V<sub>low-k</sub> (given by Prof. T. T. S. Kuo)

## 3. Pair truncation

SD pairs of both pp and hh types

& ph pairs (phonons) with spin-2, 4, 6

# Calculation of $^{100}\text{Sn}$

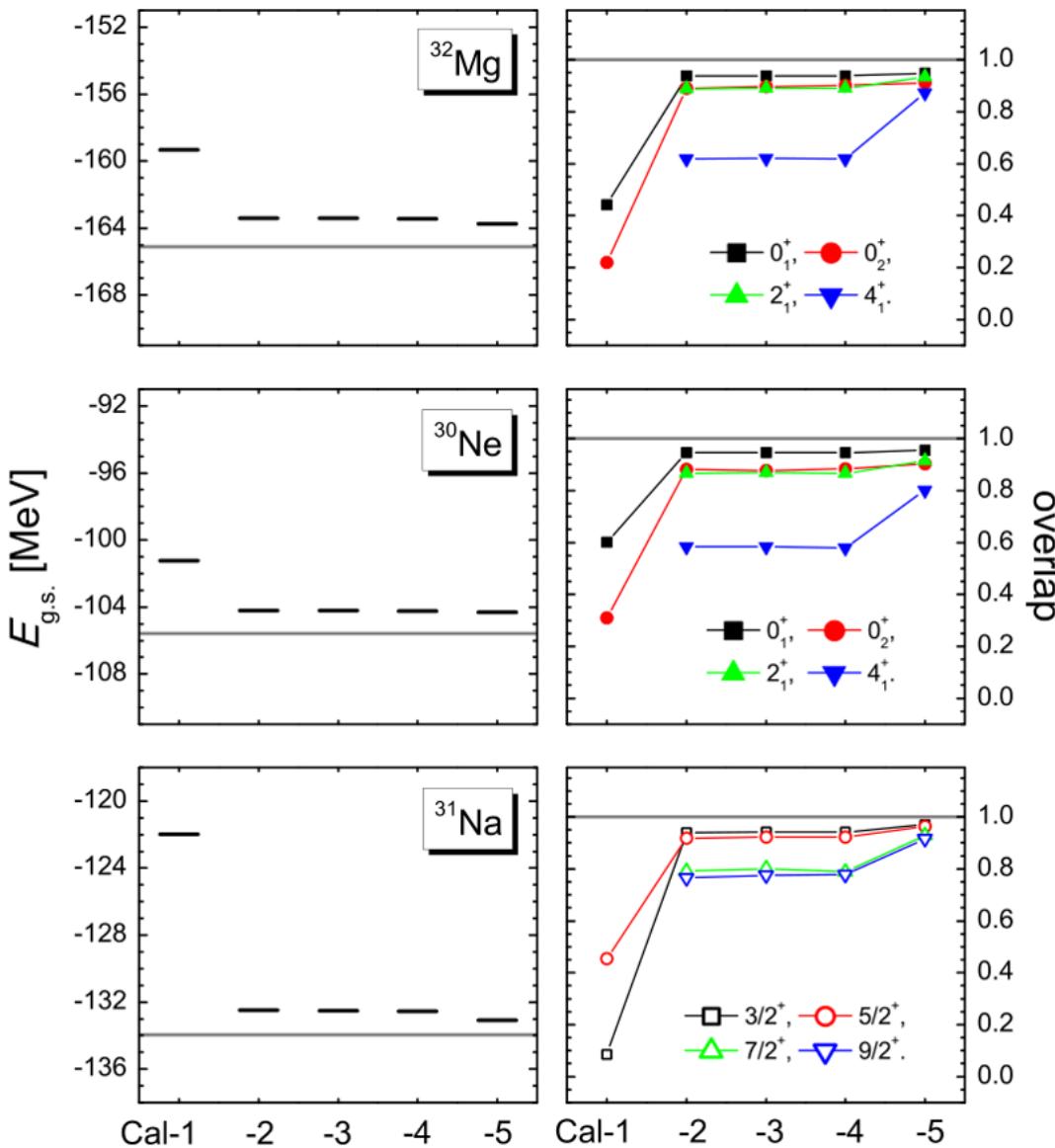


LSSM:

holes in  $0g_{9/2}$ , particles in  $0g_{7/2} 1d_{5/2} 1d_{3/2} 2s_{1/2}$ , up to 4p4h

CD-Bonn + G-matrix

# Inversion-island nuclei: validity study



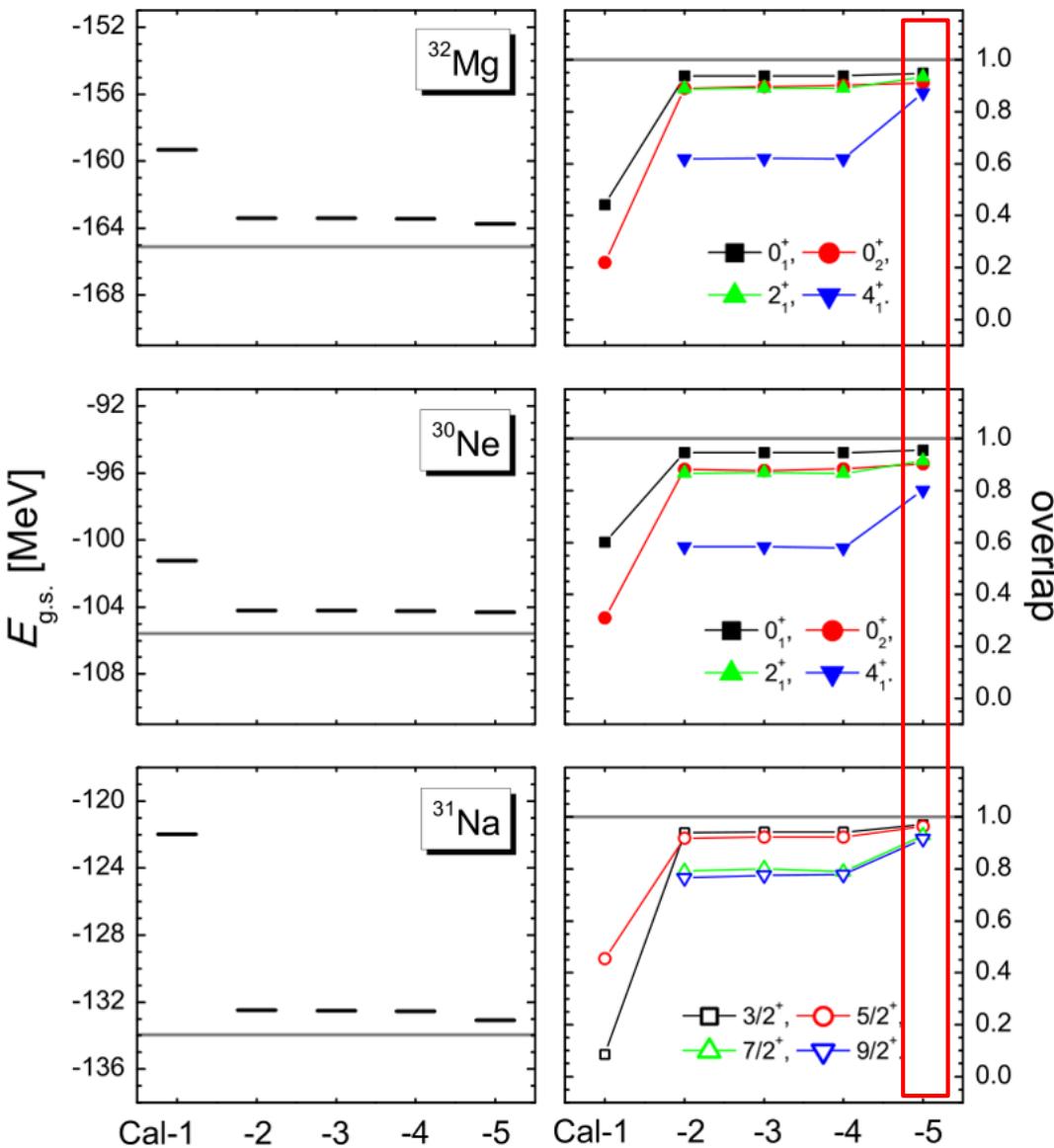
particle-hole excitations  
across **N=20** shell

sdpf-M interactions  
of Utsuno et al.

to fix structure coeff.

$$\frac{\delta}{\delta y_i} E_{g.s.} = 0$$

# Inversion-island nuclei: validity study



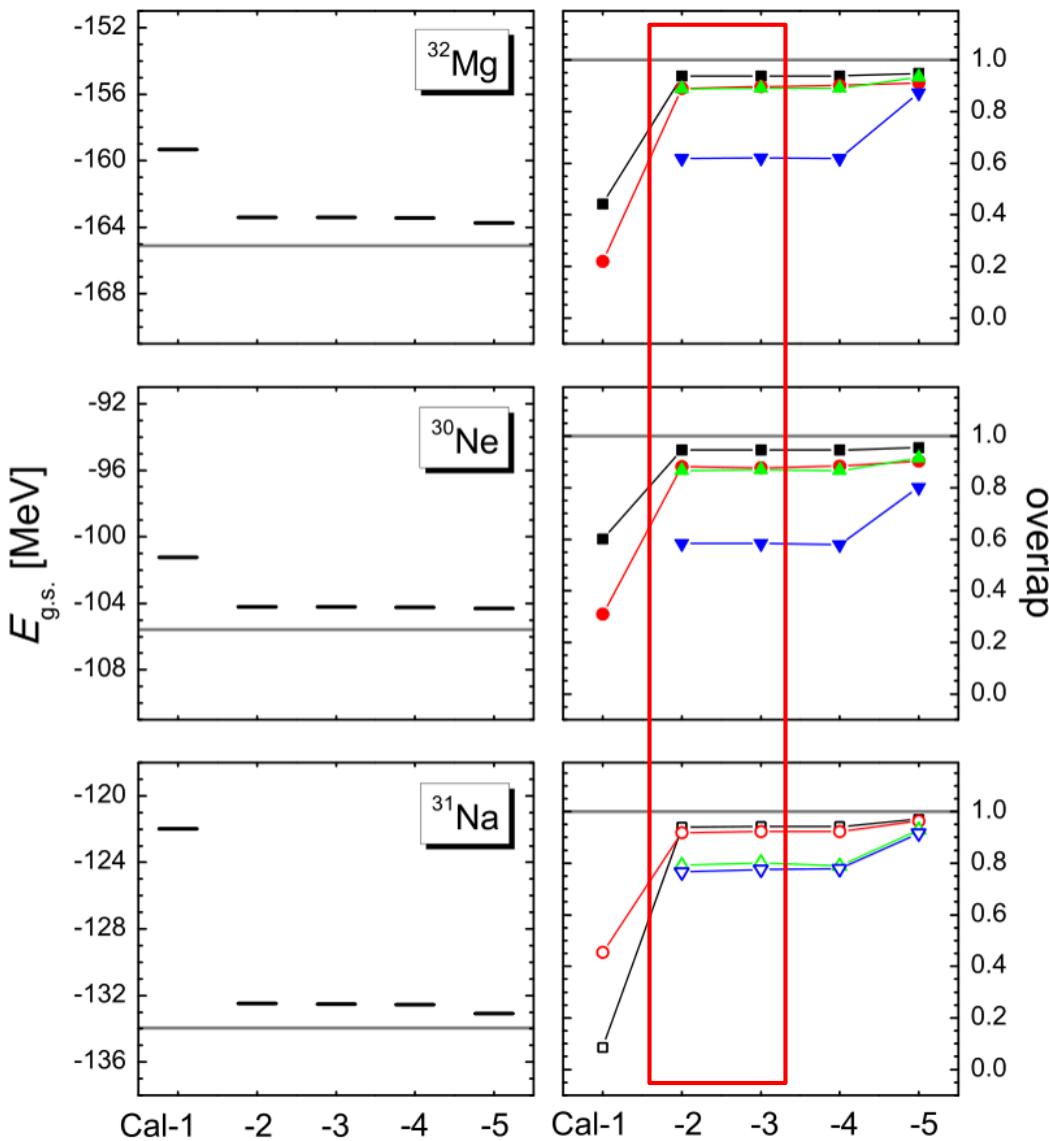
$0_1^+, 2_1^+, 4_1^+, 3/2_1^+, 5/2_1^+, 7/2_1^+, 9/2_1^+$

particle-hole config. dominant

$0_2^+$

shape-coexisting state;  
consistent with the result  
of two-neutron transfer  
reaction for Mg-32 (PRL  
105, 252501)

# Inversion-island nuclei: validity study



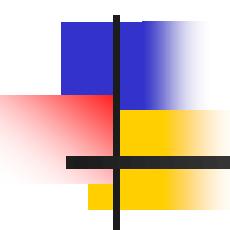
Cal-1: pp and hh  $S$  pairs

Cal-2: pp and hh  $SD$  pairs

Cal-3: Cal-2+ph  $F$  pair

Cal-4: Cal-2+ $D'$  pair

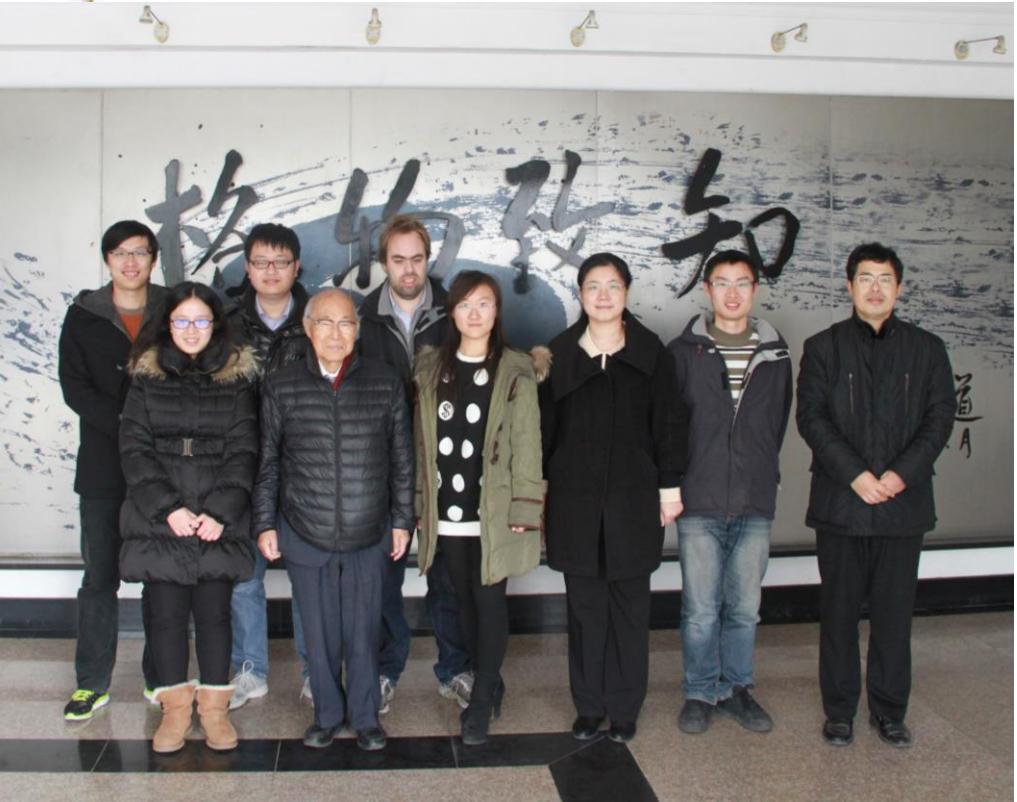
Cal-5: Cal-2+ $G$  pair



# Next plans

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- **Effects of particle-hole configurations and effective operators in descriptions of magnetic moments and E2-transition probabilities in heavy nuclei**
  
- **Descriptions for collectivity in transitional nuclei from perspective of a pair-truncated particle-hole configuration space**



MANY...MANY  
Gratitudes  
to Arima Sensei !

