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TONGJI UNIVERSITY

International Symposium on Simplicity, Symmetry and Beauty of Atomic Nuclei

26-28 September 2018, Shanghai

In honor of Professor Akito Arima's 88 year-old birthday (米寿)

Isovector and isoscalar pairing in low-lying states of *N* = *Z* nuclei

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Y. M. Zhao (*Shanghai Jiao Tong University*)
S. Pittel (*University of Delaware*)
Y. Y. Cheng (*East China Normal University*)
Y. Lei (*Southwest University of Science and Technology*)
J. J. Shen (*Shanghai Maritime University*)

- I used to be a PhD student (2010-2015) in Prof. Yu-Min Zhao's group in Shanghai Jiao Tong University . I had opportunities to collaborate with Prof. Akito Arima, is my great honor!

- The 1st time I met Prof. Arima: Honorary Professor of Shanghai Jiao Tong University in Nov. 2006.

- The 2nd time I met Prof. Arima: Summer School on Nuclear Physics (by Prof. Jie Meng) in Peking University, 2009.

- Prof. Arima visits Shanghai Jiao Tong University every year: lectures, academic presentations, discussions…

- Coauthor with Prof. Arima, 26 papers on Phys. Rev. C. My great honor!

Thank You and Happy Birthday Prof. Arima !

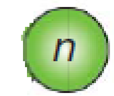
Nucleon pair: isospin singlet / triplet

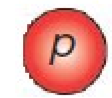
Nuclear force is charge independent approximately

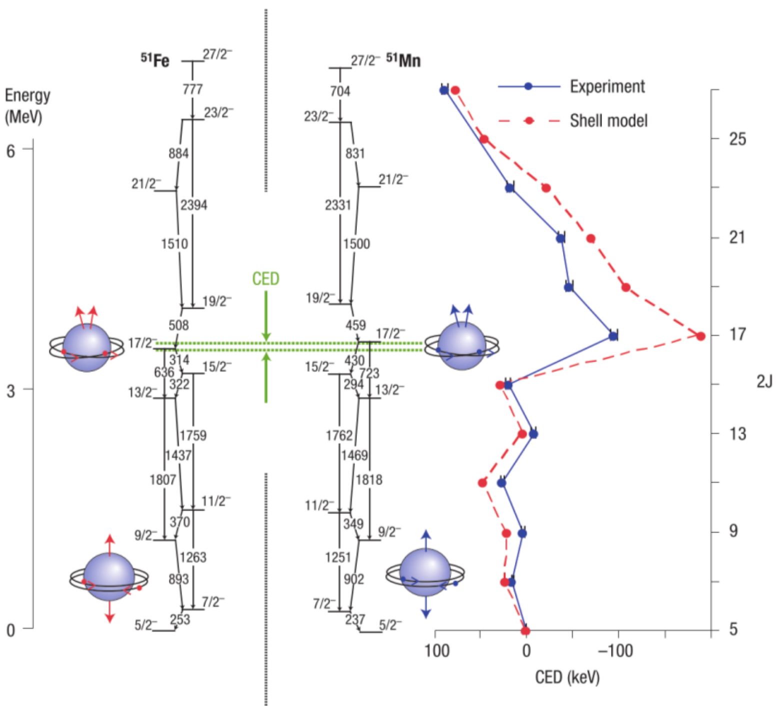
The Coulomb interaction doesn't play an important role in low-lying states

Isospin conservation

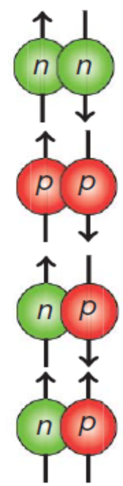
A proton and a neutron are the same kind of nucleons, with different *z*-components of isospin

$T = 1/2$, $T_z = +1/2$ for 

$= -1/2$ for 



singlet states and triplet states in isospin space



$T = 1, T_z = 1$

$T = 1, T_z = -1$

$T = 1, T_z = 0$

isospin triplet states
isovector

$T = 0, T_z = 0$

isospin singlet state
isoscalar

D. D. Warner, M. A. Bentley, and P. van Isacker, Nat. Phys. **2**, 311 (2006).

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- Purpose of the project
  Whether the isoscalar *pairing* is important or not?
  What is the definition of “pairing” or “pair correlation”?

- When considering nuclear pairing, many authors have used the terms as ‘‘correlation energy’’, ‘‘number of pairs’’, ‘‘order parameter’’, ‘‘pair field’’, ‘‘pairing gap’’, ‘‘pair transfer’’, ‘‘paired phase’’ but meant something different. This has lead to a certain degree of confusion, in particular with respect to the question: ‘‘Is there isoscalar np pairing?’’

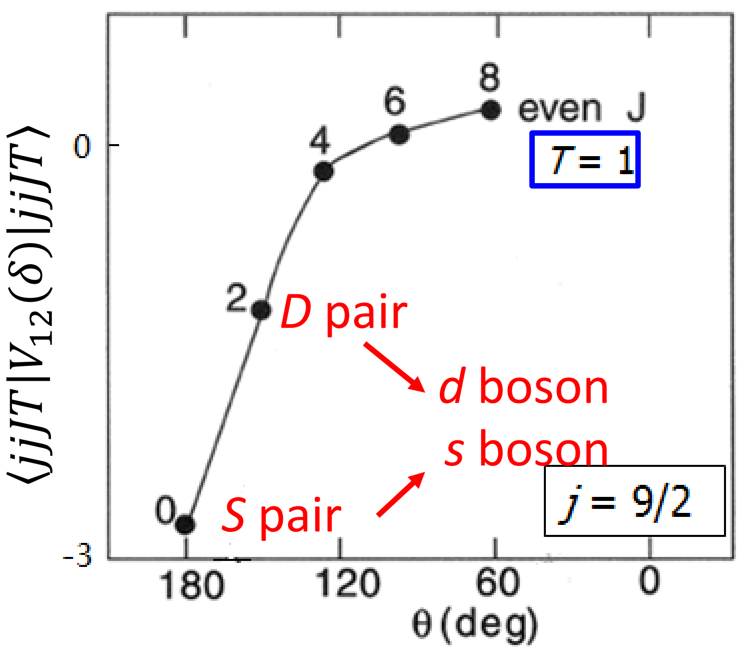
- Prof. Arima said
  Simple: pair approximation (wave function constructed by nucleon pairs)

S. Frauendorf and A. O. Macchiavelli, Prog. Part. Nucl. Phys. **78**, 24 (2014)

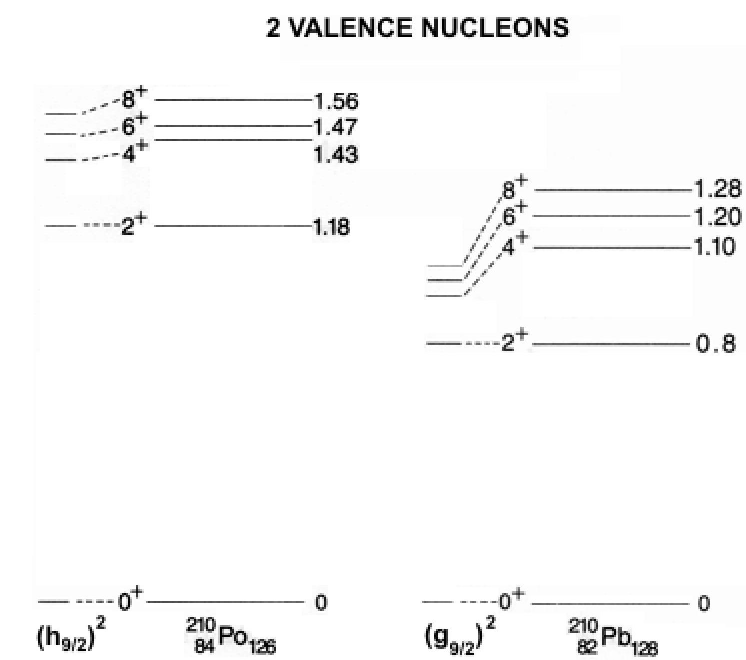
Interactions with $T = 1$

attractive and short ranged: δ interaction

$$V_{12}(\delta) = -V_0\,\delta(r_1 - r_2)$$
$$= \frac{-V_0}{r_1 - r_2}\,\delta(r_1 - r_2)\,\delta(\cos\theta_1 - \cos\theta_2)\,\delta(\Phi_1 - \Phi_2)$$



The very famous
Interacting Boson Model !



R. F. Casten, *Nuclear Structure from a Simple Perspective.*

Isovector pairing in semimagic nuclei

- Generalized seniority scheme:

collective (correlated) J = 0 pair

$$S^{\dagger} = \sum_j y_j S_j^{\dagger} \ , \quad S_j^{\dagger} = \left(a_j^{\dagger} \times a_j^{\dagger}\right)^{J=0}$$

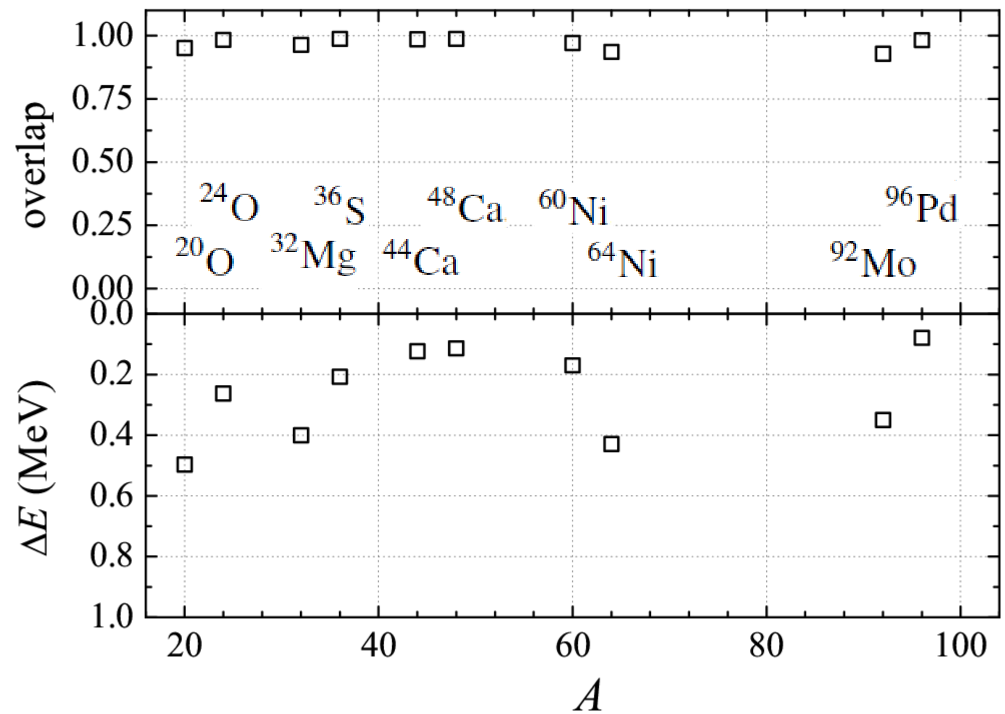
ground states of even-even semimagic nuclei: S pair wave function

$$|\varphi\rangle = \left[S^{\dagger} S^{\dagger} \cdots \qquad 0 \right\rangle$$

$|\mathrm{SM}\rangle$: shell-model wave function

overlap $\langle \varphi | \mathrm{SM} \rangle^2 > 0.92$

multi-j shell space + effective interaction



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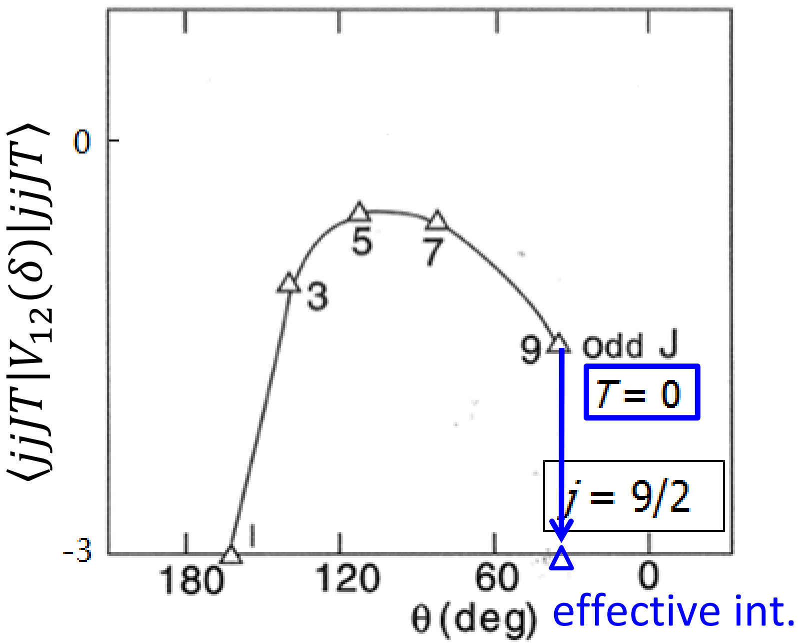
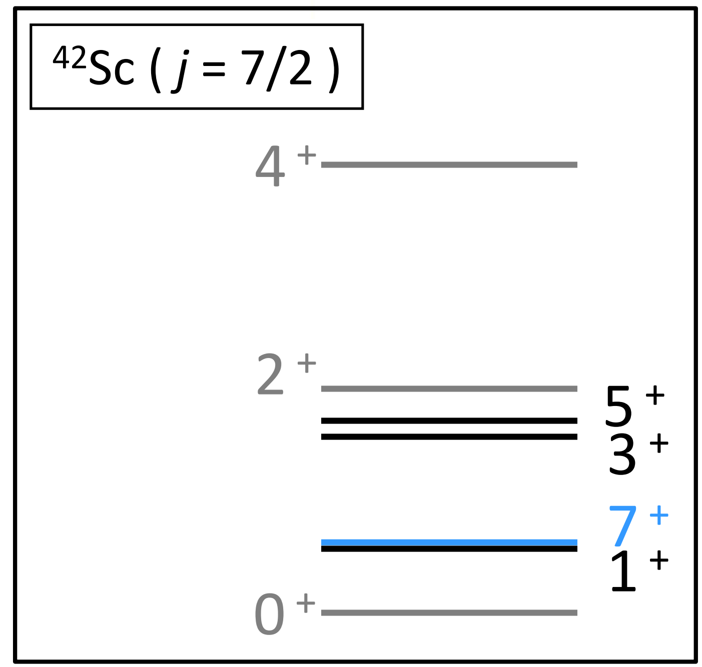
Interactions with T = 0

attractive and short ranged: δ interaction

$$V_{12}(\delta) = -V_0\,\delta(r_1 - r_2)$$
$$= \frac{-V_0}{r_1 - r_2}\,\delta(r_1 - r_2)\,\delta(\cos\theta_1 - \cos\theta_2)\,\delta(\Phi_1 - \Phi_2)$$

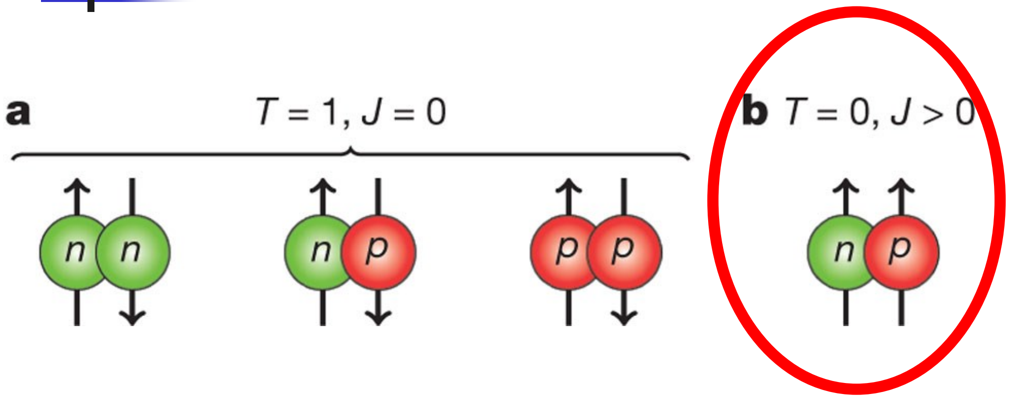




Whether or not can a pair correlated state or a pair condensation state be formed for N = Z nuclei?

R. F. Casten, *Nuclear Structure from a Simple Perspective.*

Isoscalar spin-aligned pair of the $g_{9/2}$ orbit



In $N = Z$ nuclei, isoscalar spin-aligned proton-neutron pairing might be very important（i.e., $J = J$max）

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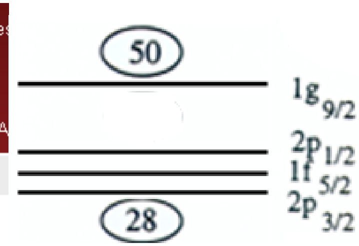
Evidence for a spin-aligned neutron–proton paired phase from the level structure of ^{92}Pd

B. Cederwall, F. Ghazi Moradi, T. Bäck, A. Johnson, J. Blomqvist, E. Clément, G. de France, R. Wadsworth, K. Andgren, K. Lagergren, A. Dijon, G. Jaworski, R. Liotta, C. Qi, B. M. Nyakó, J. Nyberg, M. Palacz, H. Al-Azri, A. Algora, G. de Angelis, A. Ataç, S. Bhattacharyya, T. Brock, J. R. Brown, P. Davies, A. Di Nitto, Zs. Dombrádi, A. Gadea, J. Gál, B. Hadinia, F. Johnston-Theasby, Joshi, K. Juhász, R. Julin, A. Jungclaus, G. Kalinka, S. O. Kara, A. Khaplanov, J. Kownacki, G. Rana, S. M. Lenzi, J. Molnár, R. Moro, D. R. Napoli, B. S. Nara Singh, A. Persson, F. Recchia, M. Sandzelius, J.-N. Scheurer, G. Sletten, D. Sohler, P.-A. Söderström, M. J. Taylor, J. Timár, J. J. Valiente-Dobón, E. Vardaci & S. Williams Show fewer authors

Affiliations | Contributions | Corresponding author

Nature 469, 68–71 (06 January 2011) | doi:10.1038/nature09644



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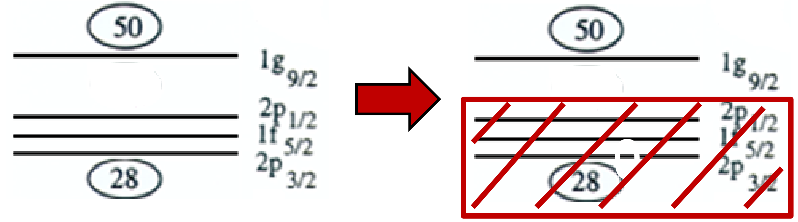
PRL 107, 172502 (2011) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2011

16^+ Spin-Gap Isomer in ^{96}Cd

B. S. Nara Singh,[1] Z. Liu,[2] R. Wadsworth,[1] H. Grawe,[3] T. S. Brock,[1] P. Boutachkov,[3] N. Braun,[4] A. Blazhev,[4] M. Górska,[3] S. Pietri,[3] D. Rudolph,[5] C. Domingo-Pardo,[3] S. J. Steer,[6] A. Ataç,[7] L. Bettermann,[4] L. Cáceres,[3] K. Eppinger,[8] T. Engert,[3] T. Faestermann,[8] F. Farinon,[3] F. Finke,[4] K. Geibel,[4] J. Gerl,[3] R. Gernhäuser,[8] N. Goel,[3] A. Gottardo,[2] J. Grębosz,[9] C. Hinke,[8] R. Hoischen,[3,5] G. Ilie,[4] H. Iwasaki,[4] J. Jolie,[4] A. Kaşkaş,[7] I. Kojouharov,[3] R. Krücken,[8] N. Kurz,[3] E. Merchán,[3] C. Nociforo,[3] J. Nyberg,[10] M. Pfützner,[11] A. Prochazka,[3] Zs. Podolyák,[6] P. H. Regan,[6] P. Reiter,[4] S. Rinta-Antila,[12] C. Scholl,[4] H. Schaffner,[3] P.-A. Söderström,[10] N. Warr,[4] H. Weick,[3] H.-J. Wollersheim,[3] P. J. Woods,[2] F. Nowacki,[13] and K. Sieja[13]

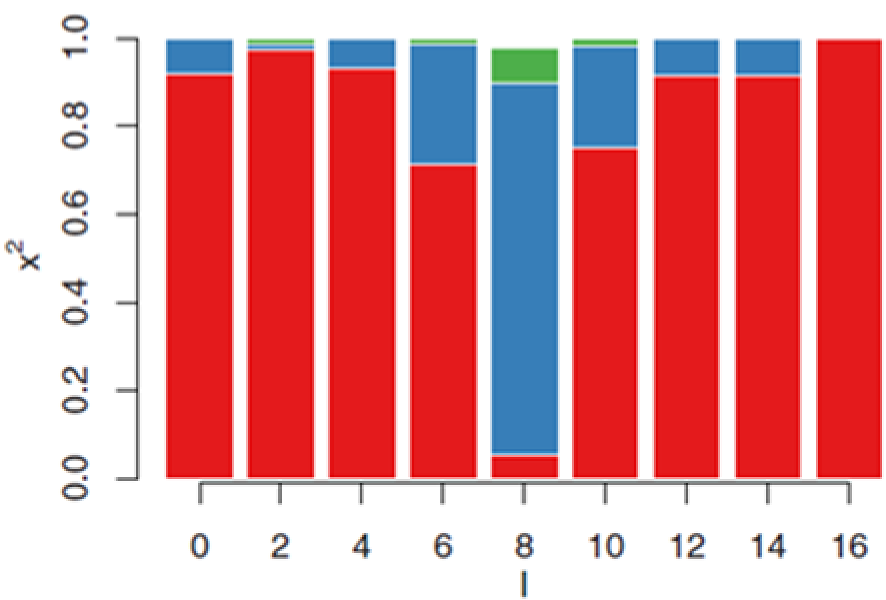
Isoscalar spin-aligned pair of the $g_{9/2}$ orbit

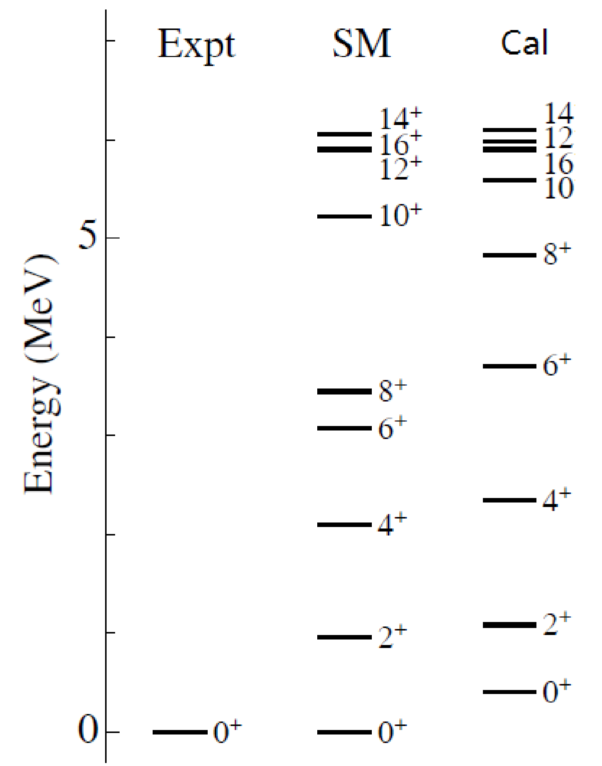
Single $g_{9/2}$ shell calculation for **^{96}Cd**,



the spin-aligned pair: $A^{(9,0)+} = \left(a_j^+ \times a_j^+\right)^{J=2j=9, T=0}$

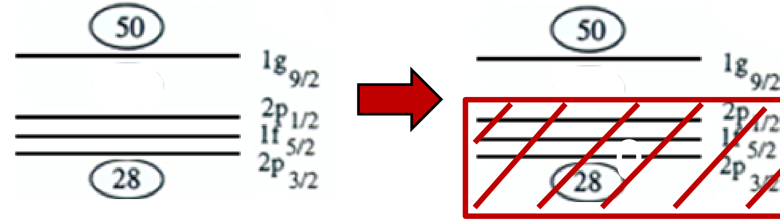
pair condensation: $|\varphi\rangle = (A^{(9,0)\dagger} \times A^{(9,0)\dagger})^{(I,0)}|0\rangle$





C. Qi, J. Blomqvist, T. Back, B. Cederwall, A. Johnson, R. J. Liotta, and R. Wyss, Phys. Scr. T 150, 014031.
P. van Isacker, Int. J. Mod. Phys. E 22, 1330028.

Isoscalar spin-aligned pair of the $g_{9/2}$ orbit

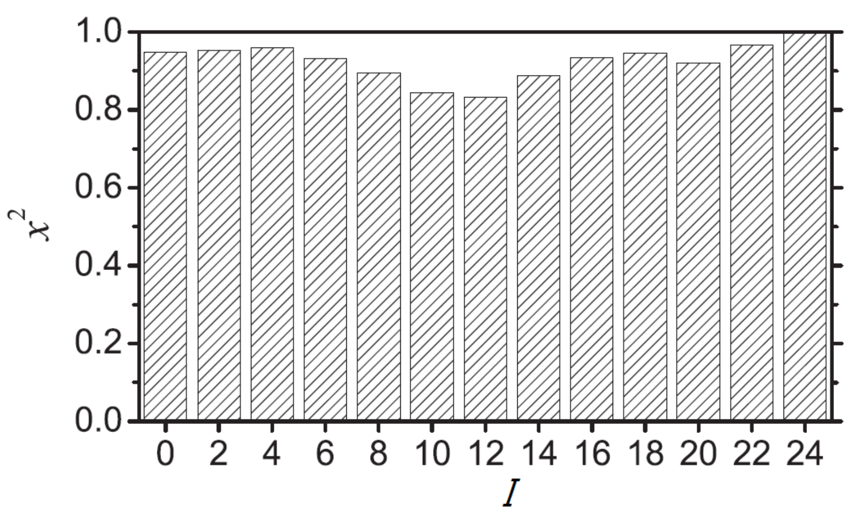


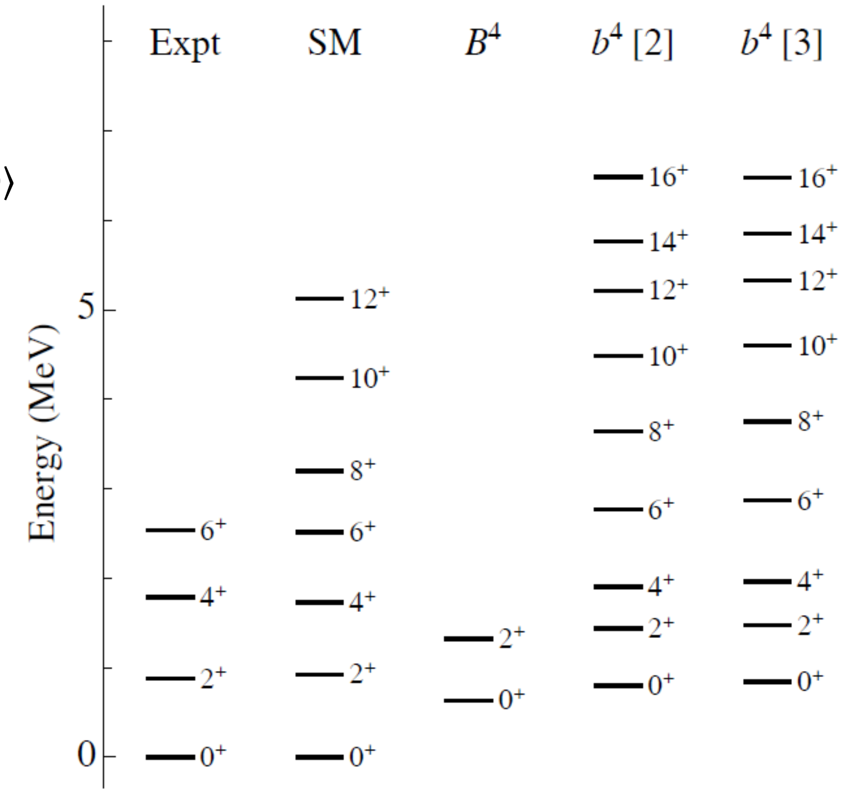
Single $g_{9/2}$ shell calculation for **^{92}Pd**,

the spin-aligned pair: $A^{(9,0)+} = \left(a_j^{+} \times a_j^{+}\right)^{J=2j=9, T=0}$

pair condensation basis:

$$|\varphi\rangle = (((A^{(9,0)\dagger} \times A^{(9,0)\dagger})^{(J_2,0)} \times A^{(9,0)\dagger})^{(J_3,0)} \times A^{(9,0)\dagger})^{(I,0)}|0\rangle$$





P. van Isacker, Int. J. Mod. Phys. E 22, 1330028.
G. J. Fu, J. J. Shen, Y. M. Zhao, and A. Arima, Phys. Rev. C 87, 044312.

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NPA: a pair-truncation model for multi-*j* shells

- Shell model, full correlated many-body solution

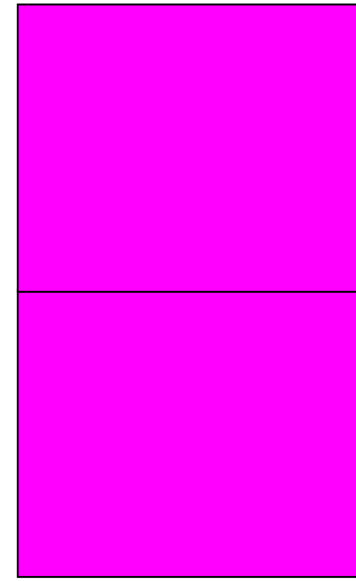
$$|\varphi\rangle = C^{+}_{j_1 m_1} C^{+}_{j_2 m_2} \cdots C^{+}_{j_n m_n} |0\rangle$$

- Nucleon-pair approximation (NPA) with isospin

$$A^{r\dagger} = \sum_{ab} y(abr) A^{r\dagger}(ab) \ , \quad A^{r\dagger}(ab) = \left(C_a^{+} \times C_b^{+}\right)^{r}$$

where r is short for J_r, T_r, and π_r. The pair wave function $|p\rangle$:

$$\boxed{|p\rangle = \left(\cdots \left(\left(A^{(r)+} A^{(r)+} \right)^{(J_2)} A^{(r)+} \right)^{(J_3)} \cdots A^{(r)+} \right)^{(J_N)}_{M_N} |0\rangle.}$$

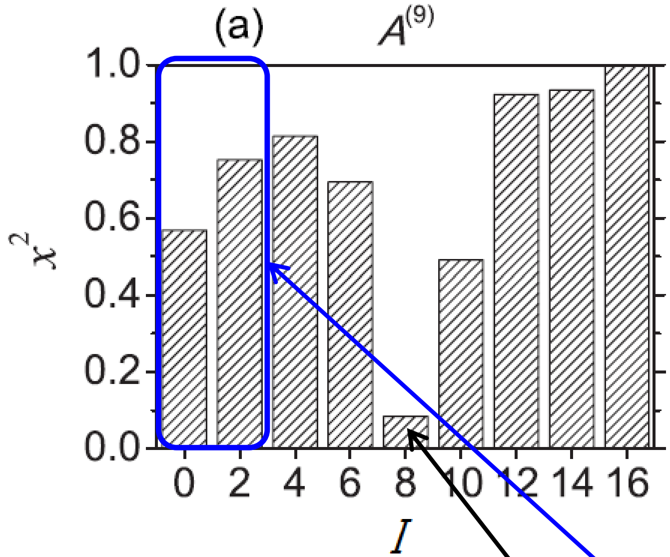


- Pair truncation: the building block is (*J*,*T*) pairs

Validity: overlap between pair (NPA) wave functions and SM wave functions
many-*j* shell calculation

G. J. Fu, Y. Lei, Y. M. Zhao, S. Pittel, A. Arima, Phys. Rev. C 87, 044310 (2013).
Y. M. Zhao and A. Arima, Phys. Rep. 545, 1 (2014)

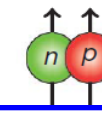
^{96}Cd: dual description

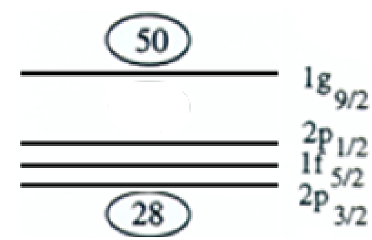


S **pair:** $J=0, T=1$

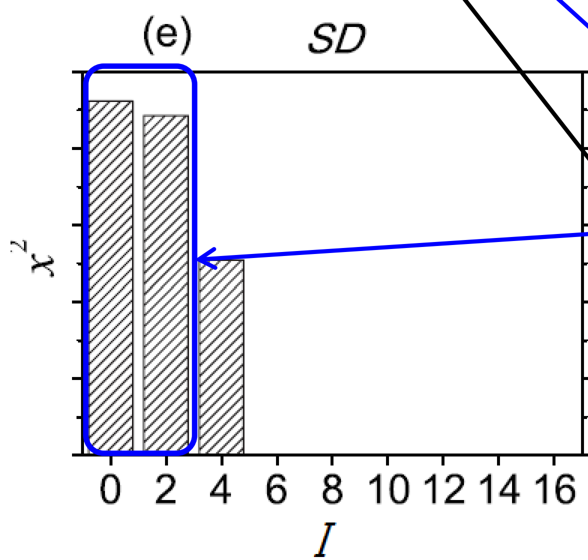
D **pair :** $J=2, T=1$

$A^{(9)}$ **pair :** $J=9, T=0$





+ JUN45



Dual description:
Non-orthogonality feature of nucleon-pair basis!

Seniority two state

G. J. Fu, J. J. Shen, Y. M. Zhao, and A. Arima, Phys. Rev. C 87, 044312 (2013).

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Pair wave function in g.s. of *N*=*Z* even-even nuclei

- Study $N = Z$ even-even nuclei: many-j shells + effective interaction

$$\left[\left(\left(A^{(J,T)+} \times A^{(J,T)+}\right)^{(J_2,T_2)} \times A^{(J,T)+}\right)^{(J_3,T_3)} \times \cdots \quad ^{+}\right]^{(J_{n/2},T_{n/2})} |0\rangle$$

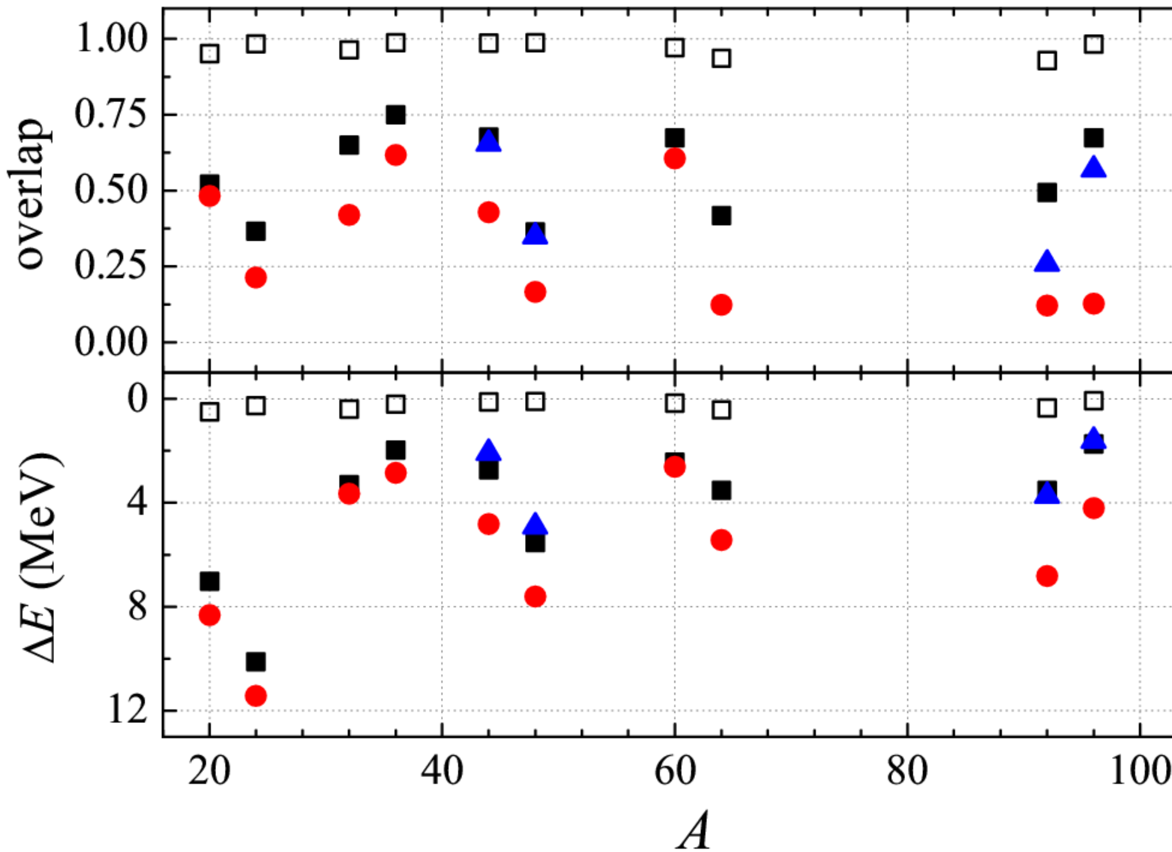
^{20}Ne, ^{24}Mg, ^{32}S, ^{36}Ar : USDB
^{44}Ti, ^{48}Cr : GXPF1
^{60}Zn, ^{64}Ge, ^{92}Pd, ^{96}Cd : JUN45

For $N = Z$ nuclei:

- ■ *S* pairs (J=0, T=1)
- ● *P* pairs (J=1, T=0)
- ▲ Spin-aligned pairs (J=2j, T=0)

For semimagic nuclei:

- □ *S* pairs (J=0, T=1) (> 0.92)



G. J. Fu, Y. M. Zhao, and A. Arima, Phys. Rev. C 91, 054322 (2015)

$T = 0$ states of $N = Z$ odd-odd nuclei

- Low-lying $T = 0$ states of 3 valence protons and 3 valence neutrons: many-j shells + effective interaction

^{22}Na, ^{34}Cl: USDB; ^{46}V: GXPF1; ^{62}Ga, ^{94}Ag: JUN45

- S broken pair: 2 isovector $J = 0$ pairs + 1 isoscalar pair

$$\left((A^{(0,1)\dagger} \times A^{(0,1)\dagger})^{(0,0)} \times A^{(\mathcal{J},0)}(j_1 j_2)^{\dagger}\right)^{(\mathcal{J},0)} |0\rangle$$

SD broken pair: 2 isovector $J = 0/2$ pairs + 1 isoscalar pair

$$\left((A^{(0,1)\dagger} \times A^{(0,1)\dagger})^{(0,0)} \times A^{(\mathcal{J},0)}(j_1 j_2)^{\dagger}\right)^{(\mathcal{J},0)} |0\rangle$$

$$\left((A^{(2,1)\dagger} \times A^{(2,1)\dagger})^{(0,0)} \times A^{(\mathcal{J},0)}(j_1 j_2)^{\dagger}\right)^{(\mathcal{J},0)} |0\rangle$$

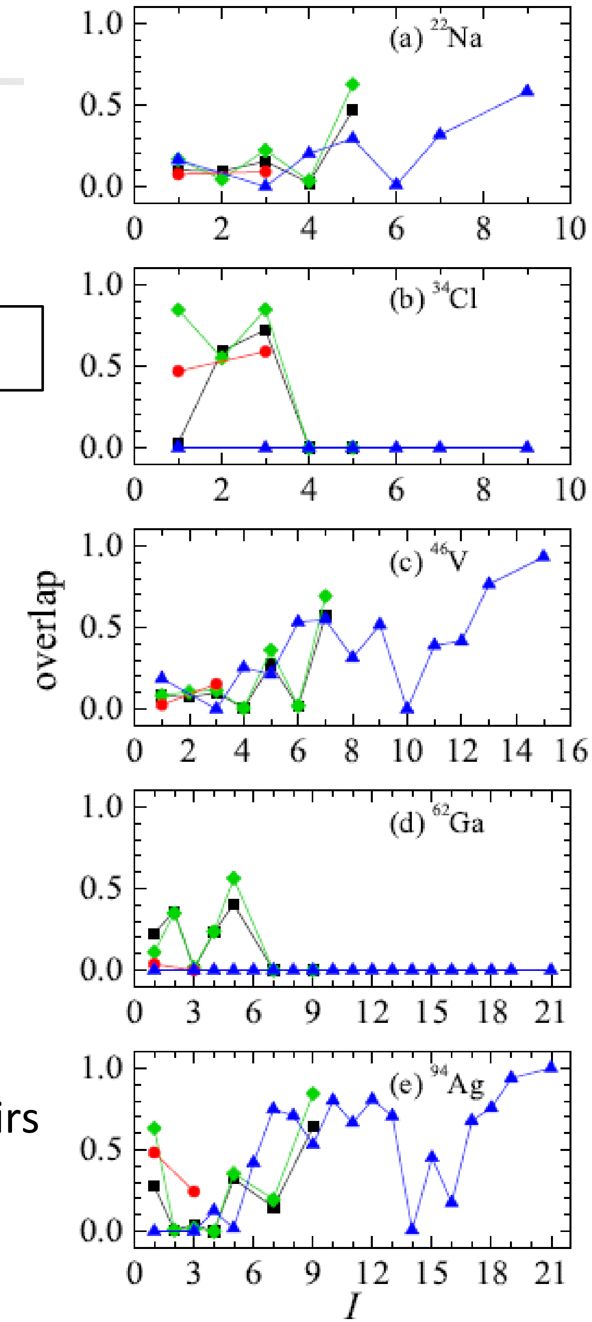
isoscalar spin-one pair condensation: 3 scalar $J = 1$ pairs

$$\left((A^{(1,0)\dagger} \times A^{(1,0)\dagger})^{(0,0)} \times A^{(1,0)\dagger}\right)^{(1,0)} |0\rangle$$

$$\left((A^{(1,0)\dagger} \times A^{(1,0)\dagger})^{(2,0)} \times A^{(1,0)\dagger}\right)^{(3,0)} |0\rangle$$

isoscalar spin-aligned pair condensation: 3 scalar $J = 2j_{\max}$ pairs

$$\left((A^{(2j_{\max},0)\dagger} \times A^{(2j_{\max},0)\dagger})^{(I',0)} \times A^{(2j_{\max},0)\dagger}\right)^{(I,0)} |0\rangle$$



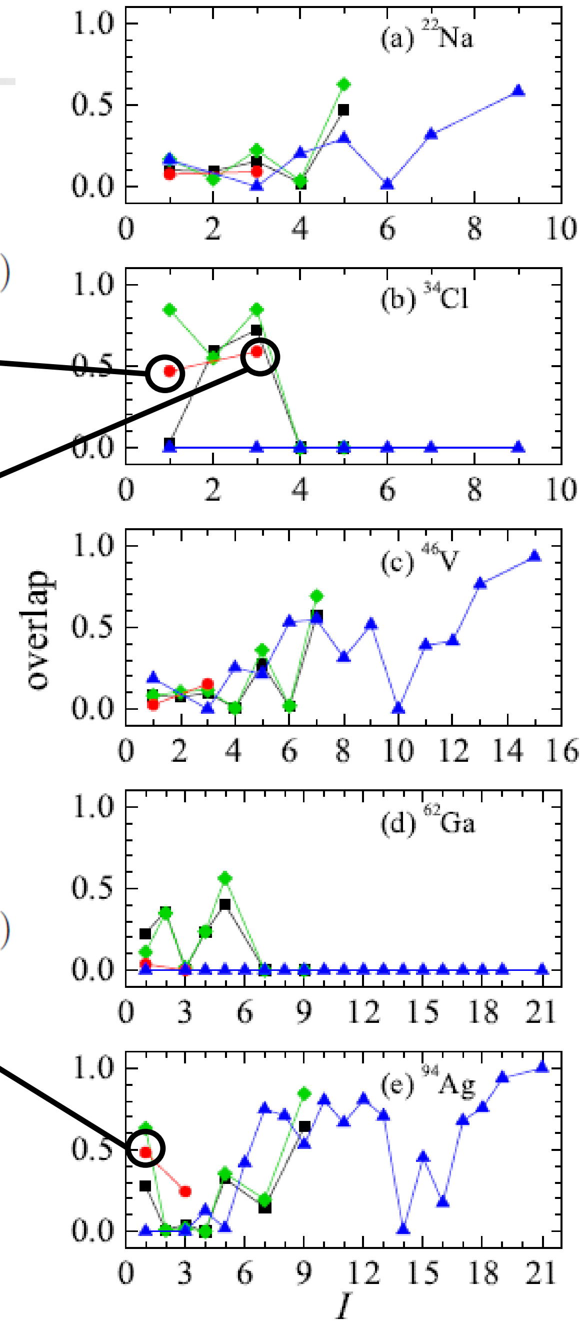
G. J. Fu, Y. M. Zhao, and A. Arima, Phys. Rev. C 97, 024337 (2018)

$T = 0$ states of $N = Z$ odd-odd nuclei

$$A^{(1,0)\dagger} \approx 0.92A^{(1,0)\dagger}(d_{3/2}d_{3/2}) - 0.35A^{(1,0)\dagger}(d_{3/2}s_{1/2}) - 0.12A^{(1,0)\dagger}(d_{3/2}d_{5/2})$$
$$\approx 0.76A^{(2,1,1,0)\dagger}(dd) + 0.35A^{(2,1,1,0)\dagger}(ds) + 0.51A^{(1,0,1,0)\dagger}(dd),$$
$$-0.17A^{(0,1,1,0)\dagger}(dd) + 0.10A^{(0,1,1,0)\dagger}(ss)$$

$$A^{(1,0)\dagger} \approx 0.99A^{(1,0)\dagger}(d_{3/2}d_{3/2}) + 0.06A^{(1,0)\dagger}(d_{3/2}s_{1/2})$$
$$\approx 0.75A^{(2,1,1,0)\dagger}(dd) + 0.58A^{(1,0,1,0)\dagger}(dd) - 0.22A^{(0,1,1,0)\dagger}(dd)$$
$$- 0.06A^{(2,1,1,0)\dagger}(ds)$$

$$A^{(1,0)\dagger} \approx 0.98A^{(1,0)\dagger}(g_{9/2}g_{9/2}) + 0.14A^{(1,0)\dagger}(p_{1/2}p_{3/2}) - 0.10A^{(1,0)\dagger}(p_{3/2}p_{3/2})$$
$$\approx 0.72A^{(1,0,1,0)\dagger}(gg) - 0.47A^{(2,1,1,0)\dagger}(gg) + 0.47A^{(0,1,1,0)\dagger}(gg)$$
$$- 0.18A^{(0,1,1,0)\dagger}(pp).$$



T = 0 states of N = Z odd-odd nuclei

$J = 3$, $T = 0$ pairing for the 9^+ state of ^{62}Ga, *overlap > 0.92*

$$
\begin{aligned}
A^{(3,0)^\dagger} &\approx 0.96A^{(3,0)^\dagger}(p_{3/2}p_{3/2}) + 0.25A^{(3,0)^\dagger}(p_{3/2}f_{5/2}) + 0.13A^{(3,0)^\dagger}(p_{1/2}f_{5/2}) \\
&\approx 0.96A^{(2,1,3,0)^\dagger}(pp) + 0.23A^{(4,1,3,0)^\dagger}(pf) + 0.14A^{(3,1,3,0)^\dagger}(pf)
\end{aligned}
$$

Future: propose a fingerprint for experiments?

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Summary

Isovector J=0 pairing is very important, especially, in low-lying states of semi-magic nuclei. The isoscalar pairing study began in 1960s or earlier. However, due to the complexity in both theories and experiments, no conclusive evidence was found.

- We calculate pair wave functions by using the nucleon-pair approximation of the shell model. It is shown that **the isoscalar spin-aligned pair truncation plays a role** for low-lying states of ^{22}Na, ^{44}Ti, ^{46}V, ^{94}Ag, and ^{96}Cd in multi-j shell calculation.
- **Dual description:** both isoscalar-pair approximation and isovector-pair approximation are reasonably good for low-lying states of N=Z even-even nuclei.
- Roughly speaking, **the isoscalar J=1-pair wave function is not so good**.
- **The isoscalar J=3-pair truncation for the 9^+ state of ^{62}Ga?**

Happy Birthday Prof. Arima !

Thank You !